Foundations of Physics Letters, Vol. 2, No. 3, 1989

QUANTUM MECHANICS AND RELATIVITY: ATTEMPT AT A NEW START

Mioara Mugur-Schächter

Laboratoire de Mécanique Quantique
et Structures de l'Information
Université de Reims, France

Received February 2, 1989

The approach which led Louis de Broglie to the assertion, for particles with nonzero rest mass, of the two correlated relations $p = h/\lambda$ and $W = mc^2 = h\nu$, is reexamined. A modified approach is then developed. This leads to a set of mutually coherent new relations with respect to which de Broglie's relations $p = h/\lambda$ and $W = mc^2 = h\nu$ appear as certain approximations. The mentioned set of new relations entails the prediction of specific effects which can be verified experimentally. If it is confirmed, this set of new relations might constitute the germ of a theory able to accomplish a veritable unification of relativity and microphysics.

Key words: quantum mechanics, relativity, de Broglie wavelength, corpuscular Doppler, unified theory, duality of energy.

1. INTRODUCTION

The whole structure of quantum mechanics is connected indissolubly with the acceptance of the two correlated relations $p = h/\lambda$ and $W = mc^2 = h\nu$. These relations have been introduced by Louis de Broglie, in his Thesis [1], on the basis—essentially—of considerations of relativistic invariance and variance. One might expect that such a genesis should insure a fundamental harmony between quantum mechanics and relativity. Nevertheless, for a reason that remained obscure but which is remarkably tenacious, quantum mechanics resists all the efforts to accomplish for it an entirely satisfactory coherence with relativity. In this work we identify a feature of quan-

tum mechanics—tied to a more general conceptual situation—which might be connected with this resistance, and we attempt a new start.

2. EXEGESIS

Einstein has postulated the relation $W = h\nu$ exclusively for the substance devoid of ponderable mass. In de Broglie's approach this relation is extended—as it stands—to particles of nonzero rest mass also. This extension is logically consistent with the assumption that, in the proper reference frame, the wavelike phenomenon associated by de Broglie with a heavy corpuscle takes on a non-progressive form. The structure of de Broglie's approach can be seen clearly in the following text of Ref. 2 (pp. 1–5), where de Broglie himself presents the essence of his Thesis (our translation):

> "Let us imagine a corpuscle that moves with uniform rectilinear motion along a certain direction, in the absence of any external field. We shall fix our attention exclusively on the state of movement of the corpuscle, making abstraction of its position in space. This movement will be performed along some given direction, which we choose as the z axis, and it will be defined by two quantities, the energy and the momentum, for which the relativistic expressions, as functions of the proper mass m_0 of the corpuscle, are given by the formulae
>
> $$W = \frac{m_0 c^2}{(1-\beta^2)^{1/2}}, \quad \mathbf{p} = \frac{m_0 \mathbf{v}}{(1-\beta^2)^{1/2}}, \quad \beta = \frac{v}{c}, \tag{1}$$
>
> from which the formula
>
> $$p = |\mathbf{p}| = \left(\frac{W}{c^2}\right)|\mathbf{v}| = \left(\frac{W}{c^2}\right)v \tag{2}$$
>
> is derived.
>
> In this way the state of movement is defined for a certain observer A tied to a Galilean reference frame, an observer who makes use of a time t and of the rectangular coordinates x, y, z.
>
> Consider now another observer B having, with respect to the first one, the velocity $\mathbf{v}$ with direction Oz, i.e., an observer traveling with the corpuscle. We can assume that B has chosen an axis O_0z_0 that glides along Oz and axes O_0x_0 and O_0y_0 that are, respectively, parallel to Ox and Oy. This being admitted, the coordinates x_0, y_0, z_0, t_0 of space and time of B are related to

the coordinates x, y, z, t of A by the well-known simple Lorentz transformation

$$x_0 = x, \quad y_0 = y, \quad z_0 = \frac{z - vt}{(1-\beta^2)^{1/2}}, \quad t_0 = \frac{t - (\beta/c)z}{(1-\beta^2)^{1/2}}. \qquad (3)$$

Now, for the observer B the velocity of the corpuscle is zero: So he assigns to the energy and the momentum the values

$$W_0 = m_0 c^2, \quad \mathbf{p}_0 = 0. \qquad (4)$$

According to our basic idea, we must now try to introduce a periodic element, and we shall try to define it first in the proper system of the corpuscle, that is, in the system of the observer B. Since in this system all is at rest, it is natural to define there the desired periodic element in the form of a stationary wave. Therefore we shall define the periodic element by the quantity, supposed to be a scalar,

$$\Psi_0 = a_0 e^{2\pi i \nu_0 t_0}, \qquad (5)$$

which has the form of the complex representation of a stationary wave; Ψ_0 oscillates as a function of the proper time with a frequency ν_0 characteristic of the nature of the envisaged corpuscle. We shall admit that a_0 is a constant (in general complex), so that Ψ_0 shall have at t_0 the same value at any point of the proper system of the observer B.

... What value is it convenient to assign to the proper frequency ν_0? Evidently we must try to define it starting from a quantity that characterizes the corpuscle in the proper system of B; but, in this system, only one non-null quantity is available, the energy $W_0 = m_0 c^2$. Given the role played by Planck's constant in all quantum problems, it is natural to postulate

$$\nu_0 = \frac{W_0}{h} = \frac{m_0 c^2}{h}, \qquad (6)$$

the analog of Einstein's relations for photons.

How will the periodic element defined above for the observer B manifest itself for the observer A? Supposing, which is natural here, that the element Ψ is an invariant, it will suffice, in order to obtain the expression for A, to substitute in the expression for B the value of t_0 provided by the fourth Lorentz equation (3), which entails

$$\Psi = a_0 e^{2\pi i \nu (t - z/V)} \qquad (7)$$

if one sets

$$\nu = \frac{\nu_0}{(1-\beta^2)^{1/2}} \quad \text{and} \quad V = \frac{c}{\beta} = \frac{c^2}{v}. \qquad (8)$$

Thus, for the observer A who sees the corpuscle passing with velocity $\mathbf{v}$ along Oz, the phases of the periodic phenomenon Ψ are distributed like those of a plane monochromatic wave for which the frequency ν and the phase velocity V would have the values (8).

... The comparison of the first relations (1) and (8) yields

$$W = h\nu, \qquad (11)$$

a relation which evidently must be valid in any Galilean reference frame, since the observer A is any Galilean observer."

So, in de Broglie's approach, the quantum relation in its general form (11) is a consequence, deduced—via the relativistic definitions of mechanical energy and momentum and the use of the Einstein-Lorentz transformation of time—from:

(a) the direct assumption (6) of the particular form assumed by the quantum relation for the particular observer B tied to the corpuscle and

(b) the postulation of the "stationary" form (5) for the "periodic element Ψ" associated with the proper mass m_0, as it is perceived by B, and of which the existence is posed to be an invariant.

Here enters a crucial remark:

The degree of reality assigned by de Broglie to the wave-like aspect of the "periodic element Ψ" associated with a piece of energy of proper mass m_0 is of the same order as the degree of reality of velocity or of a magnetic field: a characteristic entirely generated by the relative state of observation and which entirely vanishes in the proper frame of reference, while "intrinsically existing" aspects, like mass or spacetime position, subsist in the proper frame of reference, with proper values. The "periodic element Ψ" itself—its existence—*is* posited to be an invariant [this alone permits the vital transition from (5) to (6) and (7)], but its *wavelike* aspect is *not* conceived by de Broglie as being an intrinsic aspect. Louis de Broglie conveys in a very striking way the peculiar view he holds concerning this question (Ref. 2, pp. 3–4):

> "We can form a representation of the repartition of the values of Ψ_0 by imagining an infinity of small clocks disposed at all the points of the proper system of the corpuscle, mutually synchronized and possessing a period $T_0 = 1/\nu_0$. These small clocks

somehow figure in each point the "phase" of the periodic phenomenon, which is everywhere the same for the observer B at a same moment t_0 of his proper time ....

... In consequence of the relativistic phenomenon of the slowing down of moving clocks, each one of these clocks appears to the observer A as having a *diminished* frequency

$$\nu_H = \nu_0\sqrt{1-\beta^2}, \tag{9}$$

but the distribution of the *ensemble* of all the phases of all the clocks is given for A by the formula (7), that is, it coincides with the distribution of the phases of a plane monochromatic wave of which the frequency ν and the phase velocity V are given by (8).

By comparing the expressions (8) and (9), one can note the essential difference between the apparent frequency ν_H of an individual moving clock, which is *diminished* by the influence of motion, and the frequency ν of the associated wave, which is *increased* by this influence. This difference between the relativistic variations of the frequency of a clock and the frequency of a wave is essential: It strongly drew my attention, and it is by meditating on it that I became oriented in my researches.

What precedes can be summarized by saying that the corpuscle assimilated to one of the small clocks glides with respect to the phase of the wave with a velocity $V - v = c(1-\beta^2)/\beta$, so that it shall remain constantly in phase with the wave.

Let us reconsider this last idea in a more precise form. Among the infinity of small clocks imagined above, suppose that one of them plays a particular role. This will be the regulating clock which we shall identify with the corpuscle, while the other clocks represent the phase of the wavelike phenomenon for which the corpuscle is the center. In the proper system, all the clocks are immobile and have the same frequency ν_0. In the system of the observer who sees all the clocks passing by with velocity v, the ensemble of the phases of these clocks is given by the factor $\nu(t - z/V)$, defined as in (8). During a time dt, the regulating clock performs a displacement $v\,dt$ in the sense of Oz, and its indication undergoes a variation $\nu_0(1-\beta^2)^{1/2}dt$. The phase of the wave at the point where this clock is located undergoes a variation

$$\frac{\nu_0}{(1-\beta^2)^{1/2}}\left(dt - \frac{v\,dt}{V}\right).$$

Since these two changes must be equal, we must have

$$\sqrt{1-\beta^2} = \frac{1}{(1-\beta^2)^{1/2}}\left(1 - \frac{v}{V}\right), \qquad \text{where } \beta^2 = \frac{v}{V}, \tag{10}$$

in agreement with the second relation (8)."

And in the Thesis (Ref. 1, pp. 21 and 23) one reads:

"Are we obliged to suppose that the periodic phenomenon is localized inside the piece of energy? This is by no means necessary, and it will appear in III that it is no doubt displayed in a big portion of space. Moreover, what should be understood by the interior of a piece of energy? The electron is for us the type of an isolated piece of energy, that one which we believe, maybe erroneously, to know best; now, according to our received conceptions, the energy of the electron is displayed in all of space with a very strong condensation in a tiny region of which the properties, moreover, are rather poorly known to us. What characterizes the electron as an atom of energy is not the small place it occupies in space—I repeat that it fills it entirely—but the fact that it cannot be cut into parts, that it cannot be divided, that it forms a *unity* ....

... It is now indispensable to reflect on the nature of the wave of which we conceived the existence. The fact that its velocity $V - c/\beta$ is necessarily larger than c (β being always less than 1, since otherwise the mass would be infinite or imaginary), shows us that a wave transporting energy is out of question."

So de Broglie's famous "wave" was not conceived initially as an intrinsically existing *wave*! Much more abstractly, it has been conceived initially as an intrinsically existing—and infinitely extended—"periodic element" taking on the global *appearance* of a wave only in frames of reference distinct from the proper one, as a consequence of the influence produced on observation by the relative motion. This might be at the origin of the designation "phase wave" chosen by de Broglie.

Let us now finish the quotation:

"Defining as usually the wavelength by the formula $\lambda = V/\nu$, one finds the value

$$\lambda = \left(\frac{c^2}{v}\right)\left(\frac{h}{W}\right) = \frac{h}{p}. \tag{12}$$

We have thus found the two fundamental formulae (11) and (12) that define the frequency and the wavelength of the wave associated with the corpuscle, starting from its energy and momentum. For velocities that are small with respect to that of the light in vacuum, the formula (12) acquires the approximate form

$$\lambda = \frac{h}{mv}. \tag{13}$$

For a particle with velocity c (or undistinguishable from c), we have

$$v = V = c, \quad W = h\nu, \quad p = \frac{h\nu}{c}. \tag{14}$$

So one finds indeed that the fundamental formulae of the theory of quanta of light (Einstein, 1905) are valid for photons."

While the fact that (12) entails (13) is obvious indeed, the last proposition quoted above is misleading. It suggests that de Broglie's formulae (12) somehow would entail the formulae (14) as valid for photons, while in fact the radical $(1-\beta^2)^{1/2}$ becomes 0 when v becomes c, and when this happens, in order to avoid divergence, one has to set also $m_0 = 0$. So the expression $W = mc^2 = (m_0/(1-\beta^2)^{1/2})c^2$ becomes undetermined. Therefore, the second relation (14) cannot be derived from (11) written as $W = h\nu = mc^2$; it has to be postulated independently.

Mutatis mutandis, the same remark holds concerning the connectability of (12) with the third relation (14).

So, the "quantum relation" (11) introduced by de Broglie is neither confirmed nor invalidated by (14). Between the domain of validity of Einstein's relations (14) and the domain of validity of de Broglie's relations (11) and (12) there is, for the moment, a solution of continuity. The formal unity between the representation of the photonic domain (electromagnetism) and the representation of the mechanical domain (macroscopic mechanics and quantum mechanics) is not yet worked out; it is just supposed to exist. The graphic unity between Einstein's postulate and de Broglie's one does not create a conceptual unity: *So far, Einstein's postulate $W = h\nu$ and de Broglie's postulate $W = mc^2 = h\nu$ are two logically independent postulates.*

3. HYPOTHESIS

Our subsequent development is founded on the following remark:

The fact that in the proper frame of reference the relative velocity of the observer becomes zero does *not* entail that there "everything is at rest," nor does it suggest that "it is natural to define there the desired periodic element in the form of a stationary wave." Indeed, in the proper frame of reference de Broglie's clocklike process interior to the localized piece of energy m_0c^2 is *progressive* in time; it develops in time notwithstanding the fact that the mechanical velocity of the mass m_0 is perceived to be zero. So, a perturbation extended in space that would be *generated by this process* might appear inside the proper frame of reference as being generated *progressively*. In this case, this perturbation could appear as also *spreading progressively through space*, i.e., as taking on the form of a progressive wave. Anyhow, nothing whatsoever forbids one from envisaging that the "periodic element" associated with a mass m_0 might have a wave-like character inside the proper frame of reference, too. The theory of relativity permits this hypothesis at least as much as it permits de Broglie's hypothesis (5). However, it seems that this possible alternative hypothesis has never been explored.

On the other hand, de Broglie's corpuscular "wave"—which is not intrinsically a wave—is a very abstract concept. It is difficult to give it a physical interpretation. Inside the *proper* frame of reference, where no observational effect of relative motion can emerge, what could physically correspond to an extended *non*-wavelike periodicity of proper frequency ν_0, animating as a sole block the entire space surrounding the localized piece of energy m_0c^2, such that, at any given time t_0, the same phase would prevail everywhere, independently of the distance to the corpuscle, like an infinitely extended feeble pulsation? In a certain sense, such an infinitely extended non-progressive pulsation implies action at a distance, thereby even *contradicting* relativity: Indeed, if it is referred to the standard form

$$\Psi(x,t) = ae^{2\pi i\nu_w(t-x/V)}$$

of a progressive wave, de Broglie's form (5) can be obtained from it only by setting $V = \infty$. Passing now to a frame of reference that travels with respect to the corpuscle with a non null velocity v, what does insure the continuous harmony of phases between the phase of the clocklike frequency ν_H of the piece of energy mc^2 and the non-clocklike frequency ν of the surrounding distribution of phases, while the corpuscle seems to glide through this, apparent, distribution? Is that a physical, an energetic interaction taking place along the frontier between the corpuscle and the wave from the surrounding space? And if it is not an energetic interaction, what else could it be? How is the corpuscular wave *generated*?

In what follows we want to explore the hypothesis that the periodic phenomenon associated with a piece of localized energy mc^2 possesses a progressive character inside the proper frame of reference. We want to explore whether this assumption permits a more concrete, a more physical understanding of the periodic element associated with heavy energy.

Since the pair of relations $W = mc^2 = h\nu$ and $p = h/\lambda$ are logically consistent with the nonprogressive representation (5) in the proper frame of reference, we are prepared to be led to some modification of these relations, involving them as limits.

The consequences of the specified hypothesis, whatever they might be, cannot be rejected before they are known and examined.

4. THE STUDY OF A CORPUSCULAR WAVE WITH INTRINSICALLY PROGRESSIVE FORM

4.1. Representation in the Proper Frame of Reference

From now on, in order to distinguish clearly between corpuscular wave frequencies and clocklike frequencies, we shall index the first ones by a w (wave) and the second ones by a c (clock). The relative velocity of the reference frame is denoted by $v = u$. We consider only one spatial dimension, designated by x.

We admit the Einstein-Lorentz transformations, the definition $W_0 = m_0c^2$ of the energy *in the proper frame of reference*, and the general relation (2), $p = (W/c^2)u$.

The transformation law (1),

$$W = \frac{m_0c^2}{(1-\beta^2)^{1/2}} = \frac{W_0}{(1-\beta^2)^{1/2}},$$

which, according to the theory of relativity, characterizes the energy of a macroscopic heavy body, is *not* assumed *a priori* for microscopic heavy systems also, because this transformation law is connected with the quantum relation $W = h\nu$, which will *not* be assumed.

Consider now a corpuscle of proper mass m_0. Like de Broglie, we assume as a first hypothesis:

—(H_1) The proper mass m_0 involves a localized clocklike periodicity of proper frequency ν_{0c}.

Our second hypothesis is opposed to de Broglie's form (5):

—(H_2) In the proper frame of reference, the periodic phenomenon associated with the piece of energy m_0c^2 admits a representation by a wave function having the standard form of a *progressive* plane wave (in this first approach, the amplitude factor is supposed to be a constant, as in de Broglie's treatment). We thus write

$$\Psi_0(x_0, t_0) = a_0 e^{2\pi i \nu_{0w}(t_0 - x_0/V_0)}, \tag{5'}$$

where a_0 denotes the amplitude and ν_{0w} and V_0 are, respectively, a ave frequency and the corresponding proper value of the phase velocity, posited to be in general *finite*. However, as a particular case, it is permissible to envisage also the limiting situation $V_0 = \infty$, which then leads back to de Broglie's treatment.

Finally, we assume:

—(H_3) In the proper frame of reference the wave frequency ν_{0w} and the proper clock frequency ν_{0c} have the same numerical value

$$\nu_{0w} = \nu_{0c},$$

the proper mass m_0 being connected with this value accordingly to the equations

$$[\nu_{0c} = \nu_{0w}] = \frac{W_0}{h} = \frac{m_0c^2}{h}. \tag{6}$$

Though (6) extends the quantum relation $W = h\nu$, postulated as it stands by Einstein for photons, to a piece of heavy energy m_0c^2 as well, we do assume that the extension is valid for the proper frame of reference, because as de Broglie remarked, in this frame indeed "only one non-null quantity is available, the energy $W_0 = m_0c^2$."

4.2. Consequences

Consider now a reference frame that is fixed in the laboratory, i.e., one that has a non-null velocity $u_x = u = dx/dt$ with respect to the corpuscle. For the observer tied to this frame, the representation (5′) changes in accordance with the Einstein-Lorentz transformations

$$x_0 = \frac{x - ut}{(1-\beta^2)^{1/2}}, \quad t_0 = \frac{t - (\beta/c)x}{(1-\beta^2)^{1/2}}, \quad \beta = \frac{u}{c}. \tag{3}$$

Accordingly (5′) now becomes

$$\Psi(x,t) = a_0 e^{2\pi i \nu_{0w}[(t-(\beta/c)x)/(1-\beta^2)^{1/2} - (x-ut)/(V_0(1-\beta^2)^{1/2})]} \tag{15}$$

The coefficient of the time coordinate t in the phase of $\Psi(x,t)$ is

$$2\pi i \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right) = \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(\frac{u+V_0}{V_0}\right),$$

while the coefficient of x is

$$2\pi i \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(\frac{uV_0 + c^2}{c^2V_0}\right).$$

In the standard form $\Psi(x,t) = ae^{2\pi i\nu_w(t-x/V)}$, the phase velocity V is the inverse of the coefficient of x inside a bracket which admits the coefficient of t as a common factor. Thus it is convenient to rewrite the coefficient of x in the form

$$2\pi i \frac{\nu_{0w}}{V_0(1-\beta^2)^{1/2}}(u+V_0)\frac{uV_0+c^2}{c^2(u+V_0)} = 2\pi i \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right)\frac{1+uV_0/c^2}{u+V_0}.$$

It is now obvious that the expression (15) acquires in the new frame of reference also the standard form of a progressive wave $\Psi(x,t) = ae^{2\pi i\nu_w(t-x/V)}$ if, while for the amplitude it is assumed that $a = a_0$, one allows the wave frequency and the phase velocity to transform according to the relativistic transformation laws stated below.

4.2.1. Relativistic Transformation Law for the Wave Frequency

We postulate

$$\nu_w = \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(\frac{u+V_0}{V_0}\right) = \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right), \tag{16}$$

which is *different* from de Broglie's transformation law

$$\nu_w = \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}.$$

However, the limiting values are the same as in de Broglie's treatment: For $u = 0$, Eq. (16) yields

$$\nu_w(u=0) = \nu_{0w} = \nu_{0c}$$

[according to (H_3)], while, for $u = c$, it gives $\nu_w(u = c) = \infty$.

But the most interesting point to note is the following:

The expression (16) depends on both u and V_0, directly, not via their squares. Since u and V_0 can be positive or negative, while in the laboratory frame of reference $V_0 < 0$ when $u > 0$, *the expression (16) suggests a "corpuscular" Doppler effect.* And if, in particular, one sets $V_0 = c$, which is true for photons, (16) becomes

$$\nu_w = \frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{c}\right), \tag{16'}$$

which, in one dimension, is the well-known relativistic representation of the usual Doppler effect. So, in our approach, the representation of the usual relativistic Doppler effect is obtained on specializing the general relativistic variation of any wave frequency, to the particular case of photonic waves: *In our view, any "Doppler effect," photonic or corpuscular, is nothing else but the physical counterpart of the general relativistic transformation law (16) entailed by the Einstein-Lorentz transformations for any wave frequency.*

Let us note that this view could *not* have emerged from de Broglie's treatment, since there the relativistic transformation law of a *corpuscular* wave frequency is found to be $(\nu_{0w}/(1-\beta^2)^{1/2})$, like that of mass, in consequence of the form (5) assigned in the proper frame of reference to the periodic element associated with the piece of heavy energy m_0c^2; the factor $(1 + u/V_0)$ which creates a link to the usual Doppler effect, is missing.

4.2.2. Relativistic Transformation Law for the Phase Velocity

We require

$$V = \frac{(V_0+u)c^2}{uV_0+c^2} = \frac{V_0+u}{1+(V_0u)/c^2}, \tag{17}$$

which gives $V(u = 0) = V_0$, of course, while for $u = c$ we always have

$$V(u=c,V_0) = \frac{(V_0+c)c^2}{(V_0+c)c} = c$$

for any V_0, no matter whether $V_0 > c$ or $V_0 < c$.

Let us pause here and note immediately a new remarkable fact:

Formally, the expression (17) has the well known structure of Einstein's law of addition of velocities,

$$v = \frac{v_0+u}{1+(v_0u)/c^2}.$$

However, there exist fundamental differences with respect to Einstein's law, which are tied to the significance assigned in (17) to the symbols u and V_0 and to the assumptions made concerning them:

—In Einstein's law for the addition of velocities, all the three velocities involved, *viz.*, u, v_0, v, are "mechanical" velocities, qualifying displacements of a heavy piece of localized energy. As such, the absolute values of all these three velocities are postulated to be limited by the value c of the phase velocity of light. This is a condition for the reality of the quantity $(1-\beta^2)^{1/2}$. As for the phase velocity c of light, which, in contradistinction to the mechanical velocities, u, v_0, v qualifies a wavelike phenomenon, it is not present itself in Einstein's formula. And its numerical value c is *postulated—independently and basically*—to be an invariant.

—In the case of our form (17), only the symbol u designates a mechanical velocity, since it concerns a frame of reference, i.e., a heavy body. For this mechanical velocity the theory of relativity does require the upper limit c, it being present in the quantity $(1-\beta^2)^{1/2}$, which has to be a real number. As for the symbols V_0 and V, they indicate values of the velocity of propagation of the phase of a supposed corpuscular wave. And these phase velocities, as such, are present themselves in the formula (17). Now, concerning the numerical values of *phase* velocities in general, the theory of relativity stipulates nothing whatsoever about them, probably because their physical nature shields them *a priori* from any possible identification with the velocity of a heavy body. A semantic content similar to that from (17) appears in relativistic electromagnetism when Einstein's law of addition of velocities is applied to the velocity of light: $c = (c+u)/[1+(cu)/c^2] = c$. But in this particular case it is postulated—independently, and specifically for photons—that $V_0 = c$.

In short, when Einstein's law of composition of velocities is applied to the phase velocity of a corpuscular wave, a wave tied to heavy energy, the expression that is obtained has a semantic content of a type which, as far as we know, has not yet been considered, at least not in the context of a systematic examination. In this sense, the expression (17) can be regarded as a "semantic generalization" of Einstein's velocity composition law. This can be explicitly indicated by the following new notation

$$U = \frac{U_0 + u}{1 + (U_0 u)/c^2}, \tag{17'}$$

where u is a mechanical relative velocity of observation and U_0 is a velocity of *any* nature, either mechanical or a phase velocity, U being its Einstein-Lorentz transform.

If U_0 is a mechanical velocity, the general law (17′) becomes Einstein's law for the composition of mechanical velocities. If U_0 is a phase velocity and $U_0 = c$, then the general law (17′) is restricted to the case considered in electromagnetism. If U_0 is a phase velocity of a corpuscular wave, unrestricted numerically, the general law (17′) reduces to the law (17). In the last two cases, u is the velocity of the source emitting or producing the wave.

Since, according to the relation (17′), the particular proper value $U_0 = c$ is an invariant, inside the framework expressed by this relation the invariance of the value c of a phase velocity—of any nature, photonic or (eventually) corpuscular—acquires a deductive expression, instead of a postulated one. Of course, this is not at all surprising, since the Einstein-Lorentz transformations, which led to the formal structure (17′), are founded on the postulation (for the *particular* case of light) of this very invariance. Nevertheless, we are in the presence of a formal fact, which, in the more general context considered here, deserves attentive further meditation: In an approach aimed toward a unified representation of all the physical phenomena, whether mechanical, quantum mechanical, or electromagnetic, it might prove convenient to choose a new axiomatics, founded on the general law (17′), and yielding as a *consequence* the invariance of the particular value $U_0 = c$ (and hence, the Einstein-Lorentz transformations).

Let us now closer examine the special case when U_0 is a phase velocity $U_0 = V_0$. Then, as is well known, in consequence of the formal structure of Einstein's law for the addition of velocities, if $V_0 > c$, we have also $V > c$. And if, on the contrary, $V_0 < c$, we have also $V < c$; while if $V_0 = c$, then $V = c$. So, *if* the (absolute) proper value V_0 of the phase velocity of the corpuscular wave is bigger than c or at least equal to it, then the expression (17) entails—as in de Broglie's treatment—that in any other frame of reference the value V of the phase velocity of the corpuscular wave is equally bigger or at least equal to c. But if $V_0 \leq c$, which nothing *a priori* keeps one from envisaging, then the form (17) entails that—in contradiction to the consequence (10) of de Broglie's hypothesis (5)—in any frame of reference we have $V \leq c$. In that case, the corpuscular wave associated with a piece of energy mc^2 could be looked upon as a physical perturbation which transports energy and is able to *directly* transmit information.

If $V_0 > c$, the equation (17) admits a solution $V = \infty$, namely for the frame of reference with relative velocity $u = -c^2/V_0$. But here this solution emerges with a significance directly opposed to that from de Broglie's treatment: an observational appearence concerning an intrinsically wavelike phenomenon.

To summarize, according to the present approach, supraluminal phase velocities for the corpuscular waves are neither a formal necessity nor a formal impossibility: Only experimental facts could establish their existence or their nonexistence.

4.2.3. The Harmony of Phases

Let us continue. In the new view, the product uV has to be written in agreement with the transformation law (17). Then, instead of de Broglie's condition (8), $uV = c^2$, we find, from (17),

$$u\,\frac{V - c^2/V_0}{1 - V/V_0} = c^2 \tag{8'}$$

(which reduces to (8) for $V_0 = \infty$). In the limit $u = c$, which entails $V = c$, (8′) reproduces de Broglie's relation for any V_0. In the limit $u = 0$, which entails $V = V_0$, (8′) becomes $0/0$. Thus nothing prevents us from adopting also in this limit de Broglie's relation $uV = c^2$. But *in general* de Broglie's relation does not hold any more. Now, in de Broglie's treatment, the relation (8), $uV = c^2$, is the condition for the harmony of phases between the clocklike perodicity, with clock frequency ν_c localized "inside" the piece of energy mc^2, and the wavelike periodicity with wave frequency ν_w of the corpuscular wave that surrounds this piece of energy. De Broglie assigned utmost importance to this harmony of phases. Does it still subsist in our treatment, without the relation $uV = c^2$? For the variation during dt of the phase Φ_c of the clock frequency, we have

$$d\Phi_c = 2\pi\nu_c\,dt = 2\pi\nu_{0c}\left(\sqrt{1-\beta^2}\right)dt.$$

For the variation during dt of the phase Φ_w of the wavelike periodicity, we have, with $(dx/dt)dt = u\,dt$ and making use of (16) and (17),

$$d\Phi_w = 2\pi\nu_w\left[dt - \frac{(dx/dt)dt}{V}\right]$$
$$= 2\pi\left(\frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\right)dt\left[\left(1+\frac{u}{V_0}\right) - u\left(\frac{u}{c^2}+\frac{1}{V_0}\right)\right].$$

Since, according to (H3), we have $\nu_{0c} = \nu_{ow}$, the condition of permanent equality of the corresponding phases, $d\Phi_c = d\Phi_w$, takes the form

$$2\pi\nu_{0c}\left(\sqrt{1-\beta^2}\right)dt = 2\pi\left(\frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\right)dt\left[\left(1+\frac{u}{V_0}\right) - u\left(\frac{u}{c^2}+\frac{1}{V_0}\right)\right].$$

But this leads to the *identity* $1 - u^2/c^2 \equiv 1 - u^2/c^2$. So, while in de Broglie's treatment the harmony of phases requires the *condition* $uV = c^2$—unexplained, as if arbitrary—with our assumptions the harmony of phases is *unconditionally* assured: It appears as a direct consequence of the Einstein-Lorentz transformations.

This is a striking result. Associated with the preceding one concerning the composition of velocities, it seems to indicate that an intrinsically progressive form of the corpuscular wave is indeed in deeper agreement with relativity than de Broglie's assumption (5).

4.2.4. The Corpuscular Wavelength

Let us determine now the wavelength λ. We start from the standard definition $\lambda = V/\nu_w$. This yields

$$\lambda = \frac{V}{\nu_w} = \left[\frac{(u+V_0)c^2}{uV_0+c^2}\right]\left[\frac{((1-\beta^2)^{1/2})V_0}{\nu_{0w}(u+V_0)}\right]$$
$$= \lambda_0\left[\frac{(1-\beta^2)^{1/2}}{1+(uV_0/c^2)}\right], \tag{18}$$

where λ_0 is the proper value of the corpuscular wavelength. Since V_0 can be positive or negative (depending on whether the observer examines the phenomena along the positive or the negative part of the axis Ox), *the expression (18) reasserts the "corpuscular" Doppler effect already asserted before by the expression (16).* This effect, if it exists, could be significant in cosmology.

4.2.5. The Quantum Relation

What does the quantum relation become in the present approach? This, obviously, is a crucial question. The examination of this question brings forth a quite unexpected situation.

As we have seen, in de Broglie's treatment, the transformation law

$$W = \frac{W_0}{(1-\beta^2)^{1/2}} = \frac{m_0c^2}{(1-\beta^2)^{1/2}}$$

admitted in the theory of relativity for the energy of a macroscopic heavy body, is assumed to be valid also for microscopic heavy bodies. This assumption, associated with the form (5) assigned in the proper frame of reference to the extended periodic element connected with a microsystem, entails for the frequency of a corpuscular wave the transformation law $\nu_w = \nu_{0w}/(1-\beta^2)^{1/2}$, which is of the same form as the transformation law admitted for the mass. Hence, if one does assume the quantum relation $W_0 = h\nu_{0w}$ in the proper frame of reference, then the general quantum relation $W = h\nu_w$ holds invariantly in any frame of reference, for a de Broglie microsystem.

But this quantum relation $W = h\nu_w = h\nu_{0w}/(1-\beta^2)^{1/2}$ does not *hold also for a photonic wave: The unification with relativistic electromagnetism, suggested by the writing $W = h\nu_w$, is illusory.*

Indeed, the transformation law admitted in relativistic electromagnetism for a photonic wave frequency is the one usually designated as the "Doppler effect," compatible with the proper form (5′); that is, the law which, in one spatial dimension, has the same form as the law (16)

$$\nu_w = \frac{\nu_{ow}}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right),$$

derived in our approach for a corpuscular wave. But this law is of another form than the transformation law admitted for mass.

This remark draws attention to a fact which—quite independently now of the foundations of quantum mechanics—concerns directly macroscopic relativistic mechanics and relativistic electromagnetism: *If one assumes the quantum relation $W = h\nu_w$ for the photonic domain, then one admits* ipso facto *that the photonic energy transforms according to a law which in one spatial dimension reduces to the form*

$$\text{const.}\ \frac{1}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right),$$

while the mechanical energy of a macroscopic heavy body is assumed to transform according to a law of a different form, namely,

$$\text{const.}\ \frac{1}{(1-\beta^2)^{1/2}}.$$

Was Einstein aware of this scandalous duality? *Is it true*?

Anyway, it seems probable that de Broglie was *not* aware of the dichotomy and that he felt compelled to accept, in the proper frame of reference, the strange and abstract form (5) for his corpuscular "wave," precisely in order to ensure unity with the transformation law posed for the energy in macroscopic relativistic dynamics, which, he believed, extended to photons via Einstein's quantum relation $W = h\nu$.

Consider now exclusively heavy systems: Should we admit that the energy of a heavy macroscopic system transforms differently from the energy of a microscopic heavy system? *A heavy macroscopic system also involves corpuscular waves.* Though quantitatively the observable effects of these might be negligible in the case of a macroscopic mass, in *principle* they do exist: Hence they should possess some expression within a rigorous representation of the behavior of macroscopic heavy bodies, in a theory claiming uniform validity for systems of any size, as the theory of relativity does.

On the other hand, as long as a radically dichotomic situation exists concerning heavy energy and non-heavy energy, what should be assumed concerning the transformation law for the energy within the quantum mechanical domain, where the observable effects of the mechanical features and those of the wavelike features have the same order of magnitude? De Broglie's treatment, in spite of the assertion $W = h\nu_w$ which suggests unity with electromagnetism, involves the choice of an assumption that the energy associated with a microscopic heavy system transforms in the way that is assumed for the energy of a heavy macroscopic body, not in the way assumed for photonic energy. But why should precisely this choice be made, rather than the other one?

In what follows, we shall neutrally outline two possible hypotheses concerning this conceptual situation (which probably do not exhaust all conceivable possibilities). Each one of these two hypotheses—when combined with a progressive proper form (5′) of the corpuscular wave—entails specific consequences for the re-expression, in corpuscular terms, of the corpuscular wavelength and of the wave function.

Hypothesis A. Let us first continue to assume, like de Broglie, that the energy transformation law (1), $W = m_0c^2/(1-\beta^2)^{1/2} = mc^2$, admitted in macroscopic relativistic mechanics, is *rigorously* valid there and, moreover, that this same law is equally valid in the domain of quantum mechanics. If this hypothesis is now combined with the proper form (5′) for the corpuscular wave, instead of with de Broglie's proper form (5), then, contrary to what happens in de Broglie's treatment, it follows that even though in the proper reference frame we do

assume the quantum relation $W_0 = h\nu_{0w} = m_0c^2$ (hypothesis (H_3)), this relation does *not* hold invariantly, since the form (5′) entails for the frequency of a corpuscular wave the transformation law (16), different from (1). Thus we have now to distinguish between "corpuscular energy," let us denote it by W_c, and "photonic energy," which we denote by W_ℓ (ℓ = light). Then the hypothesis A entails that the energy and the wave frequency ν_w of a microscopic heavy system are connected with one another through a "modified quantum relation," different from the photonic quantum relation $W_\ell = h\nu$, namely

$$W_c = mc^2 = \frac{m_0c^2}{(1-\beta^2)^{1/2}} = \frac{h\nu_{ow}}{(1-\beta^2)^{1/2}} = \frac{h\nu_w}{(1+u/V_0)}. \qquad (11')$$

But it is most important to realize very clearly the fact stressed by the intermediary writing from (11′): *Numerically* the relation (11′) asserts the *same* thing as de Broglie's quantum relation, namely, that the following equality does hold:

$$\left[W_c = \frac{m_0c^2}{(1-\beta^2)^{1/2}}\right] = \frac{h\nu_{0w}}{(1-\beta^2)^{1/2}}.$$

Only the formal expression of the connection between energy and wave frequency changes, in consequence of the fact that, according to hypothesis A,

$$\frac{\nu_{0w}}{(1-\beta^2)^{1/2}} \neq \nu_w,$$

$$\frac{\nu_{0w}}{(1-\beta^2)^{1/2}} = \frac{\nu_w}{(1+u/V_0)},$$

so that with the form (1) for the energy, one has

(microscopic heavy energy) $\neq h \times$ corpuscular wave frequency).

Hypothesis B. At the time when the macroscopic relativistic mechanics was constructed, quantum mechanics did not exist. Accordingly, Einstein *did not refer* his arguments and his axiomatization to parameters qualifying a corpuscular wave associated with a given mass. But nowadays we know that such a wave does exist for any elementary mass, while macroscopic masses are composed of microscopic ones. So, nowadays, as we have stressed, *in a macroscopic relativistic "mechanics," yielding a description which is in* principle *rigorous and complete, there should exist, essentially built into it, at least a* reference *also to the corpuscular wave associated with each microscopic mass.*

Furthermore, Einstein's relation (1) $W = m_0c^2/(1-\beta^2)^{1/2} = mc^2$, asserted for macroscopic heavy bodies, becomes 0/0 when $\beta = u/c = 1$, hence $m_0 = 0$, which is "the photonic limit." Thus Einstein was free to set for photons—quite independently in fact—his relation $W = h\nu_w$. The two theories obtained with these two definitions for the energy, macroscopic relativistic mechanics and relativistic electromagnetism, are founded on one same spacetime topology and kinematics, derived from the invariance of the value c of the velocity of light. But, since the two quantities $h\nu_w$ and mc^2 have different Einstein-Lorentz transformations laws, the unification of the corresponding *dynamics* is—in the principle—*not* achieved: *The problem that has been raised by the Michelson-Morley experiment, still exists to a certain degree: It has migrated to the frontier between the microscopic and the macroscopic level of conceptualization.*

This problem is rooted in the definition of the fundamental transformation law for the mass. Now, when one examines the derivation of this transformation law [see, for example, Bohm in *The Special Theory of Relativity* (Ref. 3, pp. 81–90)], one finds that at least this particular derivation *continues to hold* if, instead of $m/m_0 = 1/(1-\beta^2)^{1/2}$, one sets

$$\frac{m}{m_0} = \frac{1}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right). \tag{19}$$

[In the above quoted derivation this is so because the relation (18–25), p. 86, does not depend explicitly either on the relative velocity of the two frames of reference under consideration (with Bohm's notations, $u = V'$), or on our V_0 (the proper value of the phase velocity of the corpuscular wave associated with the mass m_0), because this last parameter is not even envisaged. So, one can add in the second member of Bohm's relation (18–26), p. 87, *any* function $f(u, V_0)$, without affecting the subsequent results. It suffices then to add $\ln(1 + u/V_0)$ in order to obtain the law (19).]

In this conceptual situation it seems both possible and natural to envisage the following hypothesis:

Numerically, the transformation law for energy assumed in macroscopic relativistic mechanics, $W = m_0c^2/(1-\beta^2)^{1/2} = mc^2$, is—*usually and with a very high accuracy*—valid *there*, possibly as a mean following from destructive composition of the contributions from the waves produced by the different constituent microscopic masses. But, in contradistinction to the hypothesis A, we shall now admit that *fundamentally*, for microscopic heavy systems, in the domain of quantum mechanics where "one" or several "coherent" elementary masses are considered, *the rigorously correct transformation law for mass is (19).*

Then one has now to distinguish between: (1) the mass and the energy of a macroscopic heavy body, *considered as approximations*—let us denote them, respectively, by

$$m_M = \frac{m_0c^2}{(1-\beta^2)^{1/2}} \quad \text{and} \quad W_M = m_Mc^2$$

—and (2) the mass and energy of a microscopic heavy body, which we denote, respectively, by

$$m_\mu = \frac{m_0c^2}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right) \quad \text{and} \quad W_\mu = m_\mu c^2.$$

So, according to this hypothesis, *the heavy microscopic energy transforms according to the law assumed in relativistic electromagnetism for the energy of a photon, not according to the law admitted in relativistic mechanics for the energy of a macroscopic heavy body.*

Then W_μ obeys to the following form of the quantum relation:

$$\begin{aligned} W_\mu = h\nu_w &= \frac{h\nu_{0w}}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right) \\ &= \frac{m_0c^2}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right) \\ &= m_\mu c^2. \end{aligned} \tag{11''}$$

The first equality from (11″) has the same appearance as the formula introduced by de Broglie, but it has a different content, the one explicitly expressed by the last equalities of (11″).

Since we have supposed that the relation $p = (W/c^2)u$ holds in any case, for the momentum of a microscopic heavy system the present assumptions entail the modified transformation law

$$p_\mu = \left[\frac{m_0}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right)\right]u = m_\mu u = \left(\frac{W_\mu}{c^2}\right)u. \tag{20}$$

It can be *a priori* conceived that in certain limiting cases (for instance, in certain coherence phenomena that transpose to the macroscopic, directly observable level, the fundamental microscopic behaviors, as happens in superconductors) the rigorous transformation laws (19), (11″), and (20) might hold also for a macroscopic heavy system.

The hypothesis B seems much more probable than the hypothesis A.

4.2.6. The Corpuscular Wavelength in Corpuscular Terms

In (18) the corpuscular wavelength is expressed exclusively in terms of wave quantities. In order to now relate this expression also with corpuscular properties, we must make use of the definitions connecting the frequency of the corpuscular wave with the corpuscular energy and momentum. These connections depend upon the hypothesis adopted for the quantum relation.

Hypothesis A. According to hypothesis A, we have for any heavy body, macroscopic or microscopic,

$$W_c = \frac{m_0c^2}{(1-\beta^2)^{1/2}} = mc^2 = \frac{W_{0c}}{(1-\beta^2)^{1/2}},$$

$$p_c = \left(\frac{W_c}{c^2}\right)u = \left(\frac{m_0}{(1-\beta^2)^{1/2}}\right)u = mu.$$

Thus the quantum relation for a heavy (microscopic) body expresses (11′) as

$$W_c = mc^2 = \frac{m_0c^2}{(1-\beta^2)^{1/2}} = \frac{h\nu_{0w}}{(1-\beta^2)^{1/2}} = \frac{h\nu_w}{(1+u/V_0)},$$

which is different from the quantum relation $W_\ell = h\nu_w$ for photons. This yields successively

$$\lambda = \frac{hc^2V_0}{\left[m_0c^2/(1-\beta^2)^{1/2}\right]uV_0 + \left[m_0c^2/(1-\beta^2)^{1/2}\right]c^2}$$

$$= \frac{hc^2V_0}{p_cc^2V_0 + W_cc^2} = \frac{h}{p_c + (W_c/V_0)}.$$

So, with the hypothesis A, instead of de Broglie's relation (12) $p = h/\lambda$, we now have the modified form

$$\lambda = \frac{h}{p_c + (W_c/V_0)} = \left(\frac{h}{W_c}\right)\left(\frac{Vc^2}{uV + c^2}\right). \qquad (12')$$

For $u = c$, the energy W_c and the corpuscular momentum become infinite, so that $\lambda(u = c) = 0$, as in de Broglie's treatment. But for $u = 0$, we now find a *finite* maximal value of the corpuscular wavelength, namely,

$$\lambda(u=0) = \lambda_0 = \frac{V_0}{\nu_{0w}} = h\left(\frac{V_0}{m_0c^2}\right) = h\left(\frac{V_0}{W_{0c}}\right) < \infty, \qquad (12'')$$

while de Broglie's treatment yields $\lambda(u = 0) = \lambda_0 = \infty$. The difference $\lambda - \lambda_B$ (where $\lambda_B = h/p$ denotes de Broglie's wavelength) tends toward 0 when u tends toward c, and its maximum value is $\lambda_0 = h(V_0/W_{0c})$.

Hypothesis B. According to the hypothesis B, we have *two* different laws of variation for the energy of a *heavy* body: for a *macroscopic* heavy body, the approximate law

$$W_M = mc^2 = \frac{m_0c^2}{(1-\beta^2)^{1/2}}$$

and, for a *microscopic* heavy body, the rigorous law

$$W_\mu = m_\mu c^2 = W_\ell = h\nu_w = h\left(\frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\right)\left(1 + \frac{u}{V_0}\right)$$

$$= \frac{m_0c^2}{(1-\beta^2)^{1/2}}\left(1 + \frac{u}{V_0}\right),$$

which is associated with

$$p_\mu = \left(\frac{W_\mu}{c^2}\right)u = \left[\left(\frac{m_0}{(1-\beta^2)^{1/2}}\right)u\right]\left(1 + \frac{u}{V_0}\right) = m_\mu u.$$

(The relation $p = (W/c^2)u$ holds in any case.)

Then the definition $\lambda = V/\nu_w$ gives, directly from $\nu_w = W_\mu/h$,

$$\lambda = \frac{V}{\nu_w} = h\left(\frac{V}{W_\mu}\right) = \left(\frac{h}{p_\mu}\right)\left(\frac{Vu}{c^2}\right). \qquad (12''')$$

For the limiting values $u = c$ and $u = 0$ the expression (12‴) has the *same* content as the expression (12′) obtained with the hypothesis A; namely, it yields, respectively, $\lambda(u = c) = 0$ and

$$\lambda(u=0) = h\left(\frac{V_0}{W_{0\mu}}\right) = h\left(\frac{V_0}{W_{0M}}\right) = h\left(\frac{V_0}{m_0c^2}\right).$$

If $V = V_0 = c$, then we have

$$\lambda = \left(\frac{h}{p_\mu}\right)\left(\frac{u}{c}\right).$$

4.2.7. The Wave Function in Corpuscular Terms

Hypothesis A. If the phase of the standard form $\Psi(x,t) \doteq ae^{2\pi i\nu_w(t-x/V)}$ is rewritten in the corpuscular terms of the energy W and the momentum p, by making use of the modified relations (11′) and (12′) characteristic of the hypothesis A, one finds—instead of the well-known de Broglie's form $\Psi(x,t) = ae^{(2\pi i/h)(Wt-px)}$—the modified form

$$\begin{aligned}\Psi(x,t) &= ae^{2\pi i(W_c/h)[(1+u/V_0)t-x(u/c^2+1/V_0)]}\\ &= ae^{(2\pi i/h)[W_ct(1+u/V_0)-x(p_c+W_c/V_0)]}.\end{aligned} \tag{21}$$

The equation corresponding to (21) cannot be identified immediately.

Hypothesis B. With the relations (19), (11″), (20), (12‴) which characterize the hypothesis B, instead of the form (21) obtained above, one finds now immediately the very simple form (well known from electromagnetism where $V = c$)

$$\begin{aligned}\Psi(x,t) = a_0e^{2\pi i(\nu_wt-x/\lambda)} &= ae^{2\pi i(W_\mu/h)(t-x/V)}\\ &= ae^{(2\pi i/h)[W_\mu t-(W_\mu/V)x]}.\end{aligned} \tag{21′}$$

The form (21′) satisfies the very simple *covariant* equation

$$\frac{\partial\Psi}{\partial t} = -V\frac{\partial\Psi}{\partial x}. \tag{22}$$

Note that the expressions (21) and (21′) concern—by construction—an *individual* system. (This will become more obvious when, in a subsequent work, the amplitude will be represented by a function of the spatial position, strongly peaked around the location of the mass.) Therefore they cannot be directly compared with the "wave packets" of the quantum mechanical formalism as it now stands, of which the individual or statistical significance is very controversial. Nor can the quantum mechanical equations of evolution be directly compared with the equation corresponding to (21) or with that corresponding to (21′). The relation between our approach and quantum mechanics is not straightforward, it has to be elaborated carefully.

4.3. Experimental Investigations

The two possible modified forms of the corpuscular wave length,

$$\lambda = \frac{h}{p_c + (W_c/V_0)} = \left(\frac{h}{W_c}\right)\left(\frac{Vc^2}{uV+c^2}\right)$$

or

$$\lambda = \frac{V}{\nu_w} = h\left(\frac{V}{W_\mu}\right) = \left(\frac{h}{p_\mu}\right)\left(\frac{Vu}{c^2}\right)$$

((12′) and (12‴) respectively) can be directly tested by corpuscular interference.

The experimental study must begin with low-energy corpuscular interferences (which is most readily performable with heavy microsystems). Indeed, both of the forementioned relations assert the most important deviations from the predictions founded on the usually accepted de Broglie form (12), $\lambda_B = h/p$, when the momentum and the energy become very small: In this case, while de Broglie's relation becomes infinite, the two modified relations *become identical* and acquire the *finite* value (12″), $\lambda(u=0) = h(V_0/W_0)$.

The study must be performed for *various* types of corpuscles, introducing various proper masses m_0. In each case, the limiting value of the corpuscular wavelength λ when the relative velocity u approaches 0 must be investigated. By comparing the result with the relation (12″), $\lambda(u=0) = h(V_0/W_0)$, where W_0, hence ν_{0w}, are *known* data, a table of the proper values $V_0 = \lambda_0\nu_{0w}$ of the phase velocities of the studied corpuscular waves can be obtained. This would elucidate two crucial questions, namely:

(a) Does the proper value V_0 of the phase velocity of a corpuscular wave depend on the proper mass m_0?

(b) Is this proper value V_0 luminal, infraluminal, or supraluminal, or in what case does it belong to one or the other of these possible categories?

With V_0 known, the values (12′) or (12‴) of λ can be calculated and confronted with the experimental data.

Alternatively, the corpuscular Doppler effect asserted by the expressions (16) and (18) can be directly investigated by corpuscular interferometry. Note that, according to a general, three-dimensional treatment, this corpuscular Doppler effect, if it exists, introduces a *dependence of the energy also on the direction.*

Finally, once V_0 is known, the two possible transformation laws for the energy of a microscopic heavy body, namely

$$\begin{aligned}W_\mu = h\nu_w &= h\left(\frac{\nu_{0w}}{(1-\beta^2)^{1/2}}\right)\left(1+\frac{u}{V_0}\right)\\ &= \frac{m_0c^2}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right)\\ &= m_\mu c^2\end{aligned}$$

or de Broglie's assumption

$$W = W_c = \frac{m_0 c^2}{(1-\beta^2)^{1/2}},$$

could be confronted, by appropriately conducted investigations on nuclear collisions and nuclear reactions.

Should the approach developed here, based on the progressive proper form (5′) of the corpuscular wave, be confirmed, the whole formalism of quantum mechanics would have to be restructured. If, in addition and more specifically, the transformation law

$$W_\mu = \frac{m_0 c^2}{(1-\beta^2)^{1/2}}\left(1+\frac{u}{V_0}\right) = h\nu_w$$

turned out to be the confirmed possibility, then the theory of relativity, too, might have to be restructured, at least relativistic dynamics in its fundamental sense.

5. CONCLUSION

The hypothesis (5′), asserting that the spatially extended periodic phenomenon associated by de Broglie with a mass m_0 has a progressive form within the proper frame of reference, seems to be in much deeper agreement with relativity than de Broglie's assumption of a "stationary" proper form (5). Instead of a mere apparent aspect of a spatially extended periodic "element," due to a non-null relative velocity of observation which creates the impression that the localized mass glides inside a wave so as to respect the "condition" $uV - c^2$, the hypothesis (5′) permits a much simpler and more physical conception. Namely, it becomes *a priori* possible to conceive of this periodic "element" as being a physical perturbation (or a group of such perturbations) generated by clocklike periodicities located inside any heavy mass, which then cannot but keep in phase—quite unconditionally—with the perturbations produced by themselves. According to this conception, *the wave-particle "duality," the "wavicle," splits apart, leaving instead a particle with inner periodicities, which produce physical wavelike perturbations (possibly more than one) which propagate with finite velocity (maybe the velocity of light?) and can be modified by obstacles and then act back on the dynamics (and possibly even on the inner state) of the particle that produced them, thereby instituting an essentially reflexive type of causality* [4, 5]. Much of this conception, in fact, is quite current already. But its logico-mathematical incorporation into our theories as they now stand is wanting, and therefore we cannot benefit from its eventual truth.

Furthermore, the assumption of the proper form (5′) entails two different sets of mutually consistent experimentally verifiable consequences, each one tied to a specific hypothesis concerning the fundamental question of the connection between macroscopic heavy energy, microscopic heavy energy, and photonic energy. If one or the other of these two possible sets of consequences were confirmed, the formalism of quantum mechanics would have to be revised. The revision could involve also the theory of relativity, and it could give rise to a new theory, more general than the theories devised up to now, absorbing them into a unified representation valid for all physical systems, photons, macroscopic heavy bodies, or microscopic heavy bodies.

In particular—and this might prove very important *in itself*—the role of *time* in the microscopic domain might become less cryptic: The discontinuous character of the "quantum jumps" might stem from the assumption, for a corpuscular wave, of a stationary proper form of the type (5), a form which possibly blurs and hides the processes of generation of the corpuscular waves and the laws that govern their changes.

But if, on the contrary, the consequences of the hypotheses explored in this paper were invalidated, the whole development from this work would have to be simply ignored.

ACKNOWLEDGEMENTS

I am profoundly indebted to Dr. François Thieffine for his criticism and numerous patient rereadings of this manuscript in its successive versions. His assistance has brought forth essential improvements. Dr. D. Evrard has drawn attention to the solution $V = \infty$ of Eq. (17). I am also very thankful to Professor Robert Vallée for agreeing to read and check my manuscript. Finally, I wish to express my gratitude toward Professor Bernard d'Espagnat for his willingness to scrutinize this manuscript and communicate to me his feeling or objections.

REFERENCES

1. L. de Broglie, *Thesis*, Paris, 1924.
2. L. de Broglie, *Tentative d'interprétation causale et nonlinéaire de la mécanique ondulatoire* (Gauthier-Villars, Paris, 1956).
3. D. Bohm, *The Special Theory of Relativity* (Benjamin, New York, 1965).
4. M. Mugur-Schächter, *Found. Phys.* **17**, 813 (1987).
5. M. Mugur-Schächter, *Found. Phys.* **18**, 461 (1988).