From Quantum Mechanics to Universal Structures of Conceptualization and Feedback on Quantum Mechanics

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Reprinted from FOUNDATIONS OF PHYSICS

Vol. 23, No. 1 January 1993

Foundations of Physics, Vol. 23, No. 1, 1993

From Quantum Mechanics to Universal Structures of Conceptualization and Feedback on Quantum Mechanics

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Received July 23, 1992; revised November 12, 1992

In previous works we have established that the spacetime probabilistic organization of the quantum theory is determined by the spacetime characteristics of the operations by which the observer produces the objects to be studied ("states" of microsystems) and obtains qualifications of these. Guided by this first conclusion, we have then built a "general syntax of relativized conceptualization" where any *description is explicitly and systematically referred to the two basic epistemic operations by which the conceptor introduces the object to be qualified and then obtains qualifications of it. Inside this syntax there emerges a general typology of the relativized descriptions. Here we show that with respect to this typology the type of the predictive quantum mechanical descriptions acquires a* precise definition. *It appears that the quantum mechanical formalism has captured and has expressed directly in a* mathematical *language the* most complex *form in which can occur a* first *descriptional phase that lies* universally *at the bottom of* any *chain of conceptualization. The main features of the Hilbert–Dirac algorithms are decoded in terms of the general syntax of relativized conceptualization. This renders explicit the semantical contents of the quantum mechanical representations relating each one of these to its mathematical quantum mechanical expression. Basic insufficiencies are thus identified and, correlatively, false problems as well as answers to these, or guides toward answers. Globally the results obtained provide a basis for future attempts at a* general *mathematical representation of the processes of conceptualization.*

"Il pourrait, en effet, être dangereux pour l'avenir de la Physique qu'elle se contente trop facilement de purs formalismes, d'images floues et d'explications toutes verbales s'exprimant par des mots à signification imprécise"—Louis de Broglie, *Certitudes et Incertitudes de la Science* (Albin Michel, Paris, 1965).

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DEDICATION

I am happy to participate in the extensive homage consisting of the four issues devoted by *Foundations of Physics* to the centenary of the birth of the founder of modern microphysics. Louis de Broglie's work and what I learned from my interactions with him have deepely marked my thought.

1. INTRODUCTION

In previous works[1,2] we have explained the spacetime structure of the probabilistic organization of quantum mechanics. We have shown that this structure involves peculiar treelike associations of probability spaces with mutually determining probability measures in them, which we have called *quantum mechanical probability trees.* These, though entailed by necessity by mere confrontation of the quantum mechanical descriptors (state vectors, dynamical operators, eigenvectors of these, etc.) with the standard Kolmogorov formulation of the abstract theory of probabilities, have been found to *transgress* this abstract theory as it now stands.

The mentioned results drew our attention to a fact of a quite *general* nature: In a theory the characteristics of the "epistemic" *operations* by which the observer introduces the objects to be studied and works out qualifications of these objects, can play a crucial role in the determination of the structural properties of the theory. Recognition of this fact motivated us to attempt a general representation of *any* chain of conceptualization where each description, systematically, is explicitly referred to the epistemic operations by which the observer produces the entity to be described and obtains for it qualifications. Starting from the *lowest* level of cognitive action, from *beneath* logic as it now stands, and creating an appropriate symbolism, we have constructed a "general method of relativized conceptualization"[3] expressed by a *relativizing epistemic syntax.* This method produces a typology of increasingly complex relativized descriptions constituting a quite general framework where it is possible to locate and to compare the relativized reconstructions of the descriptions and of the systems of descriptions (theories) of any sort.

> According to this typology the descriptions which lie—*universally*—at the *basis* of *any* chain of conceptualization, reveal a tree-like spacetime structure. If, furthermore, this basic sort of description is probabilistic, its structure is identical to that of the quantum mechanical probability trees.

The specific aim of this work is:

(a) to show that the previously obtained results entail a veritable *definition* of the descriptional type of the quantum mechanical predictive descriptions;

(b) to work out, by proofs elaborated inside the general relativizing epistemic syntax, explicit identifications of the significances (semantics) encoded in all the main quantum mechanical algorithms;

(c) to extract conclusions concerning the main problems raised by quantum mechanics.

Such an aim requires, for self-sufficiency of the exposition, at least an abbreviated restatement of the spacetime organization of the quantum mechanical formalism and of the general method of relativized conceptualization. So we begin by such restatements. These, though severely reduced to essentials, will nevertheless cover more than half of the length of the work. The new part, we hope, will be found to justify the restatements.

The approach used in this work might seem strange, erected on grounds too different from those where present-day research is based. But we beg the reader's patience and indulgence. At the end of this work he will have found out that he is endowed with a new framework where many obscurities and paradoxes evaporate like mist.

2. SPACETIME QUANTUM PROBABILITIES

The Essence of the Hilbert–Dirac Formulation of Quantum Mechanics. Quantum mechanics studies states of microsystems. These are represented by normalized kets $|\psi\rangle$ that are postulated to form a (Hilbert) vector space. From a physical point of view this formal postulate constitutes the principle of superposition: If there "exist" two states with state vectors $|\psi_1\rangle$ and $\psi_2|\rangle$, then there also "exists" any state with state vector $|\psi_{12}\rangle = \lambda_1 |\psi_1\rangle + \lambda_2 |\psi_2|\rangle$ where λ_1, λ_2 are arbitrary complex numbers.

The qualifications of the states are predictive and probabilistic. Formally they are obtained with the help of linear and, in general, non-commuting operators Ω (quantum mechanical observables). The language and the algorithms are as follows. It is asked, for instance, what is the probability $\pi(\psi, \omega_j)$ to obtain, in one measurement-evolution of the physical quantity represented mathematically by the observable Ω, performed on the state with state vector $|\psi\rangle$, a physical outcome V_j corresponding to the eigenvalue ω_j of Ω. To answer this, one performs the spectral decomposition of $|\psi\rangle$ with respect to Ω, $|\psi\rangle = \Sigma_j c(\psi, \omega_j)\,|u_j\rangle$, where $|u_j\rangle$, ω_j are, respectively, the eigenvectors and the eigenvalues of Ω, determined by the equation $\Omega\,|u_j\rangle = \omega_j\,|u_j\rangle$, $j \in J$, J an index set; $c(\psi, \omega_j) = \langle u_j | \psi \rangle$ are the expansion coefficients. The researched probability is postulated to be $\pi(\psi, \omega_j) = |\langle u_j | \psi \rangle|^2 = |c(\psi, \omega_j)|^2$.

Two distinct predictive probability measures corresponding to the same state vector and to two noncommuting observables Ω_1 and Ω_2 are calculated to be related accordingly to the equations $c(\psi, \omega_{2n}) = \Sigma_j \alpha_{nj} c(\psi, \omega_{1j})$, $j \in J$, $n \in N$ (J, N are the index sets for the eigenvalues of, respectively, Ω_1 and Ω_2; $\alpha_{nj} = \langle v_n | u_j \rangle$ are the transformation coefficients from the basis $\{|u_j\rangle\}$ of eigenvectors of Ω_1 to the basis $\{|v_n\rangle\}$ of eigenvectors of Ω_2). Which constitutes the quantum mechanical theory of transformations.

So vectors, operators, equations, and probability measures are manipulated accordingly to algorithms. Hidden beneath these algorithms, the probabilistic organization, i.e., the *correspondences* between, on the one hand, the basic quantum mechanical descriptors—state vectors $|\psi\rangle$, operators Ω, eigenfunctions and eigenvectors of these—and, on the other hand, the basic probabilistic descriptors—random phenomena and probability spaces—remains obscure. We have shown[1] that these correspondences can be established as follows.

Formal Probability Chains. Consider a pair $(|\psi\rangle, \Omega)$, where $|\psi\rangle$ is the state vector assigned at the time t to the considered microsystem S and Ω is a hermitian operator representing a quantum mechanical dynamical observable. For each such pair the quantum theory defines a family of elementary probability densities $\pi(\psi, \omega_j)$, $j \in J$ (J an index set) for the emergence of the eigenvalues ω_j of the observable Ω when a measurement of Ω is performed on S in the state $|\psi\rangle$. The corresponding probability measure $\pi(\psi, \Omega)$ is integrated in a formal "probability chain," i.e., a sequence

(random phenomenon) ⇝ [a probability space on the universe of elementary events produced by that random phenomenon]

that can be symbolized by

$$(\psi, \Omega, \{\omega_j\}) \rightsquigarrow [\{\omega_j\}, \tau, \pi(\psi, \Omega)]$$

($(\psi, \Omega, \{\omega_j\})$ is the symbol for the random phenomenon that involves the state vector $|\psi\rangle$ and the dynamical observable Ω and produces by reiteration the universe $\{\omega_j\}$ of formal elementary events; τ is the total algebra on $\{\omega_j\}$; $\pi(\psi, \Omega)$ is the probability measure on τ determined, via the law of total probabilities, by the elementary probability denstiy $\pi(\psi, \omega_j) = \langle |u_j | \psi \rangle|^2$.

Factual Probability Chains. To each formal probability chain there corresponds a factual probability chain

$$(P_\psi, M_\Omega, \{V_j\}) \rightsquigarrow [\{V_j\}, \tau_F, \pi(P_\psi, M_\Omega)]$$

(P_ψ is the operation of state preparation that produces the state with state vector $|\psi\rangle$; M_Ω is an *individual* measurement evolution for the observable Ω; V_j is the "needle position" of a macroscopic device D_Ω for measurements of the observable Ω; $(P_\psi, M_\Omega, \{V_j\})$ is the random phenomenon that involves the operation P_ψ of state preparation and the individual measurement evolutions M_Ω and which, by reiteration, produces the universe of elementary events $\{V_j\}$; τ_F is the *total* algebra on $\{V_j\}$ (F factual); $\pi(P_\psi, M_\Omega)$ is the probability measure on τ_F.

Connection. Each eigenvalue $\omega_j \in \{\omega_j\}$ from a formal chain is posited to be calculable as a function $\omega_j = f_\Omega(V_j)$ of that observed "needle position" V_j from the factual chain that is labeled by the same index $j \in J$. Furthermore, each factual elementary probability density $\pi(P_\psi, \Omega, V_j)$ is posed to be numerically equal to the corresponding formal elementary probability density: $\pi(P_\psi, M_\Omega, V_j) = \pi(\psi, \omega_j) = |\langle u_j | \psi \rangle|^2$.

Elementary Quantum Mechanical Chain Experiments. A sequence $P_\psi - M_\Omega - V_j$ has been called by us an *elementary quantum mechanical chain experiment* (eqmce). It possesses a remarkable unobservable *depth* wherefrom emerges into the observable only the extremity V_j that contributes to the construction of the factual observable universe of elementary events $\{V_j\}$. Each observable quantum mechanical event—*non*elementary—from an algebra τ_F from a factual quantum mechanical probability space contains inside its semantic substratum all the unobservable individual sequences of operations and processes forming the corresponding elementary quantum mechanical chain experiments that end up with the registration of the needle positions V_j contained in that event. So any quantum mechanical prediction concerns either an elementary quantum mechanical chain experiment, or a union of such experiments. *The elementary quantum mechanical chain experiments are the "fibers" out of which is made the factual substance of the quantum theory.* Nevertheless they are *devoid of mathematical representation.*

Quantum Mechanical Probability Trees. We *fix* now an operation of state preparation P_ψ. Consider the ensemble of all the factual probability chains determined by P_ψ *and* the set of *all* the dynamical observables $\Omega_1, \Omega_2, \Omega_3, \ldots$ defined in quantum mechanics. The probability chains from this ensemble constitute together a certain *unity*, in consequence of their common provenance P_ψ. What is the *spacetime* structure of this unity?

For all the chains from the considered unity, the spacetime support of the operation of state preparation P_ψ is the *same*, but not also for all the individual measurement evolutions M_Ω involved in this unity. The ensemble of these evolutions *splits* into sub-ensembles M_X, $M_Y, \ldots$ of mutually "compatible" processes corresponding to mutually commuting observables. Many textbooks contain very confusing considerations concerning "successive measurements" of compatible observables (versus the projection postulate). But in fact the notion of *successive* measurements simply is irrelevant for *compatible* observables: Each *one* measurement evolution from one sub-ensemble, say M_X, can be operationally defined such that *each* registration of a value V_j of the "needle position" of the corresponding macroscopic device D_X permits to calculate—from the *unique* datum V_j—via a set of *various* theoretical connecting definitions $\omega_{1j} = f_1(V_j)$, $\omega_{2j} = f_2(V_j), \ldots$, *all* the *different* eigenvalues $\omega_{1j}, \omega_{2j}, \ldots$ labeled by the *same* index j, for, respectively, *all* the observables $\Omega_1, \Omega_2, \ldots$ measurable by a process belonging to the class M_X. This entails that for all the commuting observables corresponding to *one* same class M_X the physical process leading to the registration of a value V_j of the "needle position" of the device D_X can be just one common process covering just one spacetime support, while this is *not* possible for two noncommuting observables belonging to two distinct classes M_X and M_Y: *This is what is commonly designated as "Bohr complementarity."* This entails that, globally, the unity constituted by the ensemble of all the factual probability chains corresponding to a fixed operation of state preparation P_ψ possesses a *branching, treelike spacetime structure.*

Let us symbolize this treelike structure by $\mathcal{T}(\psi)$ and let us call it a *quantum mechanical probability tree* (in short, a probability tree). So the operations of state preparation P_ψ define, on the set of all the quantum mechanical probability chains, a *partition* in probability trees. *A fortiori*, they define such a partition also on the ensemble of all the elementary quantum mechanical chain experiments out of which the factual quantum mechanical probability chains are made. Figure 1 provides a simplified example of a probability tree, with only four observables $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ and three branches. The individual measurement evolutions M_{12} (abbreviated notation) correspond to two commuting observables Ω_1, Ω_2, while M_3 and M_4 correspond to two noncommuting observables Ω_3, Ω_4. The notations $[\ ]_{12}$ and $[\ ]_3$, $[\ ]_4$ indicate the factual, observational probability spaces corresponding respectively to the measurement evolu-

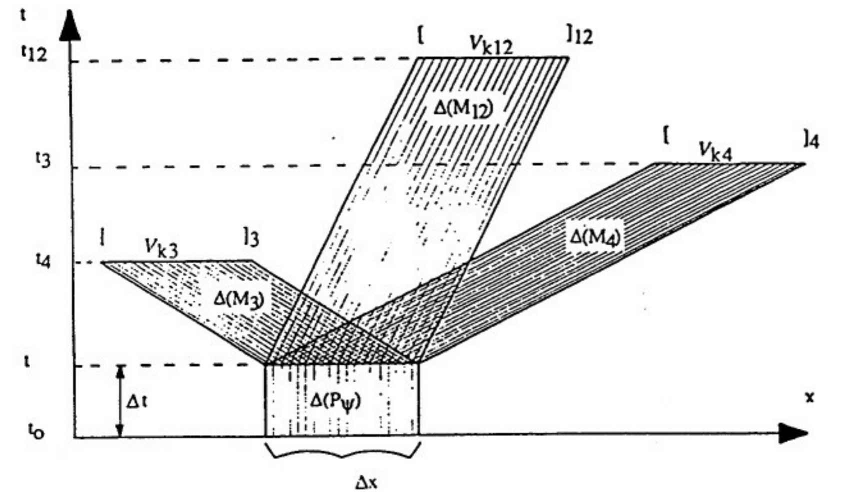

Fig. 1. A quantum mechanical probability tree $\mathcal{T}(\psi)$.

tions M_{12}, M_3, and M_4 realized on the state represented by $|\psi\rangle$. Each one of the probability spaces $[\ ]_n$, $n = 12, 3, 4$, emerges—with respect to an origin of times *reset to* 0 *after each eqmce*—at some corresponding specific time t_2 (i.e., $t_2 - t$), t_3 (i.e., $t_3 - t$), and t_4 (i.e., $t_4 - t$). The branch corresponding to Ω_1, Ω_2 (so to M_{12}) contains a very large number of fibers $P_\psi - M_\Omega - V_{n12}$ each one of which ends up with *one* needle position $V_{k12} \in \{V_{j12}\}$ that permits to calculate *two* distinct corresponding eigenvalues, $\omega_{k1} \in \{\omega_{j1}\}$ and $\omega_{k2} \in \{\omega_{j2}\}$, via two different theoretical definitions $\omega_{n1} = f_1(V_{n12})$, $\omega_{n2} = f_2(V_{n12})$. The branch corresponding to Ω_3 (so to M_3) contains a large number of fibers $P_\psi - M_\Omega - V_{n3}$ each one of which ends up with a needle position $V_{n3} \in \{V_{j3}\}$ that permits one to calculate a *unique* corresponding eigenvalue $\omega_{n3} \in \{\omega_{j3}\}$ *via* a theoretical connecting function $\omega_{n3} = f_3(V_{n3})$. Similarly the branch corresponding to Ω_4 (so to M_4) contains a large number of fibers $P_\psi - M_\Omega - V_{n4}$ each one of which ends up with a needle position $V_{n4} \in \{V_{j4}\}$ that permits one to calculate a *unique* corresponding eigenvalue $\omega_{n4} \in \{\omega_{j4}\}$ *via* a theoretical definition $\omega_{n4} = f_4(V_{n4})$. So the space $[\ ]_{12}$ is endowed with more specifications than the spaces $[\ ]_3$ and $[\ ]_4$.

In each one fiber of the tree the *initial* phase, of state preparation, covers the same spacetime domain $\Delta(P_\psi) = \Delta x \Delta t$, (the common spacetime trunk of the tree), with $\Delta t = t - t_0$. The *subsequent* phase of measurement evolution covers, for each one fiber, a spacetime domain $\Delta(M_{12})$ in the case of an evolution M_{12} corresponding to the two commuting observables Ω_1, Ω_2, or one of the two distinct spacetime domains $\Delta(M_3)$, $\Delta(M_4)$ in the case of, respectively, a measurement evolution M_3 corresponding to the observable Ω_3, or a measurement evolution M_4 corresponding to the observable Ω_4.

Most of the fundamental algorithms of the quantum mechanical calculus which combine *one* normalized state vector, with the dynamical operators representing the quantum mechanical observables, can be defined *inside*—any—*one* tree $\mathcal{T}(\psi)$ (the mean value of an observable Ω in a state with state vector $|\psi\rangle$: $\langle\psi|\,\Omega\,|\psi\rangle$, $\forall\,|\psi\rangle$, $\forall\Omega$; the uncertainty theorem, for any pair of observables: $\langle\psi|\,(\Delta\Omega_1)^2\,|\psi\rangle\langle\psi|\,(\Delta\Omega_2)^2\,|\psi\rangle \geqslant |\langle\psi|\,(i/2)(\Omega_1\Omega_2-\Omega_2\Omega_1\,|\psi\rangle| = (1/2)(h/2\pi)$, $\forall\Omega_1, \Omega_2$; the principle of spectral decomposition: $|\psi\rangle = \Sigma_j c(\psi, \omega_j)\,|u_j\rangle$, $\forall\,|\psi\rangle$, $\forall\Omega$: $\Omega\,|u_j\rangle = \omega_j\,|u_j\rangle$, with $c(\psi, \omega_j)$ the expansion coefficients, which permits one to calculate the probability density $\pi(\psi, \omega_j)$ *via* the probability postulate $\pi(\psi, \omega_j) = |\langle u_j|\psi\rangle|^2 = |c(\psi, \omega_j)|^2$; and finally, the whole quantum mechanical "transformation theory" from the basis of an observable Ω_1 to that of an observable Ω_2, $c(\psi, \omega_{2n}) = \Sigma_j \alpha_{nj}\, c(\psi, \omega_{1j})$, $\forall\Omega_1, \Omega_2$: $\Omega_1\,|u_j\rangle = \omega_{1j}\,|u_j\rangle$, and $\Omega_2\,|v_n\rangle = \omega_{2n}\,|v_n\rangle$, $\forall j \in J$, $\forall n \in N$, with J, N the index sets for the eigenvalues of, respectively, Ω_1, Ω_2 and $\alpha_{nj} = \langle v_n|u_j\rangle$ the transformation coefficients).

But as soon as the principle of superposition comes into play, the corresponding quantum mechanical algorithms cease to be embeddable into one single probability tree: *several trees have to be combined.*

Quantum Mechanical "Deterministic Probabilistic Metadependence." The quantum mechanical transformation theory ($c(\psi, \omega_{k2}) = \Sigma_j \alpha_{kj} c(\psi, \omega_{j1})$, $\forall\Omega_1, \Omega_2$: $\Omega_1\,|u_j\rangle = \omega_{j1}\,|u_j\rangle$, $\Omega_2\,|v_k\rangle = \omega_{k2}\,|v_k\rangle$, $\forall j \in J$, $\forall k \in K$, J, K index sets, $\alpha_{kj} \doteq \langle v_k|u_j\rangle$ the transformation coefficients) permits one to entirely determine, from the knowledge of the probability measure $\pi(\psi, \Omega_1)$ from one branch of a probability tree, any other probability measure $\pi(\psi, \Omega_2)$ belonging to that same tree. Indeed the equalities $|c(\psi, \omega_{k2})|^2 = |\Sigma_j \alpha_{kj} c(\psi, \omega_{j1})|^2$, $\forall j \in J$, $\forall k \in K$, are equivalent to the specification of a functional relation

$$\pi(\psi, \Omega_2) = F_{\text{QM}}[\pi(\psi, \Omega_1)]$$

between the probability measures corresponding to the two noncommuting observables Ω_1 and Ω_2. But the standard concept of functional relation between two probability measures does not singularize this particular sort of probabilistic connection between two probability measures introduced by the quantum theory. *Nor does it permit one to recover it.*[1] As the index QM emphasizes, we are in the presence of a specifically quantum mechanical functional relation.

This relation can be regarded as a "*deterministic probabilistic metadependence*" in the following sense (Ref. 1, pp. 1401–1405): According to the current theory of probabilities the concept of "probabilistic dependence" is by definition confined inside *one* probability space where it concerns *isolated* pairs of *events*. Two events are tied by a "probabilistic dependence" if knowledge of one of these events "influences" the expectations concerning the other. So the relation $\pi(\psi, \Omega_2) = F_{\text{QM}}[\pi(\psi, \Omega_1)]$ of mutual determination of the probability measures from a quantum mechanical probability tree can naturally be regarded as a "deterministic probabilistic metadependence": "deterministic" because it consists in mutual *determination*; "probabilistic" because, though this determination is a certainty about "influence," nevertheless it concerns *probabilistic* constructs; "metadependence" because it concerns, not pairs of events from one space, but *globally* pairs of probability *measures* on entire algebras of events, which, with respect to events, are *meta* entities. The notion of a probabilistic metadependence can also be upheld otherwise (Ref. 9, p. 990). Imagine a physicist who does not yet know which state vector $|\psi\rangle$ "describes" the state produced by the operation of state preparation P_ψ. So he makes various measurements on this state in order to establish probability densities that shall determine an adequate mathematical descriptor $|\psi\rangle$ (ref. 1, pp. 1408–1412). Suppose that he decides to work with two noncommuting observables Ω_1 and Ω_2, and, on the basis of some reasons, he envisages two *sets* of possible probability measures on the corresponding spectra, namely $\Sigma_1 = \{\pi(\psi, \Omega_1)\}$ and $\Sigma_2 = \{\pi(\psi, \Omega_2)\}$, respectively. (For simplicity suppose they are discrete.) The physicist now asks himself: "What is the (meta)probability for finding, by measurements, this or that probability measure from Σ_1 or this or that probability measure from Σ_2?" In the absence of any criteria for answering otherwise, he will have to presuppose equipartition on both Σ_1 and Σ_2. Suppose that he furthermore asks himself: "If for the spectrum $\{\omega_{m1}\}$ of Ω_1 the probability measure were $\pi_k(\psi, \Omega_1) \in \Sigma_1$ (k known), what would be the corresponding *conditional* probability to find this or that measure $\pi(\psi, \Omega_2)$ from Σ_2 on the spectrum $\{\omega_{j2}\}$ of eigenvalues of Ω_2?". This new question concerns now the product probability space where the elementary events are the possible associations $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ between the one measure $\pi_k(\psi, \Omega_1) \in \Sigma_1$ (assumed known) and the various unknown probability measures envisaged in the set $\Sigma_2 = \{\pi(\psi, \Omega_2)\}$. If the physicist ignores the quantum mechanical transformation theory, *he must again presuppose equipartition*, which amounts to presupposing *independence*, that is, that the probability of a joint event $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is the *product* of the probability of $(\pi_k(\psi, \Omega_1)$ (fixed) and of the probability of $\pi(\psi, \Omega_2)$ (variable inside Σ_2 and, there, *a priori* uniformly distributed). But to this question, which obviously is a *meta* probabilistic question, the quantum mechanical theory of transformations yields *another* answer. Namely, it asserts that the probability measure on the universe of elementary (meta)events $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is a Dirac dispersion-free measure that associates the probability 1 to the unique joint event $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ where $\pi(\psi, \Omega_2) \in \Sigma_2$ is related with the known measure $\pi_k(\psi, \omega_{m1}) \in \Sigma_1$ *according to the set of equations* $\pi(\psi, \omega_{j2}) = |\Sigma_m \alpha_{jm} c(\psi, \omega_{m1})_k|^2$, $\forall j \in J$, $\forall m \in M$, (J, M are *index sets*), while the probability of any other one of the considered joint events $(\pi_k(\psi, \Omega_1), \pi(\psi, \Omega_2))$ is posited 0. This means "dependence," thus specifying in *what* sense the transformation theory can be regarded as asserting "deterministic probabilistic metadependences."

The Potential-Actualization-Actualized Character of a Probability Tree. This fundamentally new type of probabilistic metadependences between the probability measures from different branches of one given probability tree *reflects the oneness of the studied state with state vector* $|\psi\rangle$ *from the common trunk of the tree*. This state that stems from a preparation operation P_ψ and somehow "is" there, nevertheless "exists" merely as a monolith of still nondifferentiated *potentialities* of outcomes of future observations—a monolith of still nondifferentiated potentialities that is just *posited* to "exist" though it has not yet produced knowledge. The assertion of its existence amounts to the assertion of a genetic unity beneath the various incompatible measurement processes of *actualization* of this or that particular set of observational potentialities, leading to this or that *actualized* observable branch-probability space $[\ ]_n$.

> *The probability tree of a state with state vector* $|\psi\rangle$ *is a unity which—with respect to the observable manifestations of the studied state—possesses a "potential-actualization-actualized character" ("potential," by what is labeled* $|\psi\rangle$*; "actualization," by the measurement evolutions* M_Ω*; "actualized," by the registered eigenvalues* $\omega_j = f_\Omega(V_{j12})$.

The quantum mechanical transformation theory involves new probabilistic features that are neither probabilistic "anomalies" nor mere numerical algorithms.

> *They are a mathematical description for the particular case of microsystems, of a peculiar type of probabilistic metaproperties entailed by the presence inside the involved random phenomena, of "objects" (the quantum mechanical "states") that are mere sets of classes of as yet nonactualized observational potentialities relative to classes of possible future processes of observation.*

This, with respect to the Kolmogorov theory of probabilities, is a *growth* of the probabilistic thinking that happened (more than 60 years ago!) inside the process of conceptualization of the microphenomena, and remained hidden there:

> *The Kolmogorov theory of probabilities presupposes random phenomena involving only actualized objects characterized by (consisting of ?) sets of actualized properties of which only the passage into knowledge is still potential.*

The Principle of Superposition: A Calculus with Whole Trees. As soon as the principle of superposition comes into play, the embeddability into one tree hits a limit. The corresponding quantum mechanical algorithms cease to be embeddable into one single probability tree: *Several trees have to be combined.* The quantum mechanical formalism contains implicit calculi with *whole probability trees.*

The principle of superposition is connected with writings of the type $|\psi_{12}\rangle = \lambda_1 |\psi_1\rangle + \lambda_2 |\psi_2|\rangle$ that combine (at least) three trees, namely those introduced by the three operations of state preparation $P_{\psi 12}$, $P_{\psi 1}$, $P_{\psi 2}$ corresponding to the three involved state vectors $|\psi_{12}\rangle$, $|\psi_1\rangle$, and $|\psi_2\rangle$. The possibility of such *linear* composition-writings, for any pair of functions ψ_1, ψ_2, is a condition *sine qua non* for the formal representability of the set of such functions by a vector space. However, and this is of basic importance, the principle of superposition only indirectly concerns the state vectors $|\psi_{12}\rangle$, $|\psi_1\rangle$, and $|\psi_2|\rangle$ themselves. Regarded as a *physical* assertion, the principle of superposition concerns *directly* the *operations of state preparation* $P_{\psi 1}$, $P_{\psi 2}$, $P_{\psi 12}$ that produce the states with state vectors $|\psi_1\rangle$, $|\psi_2|\rangle$, and $|\psi_{12}\rangle$ (Ref. 1, pp. 1405–1424), namely, it amounts to the following assertion: *If the two operations* $P_{\psi 1}$, $P_{\psi 2}$ *are realizable separately, then any operation* $P_{\psi 12}$ *that is some functional of these operations,* $P_{\psi 12} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$, *such that it produces the state* $|\psi_{12}\rangle = \lambda_1 |\psi_1\rangle + \lambda_2 |\psi_2|\rangle$,[4] *is also realizable.* On the other hand, the probability densities for the state $|\psi_{12}\rangle$, for any observable Ω,

$$\pi(\psi_{12}, \omega_j) = |\langle u_j | \psi_{12}\rangle|^2 = |\langle u_j | \lambda_1 \psi_1 + \lambda_2 \psi_2\rangle|^2 \neq \pi(\psi_1, \omega_j) + \pi(\psi_2, \omega_j)$$

"compares," in a way chosen to be *non*linear, the three probability densities $\pi(\psi_1, \omega_j)$, $\pi(\psi_2, \omega_j)$, and $\pi(\psi_{12}, \omega_j)$. Namely, it *refers* the various elementary probability densities $\pi(\psi_{12}, \omega_j)$ from the probability spaces of the *unique* tree obtained when the operation of state preparation $P_{\psi 12} = G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$ is realized, to the corresponding elementary probability densities $\pi(\psi_1, \omega_j)$ and $\pi(\psi_2, \omega_j)$ from the trees that *would* be obtained *if* the operations of state preparation $P_{\psi 1}$ and $P_{\psi 2}$ *were* realized separately.

This expresses a peculiar sort of "probabilistic *meta*-metadepen
expressing an *interaction* between the operations of preparation P
$P_{\psi 2}$ involved in the functional $P_{\psi 12}=G(\lambda_1, \lambda_2, P_{\psi 1}, P_{\psi 2})$ (R
pp. 1421–1424). In short, the well-known and so puzzling algor
injunction "the amplitudes of probability have to be added, on
probabilities interfere" corresponds to the following descriptional m

- For the composition of state descriptors a linear representa *chosen*, which permits a vector space formalism for th descriptors but offers no possibility to express interaction b the effects of separately realizable operations of state prep that are involved in one more complex operation o preparation where their effects interact;
- For the composition of the observable probability distrib corresponding to a linear combination of state vectors, linear representation is *chosen*, which permits one to expr observable consequences of the interaction between the effe separately realizable operations of state preparation th involved in one more complex operation of state preparat

This descriptional method, however, is no more than just a con system of *choices of representation*. It is most important to be clearl of the physical factual circumstances toward which these choices po

In order then to complete the mathematical quantum mech system of choices of representation, we have defined a mathem representation, via "normalized projectors," for the operations $P_{\psi i}$ o preparation and for the functionals $G(\lambda_i, P_{\psi i})$, $i = 1, 2, \ldots, n$, that co them, thus compensating for a very confusing lacuna. We have show this definition is consistent with that for dynamical operators, ent deeper specification of the terms of compatibility and commutatio from all this we have drawn nontrivial consequences concerni principle of superposition (Ref. 1, pp. 1424–1434).

In this new context it becomes strikingly obvious that the linea position writings that are founded on the "principle of superpositio to facts that are *fundamentally different* from those toward which po linear composition writings that are founded on the "principle of s decomposition":

- Each linear composition, in the purely *mathematical* se *eigen*vectors from the Hilbert space of "generalized" kets, on the basis of the principle of *spectral decomposition*, the *measurement*, on an already prepared state, of only *on* of commuting dynamical observables, and it involves on

one tree to which the studied state vector belongs, in a way *non*-referred to any other trees.

- Whereas, as just emphasized, any linear composition written on the basis of the principle of *superposition* concerns basically operations of state preparation *each* of which could generate its own probability tree but are in fact combined in only one effectively realized operation of state preparation that generates only one corresponding probability tree. Which entails
  - * *reference* relations between several whole distinct trees, in particular between *all* the probability measures from these, for *all* the quantum mechanical dynamical observables.

This distinction splits the fundamental quantum mechanical notion of "interference of probabilities" into two essentially different sorts of such interferences (Ref. 1, pp. 1412–1416):

* The *abstract* sort of interference, entailed by the transformation theory, between the square roots $c(\psi, a_j)$ of all the elementary probabilities $\pi(\psi, a_j)$ relative to an observable A, inside the expression $|c(\psi, b_k)|^2 = |\Sigma_j \alpha_{nj} c(\psi, a_j)|^2$ of the probability $\pi(\psi, b_k) = |c(\psi, b_k)|^2$ of an eigenvalue b_k of another observable B that does not commute with A, when this last probability is represented in the basis of A: an interference of just two "points of view" corresponding to A and B.
* The interference inside the elementary probabilities $\pi(\psi_{ab}, a_j)$ corresponding to a superposition state $|\psi_{ab}\rangle = \lambda_a |\psi_a\rangle + \lambda_b |\psi_b\rangle$ and to only *one* observable A, of the square roots of the elementary probabilities $\pi(\psi_a, a_j)$ and $\pi(\psi_b, a_j)$ corresponding to A and to the superposed reference state vectors $|\psi_a\rangle$ and $|\psi_b\rangle$: this last sort of interference expresses physical interactions and can produce directly observable physical effects when A is the position observable.

The Hilbert–Dirac formalism and language "unify" these two fundamentally distinct sorts of linear compositions, spectral decompositions, and superpositions of state vectors. *Ipso facto* they "unify" also the two essentially different sorts of interferences distinguished just above: such "unifications" are in fact semantic *confusions* inside a flattening concept of "superposition" in a Hilbert space of "generalized" kets. Thereby crucial significances from the quantum mechanical formalism are hidden under an opaque stratum of conceptual mud where "the interpretation problem" has floundered for more that 60 years.

Global View on the Probabilistic Organization of Quantum Mechanics. Any one observable quantum mechanical elementary event V_j is brought forth by some elementary quantum mechanical chain experiment, some fiber $P_\psi - M_\Omega - V_j$. These fibers are the semantic matter described by the quantum theory. Any fiber $P_\psi - M_\Omega - V_j$ belongs to a factual probability chain

$$(P_\psi, M_\Omega, \{V_j\}) \rightsquigarrow [\{V_j\}, \tau_F, \pi(P_\psi, M_\Omega, V_j)]$$

In its turn, any factual probability chain belongs to a probability tree $\mathcal{T}(\psi)$, the tree tied with the operation P_ψ of state preparation which starts that chain. So the probability trees define a *partition* on the set of all the chains (hence on the set of all the fibers, hence on the set of all the observable quantum mechanical elementary events V_j).

When one contemplates the landscape determined by this partition, each tree appears endowed with its own *internal* calculus (mean value of any dynamical observable Ω with respect to the state vector $|\psi\rangle$ tied with the considered tree, the uncertainty theorem for this state, the principle of spectral decomposition and the predictional probability laws for this state, and the whole quantum mechanical theory of transformations that relates the probability measures from the different branches of the tree), whereas the different trees are mutually connected by a calculus with whole trees determined by the principle of superposition and the probability law for superposition states.

This is a *hierarchical* view (fibers, chains, trees, connections between trees). It draws attention upon the following crucially important fact of a quite general nature:

> *The* spacetime *characteristics of the operations by which the observer produces the objects to be studied* (*state preparations*) *and of the processes of qualification of these* (*measurement operations*) *play a central role in the determination of the structure of the obtained descriptions.*

The following section reports on an investigation where the systematic development of the consequences of this fact leads to a "general syntax of relativized conceptualization."

3. SYNOPSIS OF THE METHOD OF RELATIVIZED CONCEPTUALIZATION: THE $[\Delta, \eta_\Delta, \diamond, D]$ SYNTAX

> Era I gave us the parabolas of Galileo and the ellipses of Kepler, motion with no explanation of motion. Era II gave us the mechanics of Newton, the electrodynamics of Maxwell and Faraday, the geometrodynamics of Einstein, and the chromodynamics of our day, law that explained change, but law without explanation of law. Out of Era III physics we have to seek nothing less than the foundation of physical law itself."—J. A. Wheeler, "Bits, quanta, meaning," in the *Caianiello Celebration Volume*, Giovanni *et al.*, eds.

3.1. The Aim

In the present formulations of the physical theories the operations by which the studied entities are produced and the operations by which these entities obtain qualifications are not, in general, represented and characterized. Even in the theory of relativity, which is *founded* upon the analysis of the operations of measurement that produce the coordinates of space and of time assigned to the studied objects, these operations themselves, though described, are not represented, they are only indicated by words from the usual language; as to the operations that introduce the studied object-entities, they are not even mentioned. This, *ab initio*, hinders the formulation of general laws and algorithms involving these two sorts of basic operations which—always—*are* involved. Inside quantum mechanics, for the very first time in the history of theories, the operations of measurement have been represented, not only formally, by some symbol, but even *mathematically*, and a calculus with such operations has been defined. However, the still more fundamental operations that introduce the studied object-entities, though they are explicitly mentioned, have not been represented, neither merely by symbols, nor, *a fortiori*, mathematically. As reported, in the works summarized in the preceding section we have compensated for this lacuna, and *this has enabled us to draw nontrivial conclusions* concerning the principle of superposition (and the quantum theory of measurements) (Ref. 1, pp. 1424–1448).

But still more basically, consider logic. This is an artificial language aiming toward universal generality via the extraction of essences. But even in logic, again, the operations that produce the object-entities and the qualificators from the considered utterances are not also *themselves* represented; they are only discussed in separate "epistemologic" approaches (Wittgenstein,[5] Russell,[6] Quine,[7] Campbell,[8] etc.). In particular, in the researches concerning quantum logic these operations, so far, have not been basically and systematically taken into account. This is particularly surprising since quantum mechanics is a theory where the operations of preparation and of measurement play a central role. We have shown in a previous work on the logical structure of quantum mechanics[9] that when this attitude is modified a new possible view on quantum logic appears where the relations between quantum probabilities and quantum logic become clear.

These remarks converged to motivate us to investigate the consequences entailed, in a general representation of the descriptions of any kind, by an explicit definition, representation, and characterization of the two basic types of epistemic operations by which the "observer" (descriptor, conceptor) introduces the object-entities to be studied and obtains for them qualifications.[3] The main results are summarized in this section. What is obtained can be regarded as a *general relativizing syntax of conceptualization* founded on only eight basic definitions and three principles. It starts *beneath* logic and it generates a typology that includes in principle relativized elaborations of all the conceivable sorts of description, whatever the degree of complexity. Thereby a quite general and maximally deeply rooted framework is obtained where it becomes possible to compare, in a definite sense, any two descriptions. This, as Russel's typology[10] did, will permit one to remove many obscurities and paradoxes.

3.2. The Kernel of the Method of Relativized Conceptualization

The concept of a description involves certain requirements of coherence and of limitation. These cannot be fulfilled without a certain *selective* and *stabilizing* attitude of the "conceptor," imposed on the one hand upon the portion of reality accepted as source of the registrations that are taken into account, and on the other hand upon the type of registrations that is researched. In order to characterize this attitude we have defined a convenient language.

3.2.1. Epistemic Operators Introducing the Object to be Examined

We denote by R, "reality"—physical, *conceptual*, whatever—the reservoir out of which *any* object of examination conceivable *at the considered time* can be produced or chosen. So the content of this reservoir is conceived here to *evolve*, by physical processes *as well as* by conceptual ones.

Definition 1: *Delimitator.* An epistemic operator Δ, defined on R, and which produces—as an object for subsequent examinations—an entity denoted η_Δ which *neither identifies with* Δ *nor includes it* but which otherwise is entirely unrestricted, will be called a *delimitator.* We write symbolically:

$$\Delta R \rightarrow \eta_\Delta \quad \text{or} \quad \eta_\Delta \leftarrow \Delta R$$

So a delimitator Δ can consist of any mode of production, out of R, of an object for future examinations. This mode can involve operations that are exclusively physical, or exclusively conceptual, or any combination of both. Furthermore it can just select a preexisting object or, on the contrary, create an object. When I point my finger toward a stone that I want to be examined I delimit by a physical act, but not creatively. When I prepare a state of an electron in order to study it I delimit by a physical operation that is creative. When I define a new notion by words in order to examine it further I delimit conceptually and creatively. When I pick up in a dictionary the definition of a chair I make use of a conceptual delimitator that selects a preexisting abstract object. If I build a program for a Turing machine in order to examine the sequence of strings that it generates, I utilize an instructional delimitation that is conceptual and creative. And so forth.

The concept of a delimitator puts an explicitly specified and symbolized generator beneath any object-entity. *Thereby it lies beneath logic.*

3.2.2. Epistemic Operators Characterizing the Researched Qualifications

Definition 2: *Aspect-View* (*Operation of Examination, Aspect, Structure of an Aspect*). The symbol $\langle g \rangle$ will indicate an *operator of examination* called *aspect-view* that is defined on the (evolving) ensemble $\{\eta_\Delta, \forall \Delta\}$ of all the conceivable entities η_Δ—the domain of $\langle g \rangle$—and can produce *via* the corresponding *operation of examination* $\langle g \rangle \eta_\Delta$, qualifications of the entities η_Δ from an ensemble of a specified type—the range of $\langle g \rangle$—structured as follows:

- the index g (permitted to take on any graphic form, a letter, a group of letters, another sign) labels *globally* a whole *discrete* and *finite* but *arbitrarily rich* class of researched qualifications globally called an *aspect g*;
- the qualifications from the aspect-g-class, pairwise distinct, are called the *values k of the aspect g*, in short *gk values*;
- the aspect g, so the aspect view $\langle g \rangle$ being considered to be defined *if and only if* a modality, physical or formal, is fully prescribed for

* accomplishing the operation of examination $\langle g \rangle \eta_\Delta$ corresponding to the aspect-view $\langle g \rangle$;
* expressing the result in terms of values gk.

If the aspect g and the corresponding aspect-view $\langle g \rangle$ *are* defined in the above-specified sense, then we *include* in the definition any object, or device, or algorithm involved by the modality defining the operation of examination $\langle g \rangle \eta_\Delta$. We transpose in symbols as follows.

$$g \supset \bigvee_k gk, \quad k \geqslant 1, \quad k \in K, \quad K\text{: an index set, } \textit{finite} \text{ and } \textit{discrete} \text{ but } \textit{arbitrarily rich}$$
$$gk \wedge gk' = \varnothing, \quad \forall (k \neq k'), \quad (k, k') \in K \tag{1}$$

which is to be read: The aspect g contains all the values gk (so the sign $\bigvee$ indicates a sort of "union"); if $k \neq k'$, the values gk and gk' of the aspect g have nothing in common (so the sign $\bigwedge$ indicates a sort of "intersection" and $\varnothing$ indicates "void"); the strict inclusion $g \supset \bigvee_k gk$ expresses that any set of aspect-values gk, even only one, generates a whole semantical dimension g that exceeds it[3] (p. 240).

The structure (1) mirrors explicitly the restrictions imposed upon any effectively realizable examination, by the nonremovable discreteness entailed by any definite *unit* or any definite set of *samples* involved in any process of qualification, as well as by the finiteness of any given human investigation. Notice that *in general no order relation is required among the values gk of an aspect g*. Notice also the distinctions and the relations between an operat*or* $\langle g \rangle$ (the aspect-*view*), the corresponding *aspect* g, and the corresponding operat*ion* of examination $\langle g \rangle \eta_\Delta$. (cf. Ref. 3 for examples and more details).

Though the set of all the conceivable aspects g is immensely rich, open and nonnumerable, in any given investigation the number of the aspects selected for being taken into consideration is necessarily *finite*. So it is adequate to form the notion:

Definition 3: *View*. We call *view* any ensemble $\{\langle g \rangle, g = 1, 2, \dots, m\}$ of a *finite* but *arbitrarily big* number m of aspect-views $\langle g \rangle$ together with all the possible groups of joint aspect-views constructible out of these. We symbolize a view *in general* by the symbol $\langle\ \rangle$ (a void open eye). When we specify its content we introduce a sign (a capital letter, or another symbol) that labels that content.

The complexity and the degree of organization of a given view is determined by the number of aspects which compose it and by the structure

assigned to the ensembles of values of these aspects: cardinal, origins, existence or not of an order, etc. In particular a view can consist of only one aspect-view, and even, as a limit, of only one aspect-view involving an aspect with only one value. But there is nothing final, nothing absolute in the distinction between view and aspect-view. An aspect-view can in general be expanded into a view by a convenient analysis in other aspect-views, and *vice versa*.

The concepts of an aspect-view and a view substitute a whole complex *structure* to the concept of a "predicate" from the present-day logic; a structure that represents the *generation* of the predications. *Thereby these concepts lie beneath logic.*

3.2.3. A Priori Arbitrary Pairings of a Delimitator and a View

No description can start without the explicit or implicit action of a certain pair $(\Delta, \langle\ \rangle)$, in succession or in simultaneity. So we define now the assemblages of a delimitator and a view:

Definition 4: *Epistemic Referential*. Any pair $(\Delta, \langle\ \rangle)$ consisting of a delimitator and a view will be called an *epistemic referential*.

3.2.4. Basic Role of the Human Actor

An epistemic referential is still a concept *devoid of autonomy*, in its genesis as well as from a functional point of view. It presupposes cognitive aims that dictate both the construction $(\Delta, \langle\ \rangle)$ and its utilizations. These arise outside the epistemic referential, in what is called here a "consciousness-functioning." So, as a closure, I define:

Definition 5: *Observer*. The basic cognant whole which emerges when a human being endowed with his consciousness-functioning equips itself with one definite epistemic referential $(\Delta, \langle\ \rangle)$, will be called an *observer*.

3.2.5. First A Posteriori Checking of a Pairing $(\Delta, \langle\ \rangle)$

Suppose an observer who makes use of his epistemic referential $(\Delta, \langle\ \rangle)$. What results can arise? The answer has a stratified structure.

Definition 6: *Relative Existence or Nonexistence*. Let $\Delta R \to \eta_\Delta$ be an entity delimited by the observer for qualification. Consider an aspect-view $\langle g \rangle \in \langle\ \rangle$ and a given value gk of the corresponding aspect g. The examination $\langle g \rangle \eta_\Delta$ either reveals to the observer the value gk, or it does not. If it does not, we write

$$[\langle g \rangle \eta_\Delta \to \varnothing / gk] \sim [\nexists gk/\eta_\Delta, \nexists \eta_\Delta / gk] \tag{2}$$

which has to be read: the examination $\Diamond\eta_\Delta$ leads to void relative to the value gk of the aspect g, or the entity η_Δ and the aspect-value gk do not mutually exist.
If (2) holds for all the values gk, we write

$$[\Diamond_g \eta_\Delta \rightarrow \varnothing/g] \sim [\nexists g/\eta_\Delta, \nexists\eta_\Delta/g] \qquad (3)$$

and we say that the examination $\Diamond_g\eta_\Delta$ leads to mutual (relative) void, or that the entity η_Δ and the aspect g do not mutually exist.
If the nonexistence (3) is realized for all the aspect-views $\Diamond_g \in \Diamond$, we write

$$[\Diamond\eta_\Delta \rightarrow \varnothing/\Diamond] \sim [\nexists\Diamond/\eta_\Delta, \nexists\eta_\Delta/\Diamond] \qquad (4)$$

and we say that the operation $\Diamond\eta_\Delta$ leads to mutual (relative) void, or that the entity η_Δ and the view $\Diamond$ do not mutually exist.
If any *succession* of *two* operations $[\Delta R \rightarrow \eta_\Delta, \Diamond\eta_\Delta]$ leads systematically to the mutual inexistence (4) we write symbolically

$$\Delta \perp \Diamond \qquad (5)$$

and we say that the delimitator Δ and the view $\Diamond$ are mutually void or orthogonal or that the association $(\Delta, \Diamond)$ that has *a priori* been taken into consideration comes out *a posteriori* to be nonsignificant [which implies all the mutual inexistences (2), (3), (4), and (5)]. Finally imagine that we let now the *observer* "vary," permitting usage, by the conceptor, of any view $\Diamond$. If then the succession of two operations $[\Delta R \rightarrow \eta_\Delta, \Diamond\eta_\Delta]$, accomplished with all the various views $\Diamond$ that we are able to conceive of, leads systematically to the mutual inexistence (5), we write symbolically

$$\Delta \perp R \qquad (6)$$

and we say that *probably* (we will never be able to test for "all" the views, which notion is but *a false absolute*) the operation of delimitation Δ and R are mutually void or orthogonal. If, on the contrary, now, the examination $\Diamond_g\eta_\Delta$ does reveal a value gk of the aspect g (or several such values), we write

$$[\Diamond_g\eta_\Delta \neq \varnothing/gk] \sim [\exists gk/\eta_\Delta, \exists\eta_\Delta/gk]$$
$$[\Diamond_g\eta_\Delta \neq \varnothing/\Diamond_g] \sim [\exists\eta_\Delta/\Diamond_g, \exists\Diamond_g/\eta_\Delta] \qquad (7)$$

and we say that η_Δ and the aspect g do mutually exist, namely via that (those) value(s) gk.

If the relation (7) is realized for all the aspects $\Diamond_g \in \Diamond$, we write

$$[\Diamond\eta_\Delta \neq \varnothing/\Diamond] \sim [\exists\eta_\Delta/\Diamond, \exists\Diamond/\eta_\Delta] \qquad (8)$$

and we say that the entity η_Δ and the view $\Diamond$ do mutually exist.
In both cases of relative existence (7) and (8), we write

$$[\exists\Delta/\Diamond], \qquad [\exists\Diamond/\Delta] \qquad (9)$$

and we say that the association $(\Delta, \Diamond)$ *a priori* taken into consideration reveals itself *a posteriori* to be indeed significant. We can then also say, *a fortiori*, that Δ is not orthogonal on R.

The definitions (2)–(4), (7), (8), express the fact that a view can qualify only an entity that can contribute by "abstraction" to the *genesis* of this view: the essentially to-and-fro and spiraling character of the abstraction-qualification processes is explicitly expressed, and the naive *one*-way representations of the evolution of the conceptualizations are banished. "The" void or "the" negation $\varnothing$ as well as the existential quantifiers $\exists$ and $\nexists$ *split* into, respectively, a whole spectrum of relativized negations and of relativized existential quantifiers. One can feel already that *the notions of delimitator and view implanted in the substratum of logic, while they will found logic, will generate a relativization of it.* Finally, the formulation that follows (4) and leads to (5) stresses the following: We systematically leave open the possibility that the reiterated use of *one same* delimitator Δ shall produce entities η_Δ which, with respect to this or that particular aspect g, might reveal either a whole ensemble $\{gk, k = 1, 2, ...\}$ of *different* values gk, or quite on the contrary—systematically—relative void: the observable manifestations (epistemological content) of a fragment of reality introduced by a given delimitator cannot be *a priori* posited to be "determined" by (predictable from) this delimitator, *nor* by this delimitator associated with this or that view. This, as will become clear progressively in what follows, is one of the major implications encapsulated in the concepts of relative existence or inexistence defined above.

3.2.6. Physical Spacetime

Consider the ensemble $\{E\mathbf{r}, \mathbf{r} \in \mathbf{R}\}$ of values $E\mathbf{r}$ indexed by the vectors $\mathbf{r} \in \mathbf{R}$ that specify, in the usual sense, the position in the physical space E. The position vectors $\mathbf{r} \in \mathbf{R}$ are supposed to be measured with respect to some space-referential and making use of some given units of length and of angle. These units, by definition, are finite, whatever their value. So $\mathbf{R}$ is here a *discrete* ensemble of indexes. Furthermore, we choose it to be *finite*.

So $\{E\mathbf{r}, \mathbf{r} \in \mathbf{R}\}$ is here a discrete and finite set which allows us to introduce a "space-aspect" $\langle E \rangle$ with a structure (1). This aspect, furthermore, is of a semantic nature such that it *does* admit the definition of an *order.*

Consider also an ensemble $\{dt, t \in T\}$ of values t of the aspect d of *physical* duration. Such values can be determined only with the help of some clock incorporating some given unit of duration. This unit, whatever it be, is necessarily finite. Hence T is a *discrete* ensemble of indexes. We furthermore choose it also to be *finite.* Then $\{dt, t \in T\}$ is a discrete and finite set that permits one to define a "duration-aspect" $\langle d \rangle$ endowed with a structure (1). Moreover, again, the aspect thus defined is of a nature such that—*most* fundamentally in this case—it *does* accept the definition of an *order.*

We emphasize that the two classes of aspects defined above do not incorporate the inner spatial and temporal aspects that a human being perceives by introspection. The inner durations are certainly more basic than the physical ones, to the implicit elaboration of which they contribute (while the prime sources of the inner spaces, in a certain very intricate sense, probably lie in the physical world). Here, however, we ignore any genetic problem concerning the concepts of space and time and we work directly with the already very complex constructs called physical space and physical duration: Since one is permitted to define any view $\diamond$ and to consider any pairings $(\Delta, \diamond)$, the method—remarkably—permits to start above the most basic level of conceptualization where the very first epistemic referentials emerge, leaving open the possibility to go down to it later.

Let us form now a *physical spacetime view* (in short, a *spacetime view*) $\langle Ed \rangle = \{\langle E \rangle, \langle d \rangle\}$ consisting exclusively of a physical space aspect and a physical duration aspect. (I make use of the indefinite article "a" because one can form an infinity of such spacetime views, differing from one another by the magnitudes of the chosen unities and the number of the considered values (i.e., by the structure and the extension of the ensembles of indexes R and T), by the choice of the origins of space and of time, by the type and the orientation of the axes used in order to form the referential.) These preliminaries serve to introduce the following

> *Principle* 1: *The Frame Principle FP.* Consider an aspect-view $\langle g \rangle$ and a *physical* entity η_Δ delimited for future examination. Whatever $\langle g \rangle$ and η_Δ be, if the entity η_Δ exists in the sense of (7) with respect to the aspect-view $\langle g \rangle$, then it also exists in the sense of (8) with respect to at least one view $\langle g \rangle \vee \langle Ed \rangle$ formed by associating the aspect $\langle g \rangle$ with a spacetime view $\langle Ed \rangle$. But the entity η_Δ is *non-existent* in the sense of (4) with respect to any spacetime view which acts *alone*, isolated from any other aspect-view $\langle g \rangle$. This feature will be expressed by saying that the spacetime views are only "frame-views" which, by themselves, are "blind." Symbolically we write

$$\exists \eta_\Delta / \langle g \rangle \rightarrow [\exists \langle Ed \rangle : \exists \eta_\Delta / \langle g \rangle \vee \langle Ed \rangle]$$
$$\langle Ed \rangle \eta_\Delta \rightarrow \varnothing / \langle Ed \rangle, \quad \forall \langle Ed \rangle, \forall \eta_\Delta \tag{10}$$

Kant[11] has asserted that the human mind is such that it cannot conceive of (physical) "existence" outside space and time, which it introduces, intuitively and subjectively, as *a priori* "frames." The principle FP *isolates* and specifies more a particular feature of Kant's conception which, we think, it would be difficult to contest. By the very nature of the functional laws of his consciousness, any mature and normal human observer has acquired a constitution such that he perceives himself as being the center of a spatial frame of reference (nonquantified) and as involving a (nonquantified) referential of time. And his behavior with respect to these referentials is that one specified in FP: As soon as he perceives or imagines a *physical* entity, *ipso facto* he introduces at least one aspect-view $\langle g \rangle \neq \langle Ed \rangle$ relatively to which the entity exists in the sense of (7) and the values of which aspect-view he *combines* with spacetime values, *thereby* locating this entity inside his spacetime. But by the use of the spacetime aspects *alone*, devoid *strictly* of any adjuvant aspect-view $\langle g \rangle \neq \langle Ed \rangle$ (color, consistency, whatever), he is unable to perceive or to *imagine* a physical entity at all. He simply cannot extract it out of the background of spacetime values which, by themselves, form a "transparent" grid. (Einstein's approach blurs the fundamental distinction between the aspect g = mass (or the *entity* "matter" that exists in the sense of (7) with respect to the aspect g = mass) and the spacetime $\langle Ed \rangle$ view alone. More in fact: it inclines to contract the view $\langle g \rangle \vee \langle Ed \rangle$, with g = mass, into the spacetime $\langle Ed \rangle$ view alone. This leads to a sort of "objectified" spacetime, a spacetime curved by ghostified masses, thereby generating *very* much confusion.) The values of the spacetime aspects are conceivable and perceptible *only* by combination with some values of some other aspect, while the values of any other aspect $g \neq Ed$ irrepressibly emerge combined with some values of spacetime, even if fugitively, even if these spacetime values can be nonspecified, and even if *a posteriori* they can be abstracted away. This is a fundamental *fact*, a law of human mind, comparable with what gravitation is in the domain of physical reality. In order to be able to take this fact into account systematically, from now on we include a spacetime view in the view $\diamond \in (\Delta, \diamond)$ involved in any considered epistemic referential $(\Delta, \diamond)$. So the minimal number of aspects in the view from any epistemic referential is from now on 3: E, d, and at least one aspect $g \neq Ed$.

When a nonphysical, a conceptual entity is considered, it is always possible, if convenient, to conceive that this entity does not exist (is not specified) with respect to the values of the spacetime-aspect involved by the utilized epistemic referential: this is precisely what happens often in present-day logic.

The spacetime views $\langle Ed\rangle$ act as "displayers," as analyzers; they maximize the power of individualization of our factual perceptions or *even of our only conceptual* representations:

> *Principle* 2: *The Principle of Individualizing Mutual Exclusion* (*PIME*). Let $\eta_\Delta \leftarrow \Delta R$ be *one* purely physical outcome of one action ΔR of a purely physical delimitator Δ. Consider a view $\langle g\rangle \vee \langle Ed\rangle$ formed by associating an aspect $\langle g\rangle$ with a spacetime view $\langle Ed\rangle$. If the entity η_Δ exists in the sense of (the first equation) (7) with respect to a set of values $gk-\mathbf{r}-t$ from $\langle g\rangle \vee \langle Ed\rangle$ where the space values $\mathbf{r}$ form a subset $\{\mathbf{r}\}$ and the gk values form a subset $\{gk\}$, then *this* entity $\eta_\Delta \leftarrow \Delta R$ cannot *also* exist, in this same sense, factually as well as only conceptually, with respect to *another* set of values $gk-\mathbf{r}-t$ where
>
> — whatever be the subset $\{gk\}$, the space values $\mathbf{r}$ form a subset $\{\mathbf{r}\}' \neq \{\mathbf{r}\}$ but t designates *one* and *same* time-value;
>
> — all the spacetime values are the *same* but the gk values form a subset $\{gk\}' \neq \{gk\}$.

Like the frame-principle, PIME expresses a psychophysical *fact* that characterizes both our factual experience and our thinking.

3.2.7. Forms of Aspect-Values

The definitions 2–9 of relative existence and the frame-principle FP, added to the general definitions 1–5, yield finally a sufficient basis for a constructed answer to the question $\langle\,\rangle\eta_\Delta \rightarrow ?$

> *Definition* 7: *Relative Description*. Consider an observer endowed with an epistemic referential $(\Delta, \langle\,\rangle)$. Let η_Δ be an entity delimited for future examination. If η_Δ does exist in the sense of (7) or (8) with respect to the view $\langle\,\rangle$, then the examination $\langle\,\rangle\eta_\Delta$ reveals to the observer a certain particular *structure* of values gk of aspects g, $\langle g\rangle \in \langle\,\rangle$: certain associations of values gk of aspects g which are permitted by the view $\langle\,\rangle$ do not arise for η_Δ; others, on the contrary, arise in certain correlated ways that define the asserted peculiar structure. This structure is called a *description of the entity* η_Δ *relatively to the view* $\langle\,\rangle$, in short, a *relative description* of η_Δ, and it is denoted by the symbol $D(\Delta, \eta_\Delta, \langle\,\rangle)$. We write
>
> $$\langle\,\rangle\eta_\Delta \rightarrow D(\Delta, \eta_\Delta, \langle\,\rangle)$$

The notation $D(\Delta, \eta_\Delta, \langle\,\rangle)$ emphasizes that any description involves a triad $(\Delta, \eta_\Delta, \langle\,\rangle)$ to which, fundamentally, it is relative. The distinction—by the *separate* specification, in the argument of D, of Δ and of η_Δ—between the relativity with respect to Δ and the relativity with respect to η_Δ, draws permanent attention upon those among the aforementioned features of this approach of which the novelty and importance are essential. Namely, that[3] (pp. 264–269):

— It would be at the same time devoid of significance, inconsistent, and often factually false, to posit *a priori* and absolutely that all the results of the reiterations of the operation $\Delta R \rightarrow \eta_\Delta$ realized with a fixed delimitator Δ are identical "in themselves" (independently of *any* view $\langle\,\rangle$) "because" the delimitator is each time the same.

— It would equally be an arbitrary restriction and a false absolutization to posit *a priori* that the reiterations of a succession of the *two* operations $[\Delta R \rightarrow \eta_\Delta, \langle\,\rangle\eta_\Delta \rightarrow D]$ certainly leads always to identical descriptions D if *both* epistemic operators, the delimitator *and* the view, are each time the same. (For instance: Suppose that the produced entity η_Δ is a physical one. The acting view $\langle\,\rangle$, by definition, includes a *finite* spacetime view. This spacetime view might possess a structure (1) (cardinals of the ensembles of indexes T and $\mathbf{R}$) *such* that it is able to cover—during *one* act of examination $\langle\,\rangle\eta_\Delta$—only a spacetime domain of which the extension is *smaller* than that one revealed later—via precisely the examinations $\langle\,\rangle\eta_\Delta$—by "the whole" entity η_Δ. If this happens, the various examinations $\langle\,\rangle\eta_\Delta$ from a sequence of reiterations of the succession of two operations $[\Delta R \rightarrow \eta_\Delta, \langle\,\rangle\eta_\Delta \rightarrow D]$ will produce descriptions that are different when the involved examinations $\langle\,\rangle\eta_\Delta$ concern *different portions* of the delimited entity η_Δ, in spite of the fact that the utilized delimitator and view are each time the same.) More generally, though all the descriptions produced by reiterations, with some *fixed* pair Δ and $\langle\,\rangle$, of the succession of the two operations $[\Delta \mathbf{R} \rightarrow \eta_\Delta, \langle\,\rangle\eta_\Delta \rightarrow D]$, *can* happen to come out to be identical, quasicertainly *this cannot happen for any choice* $(\Delta, \langle\,\rangle)$. If it did, this would seem miraculous.

In these conditions the two aforementioned tendencies to posit that Δ, or $(\Delta, \Diamond)$ entirely determine (permit to predict) the corresponding description D, have to be eradicated. Therefore it is indeed *necessary* to introduce in the argument of the symbol $D(\Delta, \eta_\Delta, \Diamond)$ a *separate* reference to each one of the three elements $\Delta, \eta_\Delta, \Diamond$.

By construction, any relative description is itself distinct from the delimitator, from the object-entity, and from the view involved by it, to all three of which it is conceptually posterior, while the three enumerated elements are distinct from each other—*always* by their descriptional *roles*, and in general also by their content. And again by construction, the concept of a relative description bears the mark of the deliberate *finitistic* character which characterizes the epistemic operators Δ and $\Diamond$: Because the ensemble of values *gk* of any aspect *g* is discrete and finite by definition, and because any view contains by definition a finite number of aspect-views, any examination $\Diamond\eta_\Delta$ can produce only a finite ensemble of qualifications.

3.2.8. Forms of Spacetime-Aspect-Values

The case, particular but very important, of the descriptions of *physical* entities, can now be singularized as follows. Consider an observer endowed with an epistemic referential $(\Delta, \Diamond)$, and let η_Δ be a physical entity delimitated for examination. In consequence of the frame principle FP (10) we have by convention $\Diamond \supset \Diamond_{Ed}$.

Definition 7′: Relative Description of a Physical Entity. If η_Δ is a *physical* entity and if it exists in the sense of (7) or of (8) with respect to the view $\Diamond \in (\Delta, \Diamond)$, the frame principle FP (10) entails that the examination $\Diamond\eta_\Delta$ reveals to the observer a "form" determined by values *gk* of the aspects *g*, $\Diamond_g \in \Diamond$, displayed on the ordered spacetime grating involved by the spacetime view $\Diamond_{Ed}$ contained in the view $\Diamond$, i.e., it reveals a configuration of $gk - \mathbf{r} - t$. We call this form a *relative description of the physical entity* η_Δ and we indicate it by the *same* symbol $D(\Delta, \eta_\Delta, \Diamond)$ used for any description.

Via the definition 7′ the PIME acts upon all the descriptions of physical entities: two *different* relative descriptions of a physical entity, performed inside one *same* epistemic referential, and both tied with one *same* time value, cannot both exist. *Prior* to any question of *truth* concerning these two descriptions—themselves regarded as objects of examination via a "truth-view"—their very factual *existences* at one same time are mutually exclusive. This is a psychophysical fact. This fact is bound to play a fundamental role in an explicit relativized reconstruction of logic. Indeed, for any pair of descriptions of which the factual existences are mutually exclusive, the definition of a logical product is devoid of semantic counterpart.(9) *This restricts strongly the basic relevance of lattice structures.*

3.2.9. Cells of Relativized Descriptional Language

A human observer, in the presence of reality, is condemned to parcelling examinations. The successivities inherent in human mind, the spatial confinements imposed by the bodily senses (whatever prolongations are adjusted to them), and the absence of limitation of what is called reality, compose together a configuration which imposes the fragmentation of the epistemic search. On the other hand, any fragment selected or produced out of the changing continuum of "reality" admits an infinity of different sorts of examinations. Furthermore, *any newly accomplished qualification multiplies the conceivable qualifications*, raising the question of the relations with itself. These confinements and these endless and changing vistas call forth hastes or panics of the mind, which entangle false problems. These knots have to be hindered. We want to build for the mind a free, an indefinitely organizing penetrability into any nook of this substance of the knowable where mind is immersed and of which mind thickens the texture by ceaseless complexifications. But how can this aim be reached? Only an appropriate *methodological* decision could meet this challenge.

Let us go back to the definition of a relative description.

Each relative description is essentially referred to one triad $(\Delta, \eta_\Delta, \Diamond)$. The relativity to this triad *limits* the capacity of information of the considered description.

Relativity and limitation are indissolubly tied to one another.

Any one relative description, we saw, is a confined cell of language able to produce only a finite number of qualifications all concerning only one class of objects (those introduced by one fixed delimitator). This confinement, however, this dam incorporated in any given description, is constantly exposed to founder under the nondominated fluxes of the epistemic actions. The human minds are exposed to whirls of implicit interrogations which generate an imperious tendency to *fluctuate* between different operations of delimitation, different object-entities, different views, a tendency to work out simultaneously several different descriptions. But as soon as several different relative descriptions are attempted simultaneously, the roles and the contents of the delimitators, the views and the object-entities involved, dispose of a ground for oscillation. And then the oscillations actually happen because it is very difficult to perceive them, so *a fortiori* to hinder them. So the different descriptions that are attempted simultaneously get mixed, and in general none of them can be achieved. Their superposition ends up in a knot of miscomprehensions that blurs and stops the conceptualization. So it is necessary to erect high and solid ramparts between two distinct descriptions. For this purpose I pose the following *methodological* "principle" (a *norm*, a rule of epistemic behavior):

> *Principle* 3: *The Principle of Separation PS.* Since *each* relative description $D(\Delta, \eta_\Delta, \Diamond)$, whatever its complexity, involves by definition one delimitator, one object-entity and one view, distinct from one another as well as from the description, as soon as *any* change either of role or of content is introduced in the triad $(\Delta, \eta_\Delta, \Diamond)$ *another* description emerges: *this other description has to be treated separately.*

In the syntax of the processes of relativized conceptualization the systematic observance of the principle of separation plays a role analogous to that played by the word "stop" or by the sign "." in the transmission of messages. The principle of separation delimits the own domain of one commenced description. Comparable to Mendeleev's laws or to Pauli's principle, it announces its saturation. It rings the bell as soon as all the qualifications have been exhausted that bear on the object-entity η_Δ delimited by the delimitator Δ acting inside that description, and which can be achieved via the view $\Diamond$ operating inside that description. It announces that from now on, if one desires to complexify further the descriptional tissue produced by the description that has been achieved, one has to start a new description, specifically appropriate for the conceived supplementary aim; that if one continues to inertly stay inside the same epistemic referential, trying to make it produce more than it can, he will hit an invisible but indestructible frontier that will manifest itself by falsely absolute impossibilities, by paradoxes, by boundary-scandals. Such descriptional frontiers, however, can be always transgressed. The separations commanded by the principle of separation are not amputating, nor definitively parceling. Quite on the contrary, they ensure a maximal and governed utilization of the capacities of conceptualization *and* of unification. For instance, consider a description $D(\Delta, \eta_\Delta, \Diamond)$. The delimitator Δ, the view $\Diamond$, and the object-entity η_Δ have been specified and on this basis there emerged qualifications of the object-entity η_Δ. But exclusively of *it*. According to the definitions introduced here, a delimitator Δ and a view $\Diamond$ cannot be qualified inside a description where they act, respectively, as a delimitator and a view. So if one researches qualifications of also this delimitator Δ or of this view $\Diamond$, one has to organize another description where this time the delimitator Δ or the view $\Diamond$ will be the object-entity, or part of the object-entity. But *nothing* hinders one to con-

struct such a description. In this sense the principle of separation permits one to penetrate *inside* a *preceding* description, to "split" it *a posteriori* in a "legal" way, to creep *beneath* it and to work out specifications concerning the epistemic operations that brought forth this description, so specifications concerning its genesis, thus enveloping it in a certain sense. The principle of separation permits one to *transgress any one-way descriptional order*; it permits reflexive, self-referential, *to-and-fro* epistemic actions. Thereby it controls self-organizing complexifications comparable to those involved in living systems.

3.2.10. Relative Metadescription

In particular, the principle of separation permits one to also raise himself up "above" a preceding description, to surpass it by reconsidering it *globally* as a new object-entity, either isolated or in connection with other entities, and by examining it by some conveniently enriched view. This possibility can be realized by the help of a relativized variant of the well-known and central concept of metadescription:

> *Definition* 8: *Relative Metadescription.* Consider a *conceptual* delimitator $\Delta^{(2)}$ which selects as an object for future qualification any ensemble $E^{(2)}_\Delta = E^{(2)} = \{D(\Delta, \eta_\Delta, \Diamond)\}$ of previously realized descriptions. Let $\Diamond^{(2)}$ be a view with respect to which all the descriptions from $E^{(2)}_\Delta$ do exist in the sense of (8), and containing any desired meta-aspects of identity-difference with respect to values of aspects from the initial views $\Diamond$, thus permitting "comparisons" between the $D(\Delta, \eta_\Delta, \Diamond) \in E^{(2)}$. The description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \Diamond^{(2)})$ will be called a *metadescription relative to the ensemble of descriptions* $E^{(2)}_\Delta = E^{(2)} = \{D(\Delta, \eta_\Delta, \Diamond)\}$.

Each relativized metadescription endows us with a new and preorganized space of conceptualization, hierarchically connected with a preceding one. Inside this new descriptional space it becomes possible to unfold a whole category of apparent problems and paradoxes that emerged on the previous (lower) level of the descriptions $D(\Delta, \eta_\Delta, \Diamond) \in E^{(2)}$, and to resolve them accordingly to *algorithms* that *connect*, that *link* them inside a conveniently enriched semantic volume where they are "explained" away as illusions produced by the squeezing of a growing semantic substance, against an unavoidable but *provisional* frontier that delimited an insufficient conceptual volume where the links were not perceptible, were hidden outside that volume. And since a description $D \in E^{(2)}$ is *any* description, it can be itself a relative metadescription. So it is possible to develop an infinite number of nonlimited hierarchies of descriptions of increasing complexity. On each new level the choices of the new delimitator and the new view amount to a free redefinition of the *direction* (the aim) of the desired new segment of conceptualization.

3.2.11. Freely Orientable Descriptional Trajectories

To sum up: The principle of separation PS separates in order to permit one to link "legally" and indefinitely, according to any descriptional aim. Combined with a definite initial relative description and a definite descriptional aim, it determines progressively, with a remarkable sort of necessity, a sequence of specifically appropriated epistemic metareferentials and corresponding relativized metadescriptions, as two points determine a line. It guides the successive introduction of "locally" enriched epistemic volumes permitting one to change *ad libitum* the focus of description or its extension (by convenient choices of delimitators), the degree of analysis or synthesis (by convenient choices of views). In this way unlimited branchings of increasingly complex and unifying descriptional structures can be developed, which can be directed beneath or above an initial description, according to the desired descriptional aim. Thereby the method of relativized conceptualization frees us from the necessity to introduce, as in Russell's or Tarski's approaches, hierarchies of vast "languages" that are semantically neutral, *nonspecific* with respect to a given, "local" descriptional aim; it permits one to construct freely orientable *descriptional trajectories* formed of hierarchic descriptional cells condensed around the successive phases of precisely *this* aim of conceptualization.

3.2.12. The Relativizing Epistemic Syntax $[\Delta, \eta_\Delta, \diamond, D]$

The kernel of the method of relativized conceptualization is now entirely exposed. It sketches out a *relativizing epistemic syntax* $[\Delta, \eta_\Delta, \diamond, D]$, the "[delimitator, object-entity, view, relative description]-syntax. This syntax generates a typology of relativized descriptions which is briefly indicated below. This typology constitutes the researched general framework where, in principle, the relativized elaborations of any descriptions or systems of descriptions can be located and compared.

3.3. Fundamental Types of Relativized Descriptions

Throughout what follows we restrict ourselves exclusively to descriptions which—basically—concern *physical* entities. (We shall examine elsewhere whether any description can be absorbed into this class).

3.3.1. The Initial Stage of Any Descriptional Chain

Our first question is this: How does human mind *penetrate* into the domain of descriptions? What are the *primary* descriptions?

Transferred Description. Consider an observer endowed with an epistemic referential $(\Delta, \langle T\rangle)$ where:

— Δ is a purely physical operation—biological or *not*—which delimits physical and as yet strictly nondescribed entities η_Δ;

— $\langle T\rangle$ is a view such that every aspect-view $\langle g\rangle \in \langle T\rangle$ involves an aspect g consisting of a union of values gk which, themselves, are features of a material object for "g-registrations" (a "g-apparatus"), *variable* with g, features that are *created* and become perceivable *on* this g-registering object, in consequence of the interactions between it and the entities η_Δ delimited by the delimitator $\Delta \in (\Delta, \langle T\rangle)$ ("measurements of the aspect g on entities η_Δ"), so in consequence of the operations of examination $\langle g\rangle\eta_\Delta$. A view $\langle T\rangle$ of the type just specified will be named "a transfer view." The epistemic referential $(\Delta, \langle T\rangle)$ will be called a *basic epistemic referential.* Any description of the physical entity η_Δ generated by a basic epistemic referential will be called a *transferred description* and will be denoted $D(\Delta, \eta_\Delta, \langle T\rangle)$.

So by definition any description generated with a basic epistemic referential involves exclusively features of registering objects *distinct* from the physical entity η_Δ delimited for examination.

At a first sight the concept of a transferred description might seem particular, and too radical. But in fact it possesses absolute priority and nonrestricted generality inside the order of cognitive elaborations: *Any* entity delimited by *any* delimitator, *if* it *does* mark the consciousness of an observer, marks it *first* via a certain particular category of transferred descriptions, namely descriptions transferred on the domains of sensitiveness of the observer's body. Kant, Poincaré, Einstein and Quine have founded famous analyses on the explicit recognition of this fact. And if—more generally now—the transferred view $\langle T\rangle \in (\Delta, \langle T\rangle)$ does not involve these biological terminals, the nearest and which cannot be eliminated, if this view is formed with registering aspects of objects that are *exterior* to the observer's body, then the corresponding description belongs to the generalized type of transferred description defined above. This description constitutes then an intermediary object $[\eta'_{(\Delta)} = D(\Delta, \eta_\Delta, \langle T\rangle)]$ which, if it is perceivable by the sensorial "views" (in our sense) of the observer's body, can *found* the access of the entity denoted η_Δ, to the observer's consciousness, marking the 0-point of a chain of conceptualization of this entity. Notice that this situation is quite *systematically* encountered in microphysics: a microsystem is never directly perceivable; it produces, on macroscopic registering devices, marks that are perceivable by the sensorial views of the observer's body. In any case, it is crucial to recognize that:

> *A transferred description is a first phase* universally *traversed by* any *representation of a physical entity.*

Now, what sort of "form"—in the sense of the general definition of the relative description of a physical entity—can a transferred description generate? The transfer-view $\langle T\rangle$ which acts in a basic epistemic referential $(\Delta, \langle T\rangle)$ contains a certain finite number m of aspects which are distinct from the two frame-aspects E and d contained in $\langle T\rangle$ and are indexed by $g=1, 2,..., m$, $m \geqslant 1$, (see the convention introduced on the basis of the frame-principle FP). In general $g>1$. Now, every aspect-view $\langle g\rangle \in \langle T\rangle$ corresponds by definition to a *physical* operation of transfer-examination $\langle g\rangle\eta_\Delta$ of η_Δ via the transfer-aspect-view $\langle g\rangle$ (a physical interaction between what is labeled η_Δ and an apparatus for g-registrations). But:

> It is not possible *in general* to realize simultaneously *all* the examinations $\langle g\rangle\eta_\Delta$ corresponding to *all* the aspect-views $\langle g\rangle \in \langle T\rangle$, on the result η_Δ of *one* single realization of the operation $\Delta R \rightarrow \eta_\Delta$ (to act on *one* single outcome $\eta_\Delta \leftarrow \Delta R$, involving a definite spacetime support, simultaneously, in *various* manners which *themselves* involve *various* spacetime supports).

Indeed: Inside *another* convenient epistemic referential, different from $(\Delta, \langle T\rangle)$ (see PS), each operation of transfer-examination $\langle g\rangle\eta_\Delta$ can *itself* hold the role of a physical object-entity that can be described by the help of some appropriate view. According to the frame-principle FP such a description involves necessarily some spacetime support $\delta r(\langle g\rangle\eta_\Delta)\,\delta t(\langle g\rangle\eta_\Delta)$ of values from some spacetime view. The current definition of the notion of "operation on" requires that with respect to this spacetime view the spatial support of the operation $\langle g\rangle\eta_\Delta$ *on the entity* η_Δ intersects the spatial support $\delta r(\eta_\Delta)$ of η_Δ at any time $t \in \delta t(\langle g\rangle\eta_\Delta)$. This is a restriction on the transfer-aspects $\langle g\rangle \in \langle T\rangle$. But the definition of a transferred description involves no restriction whatever concerning the spacetime support of the operations $\langle g\rangle\eta_\Delta$, $\langle g\rangle \in \langle T\rangle$. So in general the operations $\langle g\rangle\eta_\Delta$ can have different spatial supports. Associated with the PIME, this entails that in general the transfer-aspects $\langle g\rangle$ from the basic view $\langle T\rangle$ *separate* in spacetime. The set of the transfer aspect-views $\langle g\rangle \in \langle T\rangle$ *branches out* into a number $1 \leqslant l \leqslant m$ of subsets of aspect-

views $\langle g\rangle \in \langle T\rangle$ which, with respect to *one* realization of the epistemic action $\eta_\Delta \leftarrow \Delta R$, are *mutually incompatible*. But all the examinations via aspect-views $\langle g\rangle$ belonging to *one* of these subsets *are* realizable simultaneously on the result of one single realization of the epistemic action $\eta_\Delta \leftarrow \Delta R$, i.e., they *can* constitute together one single, more complex examination. In this sense they are mutually compatible. Let us denote by $\langle b\rangle$, $b=1, 2,..., l$, $1 \leqslant l \leqslant m$, such a more complex sub-examination, simultaneously, by all the compatible aspects from one "branch" (subset) and let us call it a "branch-view" from $\langle T\rangle$. The $1 \leqslant l \leqslant m$ mutually incompatible branch-views obtained in this way constitute a partition of $\langle T\rangle$: $\langle T\rangle = \vee_b \langle b\rangle$. From this it follows that, in order to accomplish *one* complete transferred description of "the" entity η_Δ, it is necessary to *reiterate* the operation of delimitation $\Delta R \rightarrow \eta_\Delta$ a number of times $1 \leqslant l \leqslant m$, completing it *successively* by the $1 \leqslant l \leqslant m$ mutually incompatible branch examinations $\langle b\rangle\eta_\Delta$. In other terms, in order to entirely achieve *one* transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$ one must accomplish separately, successively, all the distinct $1 \leqslant l \leqslant m$ sequences of two operations $[\Delta R \rightarrow \eta_\Delta, \langle b\rangle\eta_\Delta]$, $b=1, 2,..., l$. This leads in the end to a treelike spacetime structure of the ensemble $\{[\Delta R \rightarrow \eta_\Delta, \langle b\rangle\eta_\Delta], b=1, 2,..., l\}$ of sequences of two epistemic processes which determines one transferred description. Figure 2 represents an example with three branches.

As a whole, the structure defined above is a [potential-actualization-actualized] structure that will be called "the transfer-tree of the basic epistemic referential $(\Delta, \langle T\rangle)$." The elements of this structure, the *fibers* of which it consists, are the elementary transfer-processes [an operation of delimitation $\Delta R \rightarrow \eta_\Delta$, an operation of transfer-examination $\langle b\rangle\eta_\Delta$, a registration of a transferred value bk of the b-registration device], in short, an "elementary transfer chain" $[\Delta R \rightarrow \eta_\Delta, \langle b\rangle\eta_\Delta \rightarrow bk]$.

3.3.2. On the $[\Delta, \eta_\Delta, \langle\,\rangle, D]$ Dynamics of Conceptualization

Consider now the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. This description will certainly *not* be perceived as satisfactory, as *final*. Each branch of the tree of the basic epistemic referential $(\Delta, \langle T\rangle)$ corresponds to a registering-object specific of that branch, a $\langle b\rangle$-apparatus. So the values gk of the transferred aspects g, $\langle g\rangle \in \langle T\rangle$, are perceptible *on* the l different domains of space occupied by l different registering devices, $1 \leqslant l \leqslant m$. Furthermore, notwithstanding the fact that the origin of times is reestablished after each sequence of two operations $[\Delta R \rightarrow \eta_\Delta, \langle g\rangle\eta_\Delta]$, the gk-values produced by these sequences appear in general after *different* times $t(g)$. This entails in general different durations $t(b)$ for the different branch-descriptions $D(\Delta, \eta_\Delta, \langle b\rangle)$. In short, the form of gk-spacetime

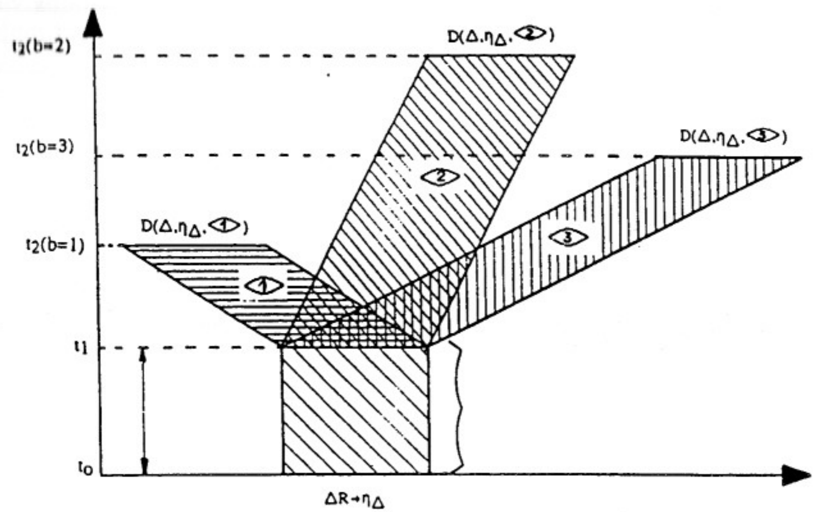

Fig. 2. The transfer tree of a basic epistemic referential (Δ, ⟨T⟩). The operation of delimitation $\Delta R \rightarrow \eta_\Delta$ (common) generates the trunk of the structure, *a monolith of nonexpressed and unknown but physically determined potentialities labeled by the symbol η_Δ and relative to the operation Δ alone.* The operation of delimitation Δ is identically reiterated for all the sequence [$\Delta R \rightarrow \eta_\Delta$, ⟨b⟩$\eta_\Delta$]. It begins at an initial moment t_0, always the same with respect to the origin of times *reestablished* after each sequence, and it lasts until a time $t_1 > t_0$. From the moment t_1 on, the spacetime supports of the epistemic operations which lead to a transferred description of the entity η_Δ *separate* into $1 \leqslant l \leqslant m$ branches, one for each one of the sub-examinations ⟨b⟩η_Δ, where several examinations ⟨g⟩η_Δ simultaneously realizable on a result η_Δ of *one* single operation of delimitation $\Delta R \rightarrow \eta_\Delta$ are combined. All the different examinations ⟨b⟩η_Δ begin at the same time t_1 when the operation of delimitation $\Delta R \rightarrow \eta_\Delta$ ends (with respect to the origin of times reestablished after each sequence [$\Delta R \rightarrow \eta_\Delta$, ⟨b⟩$\eta_\Delta$]), but each one of the examinations ⟨b⟩η_Δ *finishes* at a specific time $t(b)$, $b = 1, 2, \ldots, l$, bringing forth a value bk of a b-qualification of the utilized b-registration device. Thus each branch examination ⟨b⟩η_Δ is *a process of actualization* of *a part* of the potentialities contained in the monolith of potentialities symbolized η_Δ, namely, those bk which consist of a configuration of gk-spacetime values relative to the partial transfer view ⟨b⟩ and thus to the corresponding b-registration device. In contradistinction to the process of delimitation (creation) $\Delta R \rightarrow \eta_\Delta$ which is relative to the operator Δ alone, the process of actualization ⟨b⟩η_Δ is relative to *both* the operation of delimitation Δ *and* the view ⟨b⟩. At the top of each branch b, the operation of actualization ⟨b⟩η_Δ produces *globally* the *actualized* result consisting of the partial transferred relative description $D(\Delta, \eta_\Delta$, ⟨b⟩$)$, $b = 1, 2, \ldots, l$, "the branch description b."

values defined by a transferred description of an entity η_Δ is in general a shattered form, a form scattered on a nonconnected domain of the ordered spacetime grating ⟨Ed⟩ included in the view ⟨T⟩, a form which in general *does not even permit the definition of a law of evolution, of an own global temporal order of what is labeled η_Δ*. In such conditions *how* can we ascertain even only the *existence* of some own *significance* for the assertion that the achieved description concerns indeed an (one) "entity η_Δ," and an entity η_Δ *different* from all the operations, devices, and registering objects *whose* features—*exclusively*—contribute to that description? The description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$ tells strictly *nothing* concerning how what is labeled "the entity η_Δ" is, *itself, intrinsically*. Obviously, as soon as a transferred description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$ is achieved, we are confronted with a question of "interpretation" involving this transferred description itself and its relation with the verbal label "the entity η_Δ." In this way begins inside this method a specific sort of search on the "genesis of significance," to be compared and associated with similar inquiries developed along different lines by a continuously increasing number of other authors.

A remarkable fact comes into light from the start: The entity labeled η_Δ will *not* be kept inside the realm of the conceptualized, if, when one *reiterates* the *global* epistemic action which establishes the transferred description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$, no sort of *invariance* emerges. Indeed we find out—as we would find out that this plate is broken!—that, if no invariance whatever were brought forth by reiterations of, globally, the whole description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$, we would *a posteriori* retire to the ensemble of data symbolized by $D(\Delta, \eta_\Delta$, ⟨T⟩$)$ the qualification of "description of an entity η_Δ," even though *a priori* we did endow this ensemble of data with this qualification. So this was only a provisional, a conditional endowment, implicitly subject to subsequent tests; a kind of tactical labeling, just in order to obtain a working-ground on which to hoist up our understanding so that *afterwards* we might be able to decide which direction has to be retained for the fragment of conceptualization that we try to build. The emergence of some invariance tied with reiterations of the description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$ appears to play the role of a sort of proof of *existence* deciding whether or not what has been tentatively labeled η_Δ deserves further attention.

So, it seems, we must now examine reiterations of the considered transferred description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$, i.e., a whole set of realizations of this description. But why? Because we perceive more or less implicitly that when we define an aspect-view ⟨g⟩ corresponding to an aspect g = "variance" endowed with a value $g1$ = "invariant" and a value $g2$ = "not invariant," not only the still strictly nonqualified entity η_Δ that was the object of the transferred description $D(\Delta, \eta_\Delta$, ⟨T⟩$)$, but even this description itself, are *inexistent* in the sense of (3) with respect to this aspect. Accordingly to usual language the aspect-view $\langle g\rangle$ = "variance" exists in the sense of (7) only with respect to an entity which: (a) is already *prequalified* by some *other* aspect or view, i.e., consists of some already previously accomplished *descriptions*, not of still strictly unqualified objects; (b) consists of *at least two* descriptions, and in general of a more rich set of descriptions, so that comparisons are possible. This *imposes* indeed the study, now, of a *set* $\{D(\Delta, \eta_\Delta, \langle T\rangle)\}$ of descriptions, if we want to avoid fictitious "scandals"—paradoxes, impossibilities—generated by a nonperceived violation of the definition 7′ according to which a description of any given entity, by any given view, emerges only if this entity and this view do mutually exist in the sense of (7) or of (8). So the object of examination has changed. Then, accordingly to the principle of separation PS, *another* description has to be built in order to qualify this new object, a convenient metadescription placed on a metalevel. *The method literally ejects us on a metalevel.* We are in the presence of an illustration of the way in which the association [relative existence + PS + relativized metadescription] induces a specific sort of dynamics of relativized conceptualization.

3.3.3. The Relativities of Statisticity

Imagine then an ensemble of N reiterations of the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. *Each* description $D(\Delta, \eta_\Delta, \langle T\rangle)$, in its own turn, involves the realization of *all* the sequences of two operations $[\Delta R \to \eta_\Delta, \langle g\rangle\eta_\Delta \to \eta_{gk}]$ (where η_{gk} is an abbreviation for $D(\Delta, \eta_\Delta, \langle g\rangle)$: a gk-qualified entity η_Δ), corresponding to all the aspect-views $\langle g\rangle \in \langle T\rangle$ (grouped in mutually incompatible subsets). Let us symbolize more synthetically by the writing $[\Delta R \to \eta_\Delta, \langle T\rangle\eta_\Delta \to D(\Delta, \eta_\Delta, \langle T\rangle)]$ this set of sequences leading to *one* description $D(\Delta, \eta_\Delta, \langle T\rangle)$. And let us symbolize N reiterations of the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$ by the writing $E^{(2)} = \{[\Delta R \to \eta_\Delta, \langle T\rangle\eta_\Delta \to D_j(\Delta, \eta_\Delta, \langle T\rangle)], j = 1, 2,..., N\}$ where j labels the description produced by the jth reiteration. Now, what sort of invariance can be expected concerning this metaentity $E^{(2)}$ consisting of all the N reiterations of the description $D(\Delta, \eta_\Delta, \langle T\rangle)$?

The type of invariance which comes first into mind is the identity of all the descriptions $D_j(\Delta, \eta_\Delta, \langle T\rangle)$. However—and it is very important to realize this fully—this would be an entirely arbitrary presupposition. Some *other* sort of "invariance" might arise as well, or *none*. So, accordingly to the method applied here, the only way toward capturing perhaps a precise definition of some invariance concerning what we have provisionally labeled "one entity η_Δ" is to effectively construct and examine the pertinent metadescription of the metaentity $E^{(2)}$, *without* in any way prejudging the results that will arise. And notice that what is at stake here is huge: $D(\Delta, \eta_\Delta, \langle T\rangle)$ labels any transferred description, so any first phase of any access to knowledge of any physical entity η_Δ. In the absence of the emergence of a precise definition of some possible invariance connected with a label η_Δ, the foundation of any reasoning on the physical world dissolves, and even the foundation of any coherent language.

One realization of the succession $[\Delta R \to \eta_\Delta, \langle T\rangle\eta_\Delta \to D(\Delta, \eta_\Delta, \langle T\rangle)]$ of epistemic operations brings forth one description $D(\Delta, \eta_\Delta, \langle T\rangle)$. This by definition consists of a certain *configuration* of qualifications gk, $\forall\langle g\rangle \in \langle T\rangle$, displayed on the spacetime support of the spacetime frame-view $\langle Ed\rangle$ contained in $\langle T\rangle$. By their association with spacetime values from the spacetime frame-view $\langle Ed\rangle$, these qualifications gk generate a certain form of spacetime-gk-values. Let us label *globally* by h this form of spacetime-gk-values. We do not know whether, when the descriptional action $[\Delta R \to \eta_\Delta, \langle T\rangle\eta_\Delta \to D(\Delta, \eta_\Delta, \langle T\rangle)]$ is reiterated N times, the obtained forms $D_j(\Delta, \eta_\Delta, \langle T\rangle) = h_j$ ($j = 1, 2,..., N$, j the index of order of the reiteration) will or will not come out to be all identical. So let us introduce the notation $D_j = h$, $h = 1, 2,..., L$, $L \leqslant N$, in order to express that we leave open the possibility that the index h will vary from one reiteration of the description D to another one, thus indicating a certain number L of different results. We now define:

Individual Description or Statistical Description. Let $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ be a metadescription where:

- $E^{(2)} = \{D_j(\Delta, \eta_\Delta, \langle T\rangle) = h\}$ is an ensemble of results of N reiterations of the elaboration $[\Delta R \to \eta_\Delta, \langle\ \rangle\eta_\Delta \to D_j(\Delta, \eta_\Delta, \langle T\rangle)]$ of the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$ ($j = 1, 2,..., N$: the index of *order* of the result, $h = 1, 2,..., L$: the index of *content* of the result, $L \leqslant N$).
- The metadelimitator $\Delta^{(2)}$ is a *conceptual selector* which selects $E^{(2)}$ as object of examination.
- The metaview $\langle S\rangle^{(2)}$ is a *global statistical metaview* with respect to which $E^{(2)}$ exists in the sense of (8) and which possesses the following structure:

  $\langle S\rangle^{(2)} = \bigvee_g \langle Sg\rangle^{(2)}$, $\forall\langle g\rangle \in \langle T\rangle$, with $\langle Sg\rangle^{(2)}$: a *statistical metaview relative to* $\langle g\rangle$ possessing in its turn the following structure:

  $\langle Sg\rangle^{(2)} = \langle g\rangle \vee \langle ng\rangle^{(2)}$, $\langle ng\rangle^{(2)}$: the "metaview of g-population" corresponding to the "aspect ng of g-population," of which the values are defined as follows: From each description $D_j(\Delta, \eta_\Delta, \langle T\rangle) = h$, filter out exclusively the subconfiguration $h(g)$, $h(g)=1, 2, \ldots, L(g)$, $L(g) \leqslant L$, of the qualifications of spacetime-gk-values of the considered aspect g *alone*; then estimate, inside the ensemble of the N results $D_j = h$, the relative frequencies $n(gh)/N$ of occurrence of the different identified subconfigurations $h(g)$ where the value of the index h is bounded this time by the number $[L(g)=h(g)] \leqslant L$ (which transforms h in $h(g)$); these relative frequencies $n(gh)/N$ are the values of the aspect ng of g-population.

If the global examination $\langle S\rangle^{(2)} E^{(2)}$ produces for *all* the aspects $\langle g\rangle \in \langle T\rangle$ a Dirac (dispersion-free) distribution of the corresponding numbers $n(gh)/N$, i.e., if one finds for every aspect $\langle g\rangle \in \langle T\rangle$ one content-value h_i such that $(n(gh_i)/N)=1$ and $n(gh)/N=0$ for $h \neq h_i$, then the descriptions D_j are all identical. In this case we shall say that the initial description $D(\Delta, \eta_\Delta, \langle T\rangle)$ is an *individual transferred description of the entity* η_Δ while $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ has come out to be an equally individual metadescription, namely of the individual description $D(\Delta, \eta_\Delta, \langle T\rangle)$ of the entity η_Δ. If, on the contrary, the examination $\langle S\rangle^{(2)} E^{(2)}$ reveals for at least one aspect $\langle g\rangle \in \langle T\rangle$ a nonnull dispersion of the numbers $(n(gh)/N)$, then the descriptions D_j are not all identical. In this case we shall say that the initial description $D(\Delta, \eta_\Delta, \langle T\rangle)$ is an unstable form while the metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ is a *statistical metadescription of the initial description* $D(\Delta, \eta_\Delta, \langle T\rangle)$, or more simple, a *statistical relative description of the entity* η_Δ.

The new concepts of an individual or a statistical relative description bring into evidence all the distinct conceptual levels and all the relativities which are called into play when one tries to associate a definite significance to a physical entity labeled η_Δ that has been delimited—as yet strictly unqualified—by a purely physical operation of delimitation Δ. In particular, the definition posited above entails quite clearly that the "statisticity" or the "individuality" of a description $D(\Delta, \eta_\Delta, \langle T\rangle)$ *can appear or disappear* when the utilized view $\langle T\rangle$ is change while the delimitator Δ is kept fixed, or *vice versa*. This displaces on an entirely new ground the innumerable ancient or actual controversies—all erroneously absolutizing—concerning "the" determinism and "the" causality. However, alone, the relativizations accomplished above are still insufficient for cutting out the whole conceptual volume of this debate. The debate displays its complete volume only when furthermore an explicit and radical *distinction* is inserted between the *ontic* notion of (relative) "determination" and the *epistemic* notion of "previsibility."

The spacetime structure of the description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ is indicated in the Fig. 3. One can see that the concept of a transfer tree of a basic epistemic referential reappears as a particular instance of another more complex concept where it is explicitly connected to all the relativities of statisticity. We *re*name then the initially defined structure—more specifically—"the transfer tree of an *individual* transferred description," while the more general complexified treelike structure defined above will be called "the transfer tree of a *statistical* transferred description."

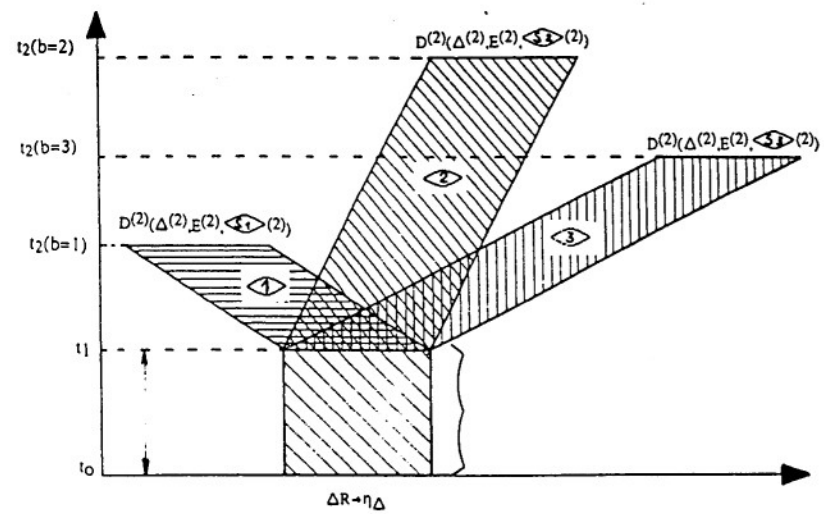

Fig. 3. The transfer tree of a statistical transferred description. First repeat the description in the caption of Fig. 2. Remember now that in order to *determine* whether a partial branch description $D(\Delta, \eta_\Delta, \langle b\rangle)$ is individual or not, this partial description has to be *reiterated* a great number of times, *globally*. An ensemble of N reiterations of the partial description $D(\Delta, \eta_\Delta, \langle b\rangle)$, b fixed, is thus obtained. This ensemble has to be examined by the g-statistical aspects $\langle Sg\rangle^{(2)} \in \langle S\rangle^{(2)}$ relative to all the aspects $\langle g\rangle \in \langle b\rangle$, to determine the respective dispersions. If all the dispersions for all the aspects $\langle g\rangle \in \langle b\rangle$ are zero, then the partial statistical metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle b\rangle^{(2)})$ as well as the partial description $D(\Delta, \eta_\Delta, \langle b\rangle)$ are individual and the branch b considered is an *individual branch of the statistical metadescription* $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. If, on the contrary, one finds a nonzero dispersion for at least one aspect $\langle g\rangle \in \langle b\rangle$, then the partial metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle b\rangle^{(2)})$ is statistical and the branch b considered is *a statistical branch of the global metadescription* $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. If all the branches $b=1, 2, \ldots, l$ are individual, then the whole metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ is an individual metadescription (of the individual description $D(\Delta, \eta_\Delta, \langle T\rangle)$ of the entity η_Δ) possessing the spacetime structure that has been previously called the transfer tree of the epistemic referential $(\Delta, \langle T\rangle)$.

3.3.4. The Minima of Individual Conceptualization: Individual Intrinsic Metaconceptualization

Suppose now that the statistical metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ appears to be an *individual* metadescription of the initial description $D(\Delta, \eta_\Delta, \langle T\rangle)$. Then by definition the N attempted reiterations j, $j=1, 2,..., N$ of the sequence of epistemic actions $[\Delta R \to \eta_\Delta, \langle T\rangle \eta_\Delta]_j$ have all led to identical transferred descriptions $D_j(\Delta, \eta_\Delta, \langle T\rangle)$. This identity, then, *is* an invariant tied with η_Δ (and relative to $\langle T\rangle$) with respect to the index of reiteration j. Namely, it is precisely the simplest sort of invariant that came spontaneously to mind but which we refused to assert *a priori*. So what is the situation now? We are already in possession of a first argument for the assertion that the label η_Δ designates "an entity": the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$ is a *stable* form. This first argument subsists even if we have found only *one* transfer view $\langle T\rangle$ with respect to which the mentioned invariance does emerge, and even if this view consists of only one aspect, with only one value. Nevertheless, and no matter whether the transfer view $\langle T\rangle$ is very simple or very complex, because the description $D(\Delta, \eta_\Delta, \langle T\rangle)$ is a transferred individual description, the spacetime form $D(\Delta, \eta_\Delta, \langle T\rangle)$ remains defined in terms of aspects of registering objects which—by definition—are all distinct of the result η_Δ of the operation of delimitation ΔR. We still know *nothing* concerning "how" the entity η_Δ is "itself." We are in possession of only a scattered form somehow "tied" with what is labeled η_Δ, and a form which, when it is considered globally, cannot be ordered by a unique time parameter. Such a form, even though it is now known to be invariant with respect to the reiterations of the epistemic action $[\Delta R \to \eta_\Delta, \langle T\rangle \eta_\Delta \to D(\Delta, \eta_\Delta, \langle T\rangle)]_j$, is irrepressibly perceived as only a *preliminary* step in the process of searching for an "interpretation" of the label η_Δ. The current language, faithfully reflected by the whole terminology introduced here, expresses this: we speak of a description which concerns *one* entity η_Δ *different from all the registering objects* which bear on them the values of the transfer aspects involved by the view $\langle T\rangle$, and which, though *individual*, is *transferred*. From the beginning on, more or less implicitly, we experience a belief that "an entity" possesses a certain "own" or "intrinsic" form that is *separable* from the apparatuses on which it produces perceptible marks. And, more or less implicitly, we posit a corresponding *a priori* decision to grasp—to construct—this "intrinsic" form. *Such* is the epistemic method that works spontaneously inside our mind. We can but recognize it as a psychological *fact*, probably tied with "adaptation" and "natural selection." So a new question arises: How can this intrinsic form decreed for the entity η_Δ be qualified? Remarkably, there exists a quite definite answer.

Intrinsic Metaconceptualization of an Individual Transferred Description. Intrinsic Model. Consider an individual transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. Let $\partial(\mathbf{r}, \eta_\Delta, t)$ be a connected space-domain on which the entity η_Δ is conceived to exist "intrinsically," i.e., independently of any observation, at a time t represented—statistically—by *the origin of the transfer times* t_1 reestablished for each pair of sequences $[\Delta R \to \eta_\Delta, \langle g\rangle \eta_\Delta]$ involved by the description $D(\Delta, \eta_\Delta, \langle T\rangle)$: $t \equiv t_1$. Let furthermore $\langle I\rangle$ be an *intrinsic view* such that any aspect i involved by this view is a functional $\Phi[D(\Delta, \eta_\Delta, \langle T\rangle)]$ of the initial transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$ of which the "values" (nonnumerical in general) are realized on the *connected* domain $\partial(\mathbf{r}, \eta_\Delta, t)$. The metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle TI\rangle^{(2)})$ where $\Delta^{(2)}$ selects conceptually for examination the ensemble $E^{(2)}$ of the two descriptions $D(\Delta, \eta_\Delta, \langle T\rangle)$ and $D(\Delta, \eta_\Delta, \langle I\rangle)$ and where the metaview $\langle TI\rangle^{(2)}$ contains all the aspects of the view $\langle T\rangle$ and of the view $\langle I\rangle$ as well as the aspects of relation between the aspects from these two views, will be called an *intrinsic metaconceptualization of the individual transferred description* $D(\Delta, \eta_\Delta, \langle T\rangle)$.
The description $D(\Delta, \eta_\Delta, \langle I\rangle)$ which corresponds to the aspects of the intrinsic view $\langle I\rangle$ *alone*—without reference to the genesis of the intrinsic aspects i from $\langle I\rangle$ as functionals of the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$—will be called an *intrinsic model of the entity* η_Δ.

An intrinsic metaconceptualization $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle TI\rangle^{(2)})$ realizes a spacetime integration of the scattered form introduced by the initial transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. The change of view $\langle T\rangle \to [\langle TI\rangle^{(2)}$ with $\langle TI\rangle^{(2)} \supset \langle T\rangle \vee \langle I\rangle$ operates a focalizing projection of the scattered transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$, onto the *connected* and *instantaneous* spacetime domain $\partial(\mathbf{r}, \eta_\Delta, t)$. The value of the time parameter $t=t_1$ which labels this domain is by construction *independent* of the index g that distinguishes from one another the different transfer aspects g, $\langle g\rangle \in \langle T\rangle$. This is so because t_1 is constructed *prior* to all the epochs $t(g)$ at which emerge, on the devices for measurements of the values gk of the aspects g, the transferred values gk which define the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$: for each examination $\langle g\rangle \eta_\Delta \in \langle T\rangle \eta_\Delta$, $g=1, 2,..., m$ from a sequence $[\Delta R \to \eta_\Delta, \langle T\rangle \eta_\Delta]$ which contributes to the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$, the measurement interaction between η_Δ and the device for measuring the values gk of an aspect g *begins* at an initial moment $t=t_1$ which is always the *same*, the *origin* of transfer times, identically redefined for each examination (Fig. 2). And *afterwards* (in order to exist) this interaction consummates some nonzero duration $[t(g)-t_1] \neq 0$

which *varies* from one examination by an $\langle g\rangle \in \langle T\rangle$ to another one. This uniqueness of the temporal qualification of the domain $\partial(\mathbf{r}, \eta_\Delta, t)$, though only of the *beginning* of the process of transfer, and only *retroactive*, suffices for permitting one now to conceive of *an intrinsic time order, of a law of intrinsic evolution underlying the transferred description* $D(\Delta, \eta_\Delta, \langle T\rangle)$. So $D(\Delta, \eta_\Delta, \langle T\rangle)$ is now "explained." The monologue runs as follows: "At a time $t = t_1$, *uniquely defined*, the entity η_Δ "possessed" on the domain $\partial(\mathbf{r}, \eta_\Delta, t)$—*connected*—the characteristics defined by the intrinsic model $D(\Delta, \eta_\Delta, \langle I\rangle)$ built by the intrinsic metaconceptualization $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle TI\rangle^{(2)})$ of the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. These characteristics were *separated* from those of any measurement device and they were *such* that via the examinations $\langle g\rangle\eta_\Delta \in \langle T\rangle\eta_\Delta$ they have produced the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. The scattered and mixed form of this transferred description is but the result of a bursting, of a pulverization of the intrinsic *integrated* form $D(\Delta, \eta_\Delta, \langle I\rangle)$ of the entity η_Δ, a pulverization produced by the transferring examinations $[\langle g\rangle\eta_\Delta] \in [\langle T\rangle\eta_\Delta]$, $g = 1, 2, \ldots, m$. These, because of the mutual spacetime incompatibility of certain examinations $\langle g\rangle\eta_\Delta$, $\langle g\rangle \in \langle T\rangle$, have obliged us to perform several different sequences $\Delta R \to \eta_\Delta$, $\langle g\rangle\eta_\Delta$ in order to obtain the transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. We succeeded in mirroring, so feebly, the intrinsic oneness of the own time of the entity η_Δ by reconstructing—on a *statistical* level—only a "common" origin of times t_1 (the final moment of the respective delimitations $\Delta R \to \eta_\Delta$) for all these different examinations $\langle g\rangle\eta_\Delta$. But the intrinsic metaconceptualization $D(\Delta, \eta_\Delta, \langle I\rangle)$ permits one now to perceive fully the unique well-ordered time of the entity η_Δ." In short, the intrinsic model $D(\Delta, \eta_\Delta, \langle I\rangle)$ corresponding to a transferred individual description is *a construct that is an invariant with respect to the group of transformations* $\{\langle g\rangle\eta_\Delta, \langle g\rangle \in \langle T\rangle\}$, with respect to which the initial transferred branch-descriptions $D(\Delta, \eta_\Delta, \langle b\rangle)$ are *not* invariant. This construct unifies all these transformations in "lumping" them together. It marks a position of saturation and of equilibrium of the significance assigned to the tentative initial label η_Δ. It makes us feel that we finally "understand" what the *a priori* label η_Δ "means." It sets an economic and stable closure upon the representation of what has been *a priori* called the entity η_Δ. This closure is perceived as satisfactory and as necessary to such a degree that its character, *ineluctably hypothetic, retroactive, and relative to an initial transferred description and to a particular intrinsic view* (no doubt admitting for a whole class of substitutions), tends to be skipped. The unavoidable initial phases of transferred description have always been left inexplicit and *a fortiori* unformalized to the maximal possible degree:

> *This is the basic character that marks all the "classical" descriptions, from physics, mathematics, etc. as well as from current language and thinking.*

Starting from the transferred data that are available for it and on which it takes support without trying to express them, the human mind always rushes as rapidly and as directly as it can toward a representation by an intrinsic model. As soon as such a representation has been attained, it is spontaneously felt to be "true," in an *absolute* and *certain* way, without reference to the transferred data on which it is founded and forgetting that it is just an economic construct, while these initial transferred data, though they are the sole cert*itudes*, are perceived implicitly as nothing more than "subjective" tools for finding the "intrinsic objective truths." Simplicity, invariance, what we tend to call "truth" and "objectivity," form an indissoluble absolutizing whole deeply imprinted in our minds by ancestral processes of adaptation. No kind of positivism will ever be strong enough for annihilating indefinitely our irrepressible tendency toward intrinsic metaconceptualization. *Nor should we desire this*:

> *There is no choice to be made between transferred descriptions and intrinsic metaconceptualization of these. There is a connection to be worked out, explicitly and systematically.*

The transferred descriptions are the *unavoidable* first phases of our processes of conceptualization, a *universal* phase, while the intrinsic metaconceptualizations of the initial transferred descriptions are a stable and economizing subsequent phase that brings us down onto a minimum of our potential of conceptualization (always a relative and provisional minimum). There is an *order* of conceptualization: Each intrinsic model $D(\Delta, \eta_\Delta, \langle I\rangle)$ presupposes some corresponding intrinsic metaconceptualization $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle TI\rangle^{(2)})$ founded on some initial transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$. But not vice versa: a transferred description presupposes no other previously accomplished description. It calls, however, for a subsequent one, with irrepressible force.

3.3.5. The Climbing Induced by Absence of Relative Individuality

What happens now if $D(\Delta, \eta_\Delta, \langle T\rangle)$ reveals itself to be a *statistical* transferred description? What significance could be associated to the assertion that we are dealing with "*one* entity" η_Δ, if the reiterations of the succession of epistemic operations $[\Delta R \to \eta_\Delta, \langle T\rangle\eta_\Delta]$, lead to descriptions $D(\Delta, \eta_\Delta, \langle T\rangle)$, that are not only transferred, but furthermore are *not* *identical*, are also *variable*, consisting of *fluctuating* ensembles of qualifications of various registering objects, all admittedly distinct from what is labeled "one entity η_Δ"? Concerning this new complexified question, the same preliminary condition which already emerged for the simplest case, tenaciously continues to impose itself: In order to admit that what had tentatively been labeled "one entity η_Δ" points toward a designatum which deserves being definitively denominated and installed into the conceptualization, it is *necessary* that *some* invariance shall manifest itself. We must find a way of screwing our perceptions into "reality." Since it has not been found concerning the first descriptional level where the basic transferred description $D(\Delta, \eta_\Delta, \langle T \rangle)$ is placed, this invariance can only concern either the metadescription $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S \rangle^{(2)})$ obtained on the second descriptional level, or some other metadescription of level higher than 2 and stemming from the epistemic action of the basic referential $(\Delta, \langle T \rangle)$. Indeed, if no sort of invariance whatever tied with the basic pairing $(\Delta, \langle T \rangle)$ would ever appear, concerning none of all the descriptional levels $1, 2, \ldots, K$ of a "sufficiently" long sequence of K levels, what would we say? We find out again—as one finds out that outside it rains!—that we would say that it finally became "practically" certain that the epistemic referential $(\Delta, \langle T \rangle)$ is unable to "prove" the "existence" of an entity η_Δ which deserves being denominated and stored into the inventory of the conceptualized, *notwithstanding* the fact that the delimitator Δ and the view $\langle T \rangle$ *do* mutually exist in the sense of (9). So, definitively this time, the qualification of "descriptions connected with an entity η_Δ" would be retired *a posteriori* to all the links of the chain of constructs $[D^{(n)}, \Delta^{(n)}, E^{(n)}, \langle S \rangle^{(n)}]$, $n = 1, 2, \ldots, K$ founded on the pairing $(\Delta, \langle T \rangle)$. Once more we would admit *reflexively* that we had invested these constructs with significance only tentatively, provisionally, under the pressure of a successively shifted and deceived hope of finding on the next descriptional level an invariant permitting one to associate some meaning with the label η_Δ, an invariant announcing that the climbing from level to level in search of a definition can finally be stopped. (But notice the relativity to the basic view $\langle T \rangle$: it *still* remains possible that the association of the same delimitator Δ with some other view $\langle T \rangle' \neq \langle T \rangle$ shall reveal a meaning assignable to the label η_Δ.) This imperious requirement that some invariant shall emerge on some descriptional level of a finite order is of the same essence as the requirement of finiteness to which the concept of definition is subjected in meta-mathematics. There like here it is *necessary* to be able to found on some stop-signal the assertion that the specification of the object to be defined has been *achieved.*

3.3.6. Relativized Probabilistic Conceptualization: Critical Remarks

The preceding remarks bring into evidence the crucial importance of the concept of probability. Indeed this concept—when it *can* be applied—expresses a *convergence* of each one of the dispersed relative frequencies which are involved in the definition of a statistical description. Such a convergence *would* constitute the researched invariant, though a far more remote and complex invariant than the relative identities that found the concept of an individual description. At this point, however, arises a preliminary problem. The "classical" concept of probability as it now stands lies, still nonextracted, inside a magma of false absolutes that stem from the fact that *its substratum of transferred descriptions has remained implicit*. The umbilical cord that ties it to the reservoir of semantic substance from which it has been extracted has been left nonspelled out. In order to incorporate "classical probabilities" into the method of relativized conceptualization it is necessary to detect these false absolutes and to clear them away, drawing into an explicit representation all the relativities involved.

The fundamental concept of the present-day theory of "objective" probabilities (Kolmogorov's formulation) is a probability space $[U, \tau, p(\tau)]$ where $U = \{e_i\}$ (with $i \in I$ and I an index set) is a "universe" (a set) of "elementary events" e_i; τ is an algebra of "events" (subsets of U) built on U; $p(\tau)$ is a "probability measure" defined on the algebra of events τ. The universe of elementary events $U = \{e_i\}$ is conceived of as generated by the reiteration of an "identically" reproducible procedure P, but which brings forth elementary events e_i that *vary* in general from one realization of P to another one. A pair $[P, U]$ containing an identically reproducible procedure P and the corresponding universe of elementary events U is called a random phenomenon. On a given universe U, a whole family of different algebras τ of subsets of U can be defined. So it is possible to form different "probability chains" [a random phenomenon] $\rightsquigarrow$ [a corresponding probability space], all stemming from $[P, U]$. In symbols

$$[P, U] \rightsquigarrow [U, \tau, p(\tau)]$$

However, the concept of "a probability *chain*" is not explicitly defined. So the unavoidable association of a considered probability space, with the random phenomenon which generates it, is very rarely explicitly mentioned and surveyed. The present-day abstract theory of probabilities is a formal system, a syntax, already remarkably precise and rich in its techniques but which is *devoid of any elaborated channels for a controlled, regulated adduction of semantic substance from the reservoir of physical and conceptual* *reality* which in this work is indicated by the letter R. The way in which the elementary events from the universe U do *operationally* emerge is usually left rather vague. The structure of what is called a reproducible procedure P is not investigated. In each application of the abstract theory of probabilities, to some specific problem, the corresponding semantic substance is injected into the formalism in an intuitively decided way, without the help of established general rules. These lacunae appear strikingly as soon as one begins to raise questions suggested by the method of relativized conceptualization:

— What is an identically reproducible procedure P? Is it exclusively an operation of delimitation, or is it some association between a delimitation and an examination by a view? It seems obvious that also some view is quite systematically involved, since it is asserted that the procedure P brings forth "different" elementary events e_i. But "different" in what sense? With respect to which view? In the absence of *any* view, the elementary events e_i cannot be perceived. They even cannot be imagined. So *a fortiori*, they cannot be compared and mutually distinguished. A delimited entity on which no view acts nor has ever acted before simply cannot penetrate into consciousness. So the index $i \in I$ necessarily refers to qualifications by values of some aspects of some view *and* these can concern only some entity η_Δ produced by some *delimitator*. This delimitator, however, we saw, cannot—alone—yield an equivalent for what is called an identically reproducible procedure P, since this involves also some view. So of what does P consist, exactly? How can its content be fully *symbolized*?

— The *unique* index i that labels the elementary events e_i is not sufficient for cutting out a conceptual receptacle able to contain the *full* specification of the qualifications of these elementary events by a view. Even in the simplest case of a view with only one aspect, the fully structured grating (1) of possible qualifications requires already *two* indexes, the aspect-index g *and* the index k devoted to the considered value of the aspect g. *The symbolic framework necessary for the expressibility of the structure*(1) *of the involved view is not constructed.* In such conditions the *expression* of the semantic substance that can be injected into the formalism is certainly amputated *systematically*.

Finally, consider the most fundamental question: Why is it that in certain cases the relative frequencies of the elementary events from a universe U do converge toward a corresponding probability measure, while in other

cases no such convergence manifests itself? *Beyond* its formal definition, what is the *significance* of the probability measure from a probability space? What *sort* of *entity* is indicated by the existence of a probability measure for the elementary events from a universe U?

3.3.7. Relativized Reconstruction Leading to a "Nonclassical" Probabilistic Structure

To work out the insertion of the fundamental probabilistic concepts into the method of relativized conceptualization, we must represent explicitly the initial, *transferred* phase of the probabilistic descriptions.

Relativized Identically Reproducible Procedure. Let us first consider the simplest case, that of a basic transfer view $\langle T\rangle = \langle g\rangle \vee \langle Ed\rangle$ which, besides the spacetime frame view $\langle Ed\rangle$, consists of only one transfer aspect view. (Since the frame view $\langle Ed\rangle$ is always available by convention, often we shall not mention it.) We place ourselves at the zero point of a chain of conceptualization: a realization of the operation $\Delta R \to \eta_\Delta$ *alone* produces a result η_Δ consisting of the purely physical determination of a certain monolith of still entirely unknown (non-expressed) potentialities. So in order to traverse from the realm of mute factuality into the realm of the communicable, we are obliged to consider *successions* $[\Delta R \to \eta_\Delta, \langle g\rangle \eta_\Delta \to D(\Delta, \eta_\Delta, \langle g\rangle)]$ of the *two* epistemic operations, $\Delta R \to \eta_\Delta$ *and* $\langle g\rangle \eta_\Delta \to D(\Delta, \eta_\Delta, \langle g\rangle)$. Each such—reproducible—succession entails as its final effect a (transferred) description $D(\Delta, \eta_\Delta, \langle g\rangle)$ of the entity η_Δ. Such a description belongs now to the realm of the observed and expressed, of the communicable. It consists by definition of a certain configuration of perceived qualifications gk (values k of the transferred aspect g) that appeared on the surface of the g-measuring device, distributed on the spacetime grating introduced by the frame view $\langle Ed\rangle \in \langle T\rangle$. We have already introduced for such a configuration a synthetic symbol h, $h = 1, 2, \ldots, L(g)$, with $L(g)$ finite in consequence of the finite number of the spacetime-gk qualifications permitted by definition for any view. So we write $D(\Delta, \eta_\Delta, \langle g\rangle) = \eta_{gh}$. Then the realized reproducible procedure P can be represented by

$$P = [\Delta R \to \eta_\Delta, \langle g\rangle \eta_\Delta \to \eta_{gh}], \qquad h = 1, 2, \ldots, L(g),\ L(g) \text{ finite}$$

This writing expresses quite explicitly the very important fact that *one* realization of what is called "the reproducible procedure P" consists of the *succession* $[\Delta R \to \eta_\Delta, \langle g\rangle \eta_\Delta \to \eta_{gh}]$ of *two* epistemic operations (supposed here to be both of a purely physical nature):

— an operation of delimitation $\Delta R \rightarrow \eta_\Delta$ of an entity labeled "η_Δ" (in consequence of the purely physical character assumed here for the delimitator Δ, the entity η_Δ—still strictly nondescribed—can even *entirely* escape, not only human perception, but also direct human *perceptibility*, as it does happen indeed in microphysics);

— an examination of the entity $\langle T\rangle \eta_\Delta$ of the entity η_Δ via the transfer view $\langle T\rangle = \langle g\rangle \vee \langle Ed\rangle$ (for the sake of simplicity we write $\langle g\rangle \eta_\Delta \rightarrow \eta_{gh}$).

The final effect is systematically a relative, observable, transferred description $D(\Delta, \eta_\Delta, \langle g\rangle) = \eta_{gh}$, with $h = 1, 2, \ldots, L(g)$.

Relativized Random Phenomenon. If, as it is here supposed, the relative description $\eta_{gh} = D(\Delta, \eta_\Delta, \langle g\rangle)$ is *not* individual, then a sufficiently large number N of reiterations $P_j = [\Delta R \rightarrow \eta_\Delta, \langle g\rangle \eta_\Delta \rightarrow \eta_{gh}]_j$, $j = 1, 2, \ldots, N$, of the procedure P (j the index of reiteration) will in general bring forth any one from the ensemble of possible distinct groups of qualifications η_{gh}, $h = 1, 2, \ldots, L(g)$, $L(g)$ finite. So the translation in our terms of the universe of elementary events $U = \{e_i, i = 1, 2, \ldots, \lambda\}$ is

$$U = \{D(\Delta, \eta_\Delta, \langle g\rangle) = \eta_{gh}, h = 1, 2, \ldots, L(g)\}$$

Notice that by the application of our method the semantically insufficient one-index differentiation of the elementary events practiced in the present-day theory of probabilities, has "automatically" transmuted into a *double* indexation of the elementary events, by g and h, permitting one to distinguish hierarchically between aspect and values of aspect.

So the relativized reformulation of the fundamental concept of a random phenomenon can be symbolized by the new writing

$$(P, U) = (\{[\Delta R \rightarrow \eta_\Delta, \langle g\rangle \eta_\Delta \rightarrow \eta_{gh}]_j, j = 1, 2, \ldots, N\},$$
$$\{\eta_{gh}, h = 1, 2, \ldots, L(g)\}) \qquad (11)$$

In this writing, the *operational* structure of the concept of a random phenomenon is entirely explicit and symbolized.

The channel $(\{[\Delta R \rightarrow \eta_\Delta, \langle g\rangle \eta_\Delta \rightarrow \eta_{gh}]_j, \ j = 1, 2, \ldots, N\}$ *for the adduction of semantic substance, from the reservoir of "reality" denoted R, into a probability space, is now explicitly represented. That is what will permit us to construct the representation of the initial, transferred phase involved in any process of probabilistic conceptualization.*

Relativized Transferred Probabilizable Space. Let us define on the universe of elementary events $U = \{\eta_{gh}\}$ from (11), the *total* algebra τ_g and let us call it the *algebra of g-events for* η_Δ. The algebra τ_g contains all the unions of elementary events from U, all the intersections of such unions, U itself, and the void ensemble. So it contains *meta*descriptions with respect to the descriptions η_{gh} from the universe U. Globally, this reservoir of relative metadescriptions is "the boolean algebra of relative descriptions generated by the elementary descriptions η_{gh}". We are now in the presence of a *relativized probabilizable chain*:

$$[\{[\Delta R \rightarrow \eta_\Delta, \langle g\rangle \eta_\Delta \rightarrow \eta_{gh}]_j, j = 1, 2, \ldots, N\},$$
$$\{\eta_{gh}, h = 1, 2, \ldots, L(g)\}] \rightarrow [\{\eta_{gh}\}, \tau_g] \qquad (12)$$

The Simplest Type of Relativized Transferred Probabilization. In the chain (12) the random phenomenon (11) is connected with a probabilizable space in the standard sense of the term. But in contradistinction to what happens in the present-day theory of probabilities the relativized reformulation (12) involves an explicitly worked out, detailed, and symbolized operational definition of the *very complex* relations between the probabilizable space $[\{\eta_{gh}\}, \tau_g]$ and the random phenomenon (11) which produces it. It goes down into the *substrata* of the conceptualization, throwing light on the genetic role played by the basic epistemic referential $(\Delta, \langle T\rangle)$ that is at work. Finally we can now define:

One Aspect Relativized Transferred Probability Space. Consider the chain (12) belonging to a (relative, transferred) statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. Select the g-population view $\langle ng\rangle^{(2)}$ from $\langle S\rangle^{(2)}$ (each value of the corresponding g-population aspect being by definition the relative frequency $n(gh)/N$ of realization of an event η_{gh} from (12)). Let $p(\tau_g)$ be a probability measure asserted on τ_g, computed, on the basis of the law of total probabilities, from an elementary probability measure, *supposed to exist*, defined on the universe of elementary events U. Namely

$$p(gh) = \lim(N \rightarrow \infty)[n(gh)/N], \qquad h = 1, 2, \ldots, L(g) \qquad (13)$$

The chain

$$[\{[\Delta R \rightarrow \eta_\Delta, \langle g\rangle \eta_\Delta \rightarrow \eta_{gh}]_j, j = 1, 2, \ldots, N\},$$
$$\{\eta_{gh}, h = 1, 2, \ldots, L(g)\}] \rightarrow [\{\eta_{gh}\}, \tau_g, p(\tau_g)] \qquad (14)$$

will be called "the probabilization with respect to the aspect-view $\langle g\rangle$ of the statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$" or *the* *probabilistic description founded on the basic epistemic referential* $(\Delta, \langle g \rangle)$ (with $\langle g \rangle \in \langle T \rangle$) and it will be symbolized by the writing $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle Pg \rangle^{(3)})$, where $\langle Pg \rangle^{(3)}$ is the meta-metaview of probability relative to the aspect-view $\langle g \rangle$, possessing by definition the structure $\langle Pg \rangle^{(3)} = \langle Sg \rangle^{(2)} \vee \langle Cg \rangle^{(3)} = \langle g \rangle \vee \langle ng \rangle^{(2)} \vee \langle Cg \rangle^{(3)}$, and $\langle Cg \rangle^{(3)}$ is the meta-meta-view of g-convergence, the values of the corresponding g-convergence aspect being by definition the limiting values $p(gh)$ defined by (13) for the populations $n(gh)/N$.

Probability Versus Certainty. So following the commands of the principle of separation, we have reached a new descriptional level, the third one with respect to the initial description $D(\Delta, \eta_\Delta, \langle g \rangle)$. For any fixed number N of reiterations of the initial description $D(\Delta, \eta_\Delta, \langle g \rangle)$, this third level of conceptualization involves *furthermore*: (a) a very big number N', $N' \neq N$, of reiterations, now, of "the" measurement of the *set* of *all* the relative frequencies $n(gh)/N$, $h = 1, 2, ..., L(g)$, (constituting *together* "one" measurement of the whole statistical *distribution* $\{n(gh)/N, h = 1, 2, ..., L(g)\}$, considered *globally*; (b) comparison of the result of each measurement of the whole statistical distribution $\{n(gh)/N, h = 1, 2, ..., L(g)\}$, with the assertion (13) of convergence. However, in consequence of the *finiteness* of any realizable pair N, N', no matter how large N and N' are and *whatever* are the results of the N' successive comparisons with the presupposed limits $p(gh)$ from (13), this presupposition remains *nonremovably* subject to a possible *a posteriori* "invalidation," while such an invalidation, in its turn, equally remains nonremovably uncertain.

Nevertheless, if on this third descriptional level an *a posteriori* invalidation of the presupposed convergence (13) would emerge *with respect* to some precision ∂, arbitrary but chosen in advance, and for some given pair of "sufficiently" big numbers N and N', arbitrary but chosen in advance, I *decide* that I would *conventionally*, *strategically*, close the exploration by *a relativized exclusion*, saying that the epistemic referential $(\Delta, \langle g \rangle)$ is rejected because finally it has been found to be "(∂, N, N')-nonsignificant" with respect to the aspect g, notwithstanding the fact that it had resisted elimination by the initial much more fundamental test of relative existence (7). This is consistent with the general attitude of *a priori* confidence and *a posteriori* back-control, and of systematic finitism, practiced in this approach. Moreover, a decision of *a posteriori* elimination of the type specified above constitutes *a relativized application of the requirement of finiteness imposed in meta-mathematics upon any definition*: the epistemic referential $(\Delta, \langle g \rangle)$ is "(∂, N, N')-banished" when the entity η_Δ produced by the delimitator Δ does not admit, via the view $\langle g \rangle$, a definition *bounded* by the trio of numbers (∂, N, N'), arbitrary but chosen in advance. Such a "(∂, N, N')-banishment" would play the role of a *relative proof of inexistence* of an interpretation for the transferred relative description $D(\Delta, \eta_\Delta, \langle g \rangle)$.

But suppose now that, on the contrary, the *a priori* asserted convergence (13) appears to be "(∂, N, N')-confirmed" *a posteriori*, i.e., the statistical distribution $\{n(gh)/N, h = 1, 2, ..., L(g)\}$ is found to be "(∂, N, N')-identical" to the posited probability law $\{p(gh), h = 1, 2, ..., L(g)\}$. In this case—again *conventionally*, *strategically*—I *decide* to consider that the probability measure $p(\tau_g)$ from (13), hence the probabilization (14), are "(∂, N, N')-true" and that the epistemic referential $(\Delta, \langle g \rangle)$ is "(∂, N, N')-significant." This decision, however, would be just a *strategic bet*, a bet expressed mathematically by what in the theory of probabilities is called the law (weak or strong) of big numbers. Only on the basis of this bet is it possible to *quit* the domain of factual statements and statistical countings, and to penetrate, with *non*individual descriptions, into the domain of "certitude," of *deductions*, but concerning probabilities: *This bet can be regarded as an illustration of what is called "the principle of induction"*: the observation [the statistical distribution $\{n(gh)/N, h = 1, 2, ..., L(g)\}$ is "(∂, N, N')-identical" to the probability law $\{p(gh), h = 1, 2, ..., L(g)\}$] has been made N' times, with N' very big; this induces a *belief* that the (meta)*individual* and *universal* proposition [there exists the probability *law* $\{p(gh), h = 1, 2, ..., L(g)\}$] is true. On this basis we decide—as just a strategy—to assert this proposition in the role of hypotheses in syllogisms, in order to accede to the *vehicle of deduction* for transporting selfconsistently truth values of propositions involving the distribution $\{p(gh), h = 1, 2, ..., L(g)\}$. This, however, cannot suppress the essential conceptual gap between statistical countings $\{n(gh)/N, h = 1, 2, ..., L(g)\}$ and the corresponding metaconcept $\{p(gh), h = 1, 2, ..., L(g)\}^{(3)}$ (pp. 291–299). (In the particular case of a dispersion-free Dirac distribution we obtain the inductive posit of quasicertainty.) This draws out the frontiers and the connections between statistics, probabilities, and logic. Knowledge of these makes clear that one can build deductions leading to certainties concerning probabilistic qualifications[9] (pp. 967–971). This indicates the way toward the framework for a relativized unification of logic and probabilities where the confusions stemming from nondistinctions between different levels of conceptualization are suppressed.

More Complex Relativized Transferred Probability Spaces. Consider now a branch b of the statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S \rangle^{(2)})$. The possible values of the branch-view $\langle b \rangle$ are by definition associations between a combination of values gk of various mutually compatible aspects g from the transfer view $\langle T \rangle$, with values rt of the spacetime frame-aspect involved by $\langle T\rangle$. And every individual examination $\langle b\rangle\eta_\Delta$ leads to a partial description consisting of a certain configuration of such associations. The description $D(\Delta, \eta_\Delta, \langle b\rangle)$ can be regarded as a simultaneous realization of several descriptions $D(\Delta, \eta_\Delta, \langle g\rangle)$, $\langle g\rangle \in \langle T\rangle$ (a logical conjunction of the propositions asserting these descriptions). Hence it is a *meta*description with respect to the descriptions $D(\Delta, \eta_\Delta, \langle g\rangle)$ considered separately. Let us make use again of the global index h for a configuration of values gk, $\mathbf{r}t$, $\forall\langle g\rangle \in \langle b\rangle$, constituting a description $D(\Delta, \eta_\Delta, \langle b\rangle)$. This index can *a priori* assume a whole ensemble of different values, $h = 1, 2,..., L(b)$ of which the cardinal $L(b) \geqslant L(g)$ depends now on the structure of the whole branch-view $\langle b\rangle$. Consider the b-statistical metaview $\langle nb\rangle^{(2)} \in \langle S\rangle^{(2)}$ corresponding to the whole branch-view $\langle b\rangle \subset \langle T\rangle$. By definition, this metaview possesses the structure $\langle nb\rangle^{(2)} = \bigvee_g \langle g\rangle$, $\forall\langle g\rangle \in \langle b\rangle$, and the values of the corresponding aspect b are the relative frequencies $n(bh)/N$, $h = 1, 2,..., L(b)$ of realization of the different configurations of values gk, $\mathbf{r}t$, $\forall\langle g\rangle \in \langle b\rangle$ globally labeled by h, $h = 1, 2,..., L(b)$ (the relative frequencies of realization of the different possible partial descriptions $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle b\rangle^{(2)})$). We define:

> ***Probabilization of a Branch.*** Consider a (relative, transferred) statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. Select the b-statistical metaview $\langle Sb\rangle^{(2)} \in \langle S\rangle^{(2)}$ corresponding to the whole branch-view $\langle b\rangle \subset \langle T\rangle$. This corresponds to a b-population view that introduces a b-population aspect with values that are the relative frequencies $n(bh)/N$ of realization of the branch-descriptions $D(\Delta, \eta_\Delta, \langle b\rangle)$. Let $p(\tau_b)$ be a probability measure on τ_b computed, via the law of total probabilities, from the elementary probability law—*supposed to exist*:
>
> $$p(bh) = \lim(N \to \infty)[n_b(h)/N], \qquad h = 1, 2,..., L(b) \qquad (13')$$
>
> The chain
>
> $$[\{[\Delta R \to \eta_\Delta, \langle b\rangle\eta_\Delta \to \eta_{bh}]_j, j = 1, 2,..., N\},$$
> $$\{\eta_{bh}, h = 1, 2,..., L(b)\}] \to [\{\eta_{bh}\}, \tau_b, p(\tau_b)] \qquad (14')$$
>
> will be called the "probabilization of the statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ with respect to the branch-view $\langle b\rangle$" or the probability-chain founded on the basic epistemic referential $(\Delta, \langle b\rangle)$, $\langle b\rangle \subset \langle T\rangle$. This same probability-chain will be also symbolized by the more compact writing $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle Pb\rangle^{(3)})$ where $\langle Pb\rangle^{(3)}$ is the *meta-metaview of probability relative to the branch-view* $\langle b\rangle$ of which the structure is by definition $\langle Pb\rangle^{(3)} = \bigvee_g \langle Pg\rangle^{(3)}$, $\forall\langle g\rangle \in \langle b\rangle$, where ($\langle Pg\rangle^{(3)}$ is the meta-metaview of probability relative to the aspect g, already defined for the chain (14)).

The algebra of events τ_b from (14') is still a Boolean algebra of relative descriptions, like that from (14). All the remarks made concerning the significance of the assertion of a probability measure $p(gh)$ concerning only one aspect-view $\langle g\rangle$, hold, *mutatis mutandis*, concerning the assertion of a probability measure $p(bh)$.

Transgression of the Classical Concept of Probability Space. Finally consider *all* the aspects from the basic view $\langle T\rangle$ involved in the epistemic referential $(\Delta, \langle T\rangle)$ on which is founded the statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. The preceding definitions of a probabilization of this description with respect to one aspect-view or with respect to one branch-view admit the following development relative to the entire view $\langle T\rangle$.

> ***Complete Transferred Probabilization.*** Consider a (relative, transferred) statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$. Consider the ensemble of all the probabilizations (14') of this description with respect to all the $l \leqslant m$ mutually incompatible branch-views $\langle b\rangle \subset \langle T\rangle$. This ensemble will be called the *probabilistic description of the entity* η_Δ *with respect to the transfer-view* $\langle T\rangle$ and will be symbolized by the writing $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle P\rangle^{(3)})$, where $\langle P\rangle^{(3)}$ is the *meta-metaview of probability relative to the whole transfer-view* $\langle T\rangle$ possessing the structure:
>
> $$\langle P\rangle^{(3)} = \bigvee_g \langle Pg\rangle^{(3)} = \bigvee_g [\langle g\rangle \vee \langle S\rangle^{(2)} \vee \langle Cg\rangle^{(3)}] \text{ with } g = 1, 2,..., m.$$

The preceding definition unites into one single concept the ensemble of all the probability-chains of type (14) or (14') stemming from one same delimitator. But it is essential to be clearly aware of the fact that the similitudes which tie to one another like a leit-motif the verbal expressions of the concepts of probabilization relative to one aspect-view, to a branch-view, or to a complete transfer-view, emerge on a ascending spiral of conceptualization. *Each level introduces its specificities* and some of these can be quite radically innovating. For instance, the final level (the 5th one already) introduces an essentially new logico-algebraic structure: The algebra of events (relative descriptions) involved in a complete probabilization of a statistical description is a *non*-Boolean union of the mutually *incompatible* algebras from the different branch-probabilizations, and thus a *non*-Boolean algebra of relative descriptions. Indeed the probabilistic description $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle P\rangle^{(3)})$ inherits the treelike spacetime structure

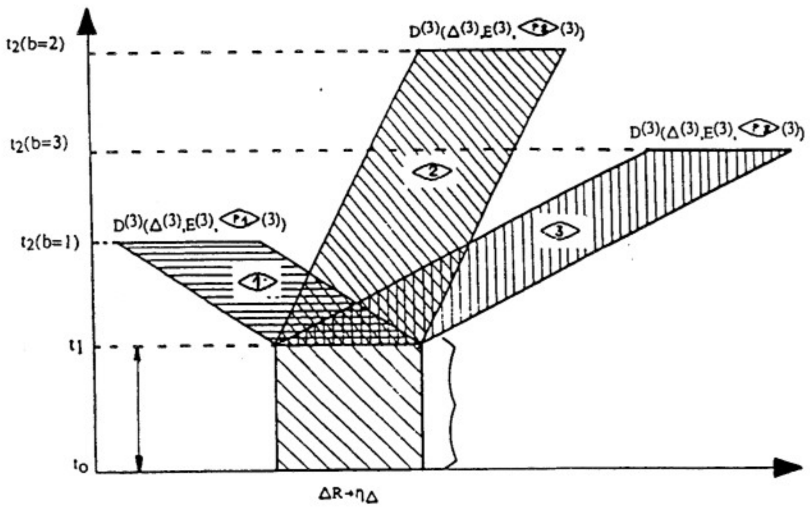

Fig. 4. The probability tree of a transferred probabilistic description. Let us examine all the possible sorts of probability trees. In the most general case the probability tree of an epistemic referential (Δ, $\langle T\rangle$) possesses a certain number of distinct branches, finite and bigger than 1. Each branch is generated by a branch-view $\langle b\rangle$ that contains a certain number bigger than 1 of mutually compatible aspect views $\langle g\rangle \in \langle T\rangle$ leading to a common probabilized space of the type contained in the chain (14′), located at the top of this branch. This most general case contains as particular cases *all* the types of relative description discerned before. Indeed: To begin with, the probability tree of the basic epistemic referential contains by construction the corresponding statistical description $D^{(2)}$. The other sorts of descriptions are reobtained as follows. If all the aspect views $\langle g\rangle \in \langle T\rangle$ from the basic view $\langle T\rangle$ are compatible, the probability tree possesses a unique branch introducing at its top a unique probability space of type (14′). If moreover, this unique space of type (14′) contains a probability measure which is a dispersion-free Dirac measure, the space of type (14′) at the top of the tree reduces to an *individual* transferred description $D(\Delta, \eta_\Delta, \langle b\rangle)$ relative to *several* (compatible) aspects. If the branch view from the unique branch of the tree contains only one aspect view $\langle g\rangle$, but the probability measure from the corresponding chain is *not* devoid of dispersion, the unique space of type (14′) from the top of the tree reduces to a probabilization of the type (14). If, furthermore, this unique chain of type (14) contains a dispersion-free probability measure, the corresponding space of type (14) reduces to an *individual* transferred description $D(\Delta, \eta_\Delta, \langle g\rangle)$ relative to a *unique* aspect g. Finally, if the tree contains *several* branches $b = 1, 2,..., l$, but at the top of *each* branch the corresponding probability chain contains a dispersion-free measure, all the spaces of type (14′) involved by the tree reduce to individual branch descriptions $D(\Delta, \eta_\Delta, \langle b\rangle)$, $b = 1, 2,..., l$. Then the whole tree represents an *individual* transferred description $D(\Delta, \eta_\Delta, \langle T\rangle)$.

of the corresponding statistical description $D^{(2)}(\Delta^{(2)}, E^{(2)}, \langle S\rangle^{(2)})$ (Fig. 4). There exists of course an essential difference: At the top of a branch of the probabilistic description $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle P\rangle^{(3)})$, instead of the partial statistical metadescription generated by the corresponding branch-view, lies the probabilization $D^{(3)}(\Delta^{(3)}, E^{(3)}, \langle Pb\rangle^{(3)})$: *The branches of the transfer-tree have grown a peg higher; they have reached a subsequent level of conceptualization.* So we are in the presence of a new structure. We call it *the probability-tree of a transferred probabilistic description* (of a basic epistemic referential (Δ, $\langle T\rangle$).

In the transferred substratum of any "classical" probability space lies a treelike structure that has been washed out by the implicit intrinsic metaconceptualization that has led to that space.

And notice that, in the last case mentioned in the caption of the Fig. 4, notwithstanding the complete resorption of the statistical character, as also of its probabilistic character, the treelike character of the spacetime structure of the description *subsists*: The treelike spacetime structure of a transferred description is tied with—exclusively—the existence, in the acting view, of incompatible transfer-aspects.

This treelike spacetime structure is a universal feature of the initial, the transferred phase of any description, whether individual, or statistical, or probabilistic. This structure is induced by the potential-actualization-actualized character of the elementary transfer-chains $[\Delta R \to \eta_\Delta, \langle b\rangle \eta_\Delta \to \eta_{bk}]$ *that form the fibers of any transferred description.*

It marks any description which concerns a still strictly noninterpreted but physically delimited monolith of potentialities, *a priori* just labeled (here by η_Δ) in order to be able to think and to speak of it, but as yet entirely *unknown*. It explains the *genesis* of the fact that the form corresponding to a transferred description is nonconnected as soon as the acting view involves mutually incompatible aspects.

3.3.8. The "Nonclassical" Specificities of a Transferred Probabilistic Description

Consider now the most general type of probability tree that can be generated by an epistemic referential. In this case the tree contains *several* random phenomena (11) *tied to one another* by *one* same operation of delimitation $\Delta R \to \eta_\Delta$ which produces the trunk of the tree, but corresponding to different branch-views $\langle b\rangle$. These l *distinct* but *related* random phenomena generate l probability spaces which in their turn are *related* even though *distinct*. In these conditions the algebra of events from the *whole* tree is the *non*-Boolean union $\tau_h = \bigcup_b \tau_b$, $b = 1, 2, \ldots, l$ of the l Boolean *mutually incompatible* branch-algebras τ_b. This union *is not even a lattice* insofar as one requires the existence of some already actualized factual counterpart for any considered proposition, and thus for any defined logical operation. In short, we are in the presence of an algebra of events which is *non*-Boolean *and* is *probabilized*. Indeed, we are compelled to use *singular* terms concerning this algebra and its probabilization. We *must* speak of *one* probabilization of *one* algebra, notwithstanding the fact that this probabilization has been achieved with the help of a whole *ensemble* of $l > 1$ distinct probability measures contained in $l > 1$ distinct probability spaces. The probability measures from these l distinct probability spaces stem all from *one same operation of delimitation* ΔR. And their very existence has been researched as an invariant that shall give us the right to speak of "one" result $\eta_\Delta \leftarrow \Delta R$ of this operation. So these l different probability measures have to be regarded as just differently "coded" descriptions transferred of the unique monolith of potentialities *a priori* labeled "η_Δ":

(a) In a "good" mathematical representation of a probability tree of a transferred probabilistic description, a symmetric and "deterministic" functional relation

$$p(\tau_{b'}) = T[p(\tau_b)]$$

must be required between the probability measures—considered as wholes—$p(\tau_b)$ and $p(\tau_{b'}) \neq p(\tau_b)$ from two distinct branches b and b' of the tree, where T just "projectively" *transforms*, determines, deduces, the description of "η_Δ" in terms of values of the transfer-aspects that act in the branch b, into the description of "η_Δ" in terms of values of the transfer-aspects that act in the branch b': *a deterministic metadependence T between whole probability measures* has to be required.

(b) On the other hand, because of the spacetime incompatibility of the transfer processes $\langle b \rangle \eta_\Delta$ involved in distinct branches, for two *elementary* events from distinct branches or *not*—and *only* for these—the principle of individualizing mutual exclusion (PIME) hinders the definibility of a joint probability, just as it also interdicts a logical product endowed with actualized semantic counterpart (a detailed illustration is worked out in Ref. 9, pp. 977–983): On the infraprobabilistic level of conceptualization of strict individuality there act "existential" mutual exclusions.

(c) It can be easily seen that these existential mutual exclusions are the direct consequence of the *purely potential character of the object-entity* $\eta_\Delta \leftarrow \Delta R$ and that, in their turn, they entail gaps of probabilistic and logical confrontability. More radically, they entail *gaps of any sort of counterfactual qualifiability*:

* Out of one single realization $\eta_\Delta \leftarrow \Delta R$ of a monolith of mere potentialities it is not possible to draw simultaneously two distinct and existentially mutually incompatible actualizations: one single realization of a monolith of potentialities is matter for only one of two possible but mutually exclusive actualizations; the other possible actualization requires another realization $\eta_\Delta \leftarrow \Delta R$ of the monolith of potentialities labelled η_Δ.
* Such a factual situation offers no ground for counterfactual reasoning. *Counterfactual reasoning presupposes alternatives that are endowed with simultaneous factual actuality*, that are only epistemologically "inexistent," i.e. just not known. But concerning still nonexistent transferred aspects there *is* nothing to be known, there is an "essential (ontological) still nonrealized determination," an *individual* essential still nonrealized determination, and thus *a fortiori* also a probabilistic one.

So,

The set of probability values from the probability tree of a transferred description does not form a "classical polytope" in the sense of Pitowsky[12] *or Beltrametti and Maczinski*[13]*: the conditions* (b) *and* (c) *violate the concept while the condition* (a) *transgresses it.*
As to the set of propositions corresponding to the probability values from the probability tree of a transferred description, it cannot be encompassed in a "classical" Boolean structure, nor in a lattice-structure.

3.3.9. Independent Delimitators and Views and Physical Definability of Strictly Potential Entities

The treelike spacetime structure identified above is involved in *any* probabilistic description, accomplished or conceivable. Nevertheless it has remained hidden. Only by the use of epistemic operators of delimitation and of examination defined as mutually *independent* operations has it been possible to bring it into explicit evidence.

It is the requirement of independence of the operation of delimitation, with respect to any eventual subsequent examination, that has permitted one to introduce a "fragment of reality" labeled $\eta_\Delta \leftarrow \Delta R$ by a "definition" which is strictly a-descriptional, a-conceptual, a-cognitive; to introduce it by an action that creates it as a monolith of *physically well-determined potentialities* that are nevertheless entirely nondescribed and, as such, captures it in a *reproducible* way thus making it available for any possible future examinations. So also for *incompatible* future examinations which *split* the descendance of this monolith of potentialities into a branching of mutually incompatible individual actualizations, generating a treelike potential-actualization-actualized structure; all this without requiring the false absolute which consists in prejudging concerning the "individuality" or the "statisticity" of the entity labeled $\eta_\Delta \leftarrow \Delta R$, with respect to views that are not yet specified.

3.3.10. Intrinsic Metaconceptualization of a Transferred Probabilistic Description

Probabilizations of a set (universe) of previously existing intrinsic metaconceptualization of an *individual* transferred description have been and are currently performed (in the "classical" style, i.e., without any attempt at a specification in the sense of (11) of the corresponding random phenomenon). But an *explicit* intrinsic metaconceptualization of—directly—a *transferred* probabilistic description is a *very* complex elaboration. It requires two interconnected strata of intrinsic metaconceptualization: On the deepest stratum one has to produce first an individual intrinsic modelization permitting one to regard "the" entity $\eta_\Delta \leftarrow \Delta R$ as a statistical ensemble of some other "corresponding" entities. On a second more superficial stratum one has to produce then an intrinsic modelization of the *statistical* characters of this ensemble, such that the whole is compatible with the transferred probability distributions that are observed. This requires furthermore an intrinsic modelization of the processes of transfer from the involved random phenomenon (11).

If all these interconnected modelizations are achieved, explicitly or only implicitly, *ipso facto* all the branch-spaces from the initially considered transferred probabilistic description are absorbed into only one intrinsically *meta*conceptualized "classical" Kolmogorov probability space wherefrom all the vestiges of the previous phases of conceptualization can be evacuated. This leaves a simple construct cleanly cut off from the reservoir of semantic substance out of which it has been extracted. If after this one tries to specify the structures that were involved in the genesis, an illusion arises which reflects the necessary generalization from backward in time

onto the future part of the time axis. In fact this generalization is necessary as a receptacle for the structures of past conceptualizations identified by an "archaeological" investigation.

3.3.11. Concluding Remark

The process of emergence of a typology of relativized descriptions briefly indicated above offers an illustration of the way in which, by the interrelated effects of the eight definitions and three principles on which it is founded, the $[\Delta, \eta_\Delta, \Diamond, D]$-syntaxis eliminates, qualifies, separates, relates, *and* calls forth definite new sorts of object-entities and aspects.

4. LOCATION OF THE QUANTUM THEORY INSIDE THE $[\Delta, \eta_\Delta, \Diamond, D]$-TYPOLOGY OF RELATIVE DESCRIPTIONS

> "It may be premature to believe that the present philosophy of quantum mechanics will remain a permanent feature of future physical theories; it will remain remarkable, in whatever way our future concepts may develop, that the very study of the external world led to the conclusion that the content of the consciousness is an ultimate reality."—E. P. Wigner, in *The Scientist Speculates*, J. J. Good, ed. (Heinemann, London, 1961), pp. 284–302.

> "The principle of relativity, when added to the classical assumptions regarding isotropy and homogeneity of spacetime, suffices to derive special relativity in all its details.... The quantum revolution seems to resist a description in similar terms. What *physical* law, or set of laws, can we depict, which will explain the nature of the theoretical change?"—I. Pitowsky, *Quantum Probability-Quantum Logic* (Springer, New York, 1989).

> "Is quantum theory universally valid?"—A. Peres and H. Zurek, *Am. J. Phys.* **50**(9), (1982).

We start by formulating the fundamental theorem of which the content and the proof are already obvious to all those who have read the preceding pages:

Theorem 1. *According to the* $[\Delta, \eta_\Delta, \Diamond, D]$*-typology of relative descriptions any quantum mechanical predictive description* $(\psi, \Omega, \{\omega_j\}) \rightsquigarrow [\{\omega_j\}, \tau, \pi(\psi, \Omega)]$ *belongs to a branch of a treelike relativized transferred probabilistic description.*

Proof. Up to mere notations the spacetime treelike structure of a quantum mechanical probability tree (Fig. 1) is strictly that of the tree of a transferred probabilistic description (Fig. 4).

This is a *definition* of the specific characters of quantum mechanics: quantum mechanics is a transferred probabilistic theory. Any definition, unavoidably, has to somehow be referred. This one is referred to the $[\triangle, \eta_\triangle, \diamond, D]$-syntax. It provides now a basis for critical and constructive developments and for precise understanding of long-lasting debates. Detailed examples are given in the following section. Just below we make only a historical remark.

So, *without* having been explicitly perceived in an *integrated* way, this very fundamental structure of a probabilistic transferred description has been nevertheless represented *mathematically* inside the quantum mechanical formalism! Quantum mechanics has captured *and formalized* a *universal* phase of the processes of conceptualization, the *most basic* phase, that of transfer, of extraction from the still strictly unknown and of very first passage into the perceived and qualified. It has realized this for a particular class of physical entities—"microsystems"—but via sophisticated *mathematical* methods and directly for the most complex sort of transferred descriptions, the *probabilistic* ones. This is what confers on quantum mechanics this basic significance and importance that, we feel, is encapsulated in its algorithms.

How has it been possible for the quantum mechanical formalism to emerge? How can the phenomenon be explained?

At the time when the "classical" probabilistic theories were constructed, the individual entities $\eta_\triangle$ *were* connected with an "intrinsic individual model" (cf. the definition of an intrinsic metaconceptualization of an individual transferred description): The classical statistical mechanics represented the values of the aspects that form a thermodynamic view, as means over transfer-values of the "needle positions" of macroscopic devices, generated by interactions of these with "individual microsystems" conceived to behave intrinsically as (in essence) small balls possessing certain objectual aspects (mass, charge) and other state-aspects (position, velocity, energy, etc.). This ball-like model permitted—directly—a "classical" Kolmogorov probabilization with respect to the *whole* intrinsic view involved, including a unique Boolean algebra of events tied with a "classical" Boolean logic. The representation of the initial transferred phase of the conceptualization could be *entirely occulted.* But the ball-like individual intrinsic model for a microsystem failed in various well-known ways. And it so happened that another more satisfactory individual intrinsic model was not invented. Louis de Broglie's model, though seminal, mixed intrinsic individual features—very new and fertile—with already statistical features.[14] Moreover, though introduced by relativistic considerations, quite paradoxically it does not possess the character of a relativistic model.[15] These hybrid features generated implicit refusals that hindered de Broglie's model from grasping one's attention long enough to permit emergence of a purification. So the physicist's minds found themselves confronted with a problem of probabilization of *transferred* data with *non*null dispersions, in the *absence* of an individual intrinsic model. Thus *stopped* at the level of the primary, still transferred phase, they were compelled to overcome the thrust to always rush directly toward the formalization of an intrinsic model, just taking ground upon the transferred aspects but without representing nor formalizing them. Then, by examination and reexamination of the texture of this initial phase which this time was withstanding transcendence, human mind, like an insect that constructs there where it is planted, but an extraordinary insect, for the very first time in the history of thought has built by *collective* efforts a very precise mathematical representation of the initial transferred phase of conceptualization, for—straight—the most complex case, that of a probabilistic description. This was done without knowing explicitly *what* was being worked out! A sort of miracle.

However, this mathematical representation, the formalism of the quantum theory, bears the stigmata of its self-ignorance. The usual writings of the quantum theory as well as their verbal accompaniments reveal nothing of the general significances encrypted in them. The quantum mechanical vectors, operators, equations, probability measures, appear as a heap of conceptually mute formal tools, to be manipulated accordingly to just postulated algorithms. This entails sequels. What is not understood cannot be utilized optimally, nor changed. The quantum mechanical algorithms are stagnant and dominating like intangible idols. With respect to them the critical functioning of the mind is blocked.

We have succeeded in assigning a definite status to the quantum mechanical descriptions. But this does not suffice. The quantum mechanical algorithms need a detailed decoding that will draw into light their conceptual contents, thereby exposing them to criticism and permitting optimizations and generalizations.

5. DECODING QUANTUM MECHANICS IN TERMS OF THE $[\triangle, \eta_\triangle, \diamond, D]$-SYNTAX

Throughout what follows the notation HD means "Hilbert–Dirac." We shall identify the $[\triangle, \eta_\triangle, \diamond, D]$-meanings of the HD writings by proofs consisting of identification of terms. The decoding will unfold the conceptual dimensions involved, flattened, in the quantum mechanical writings. This will reveal fundamental meanings hidden in these writings. It will clearly appear *how* the HD formalism produces indeed mathematical representations of transferred probabilistic descriptions. Furthermore, it will appear that the quantum mechanical formalism is endowed with *a hierarchical calculus of relative similitudes*. We shall also identify basic insufficiencies. Finally, inside the $[\Delta, \eta_\Delta, \Diamond, D]$-framework the major questions (universality, indefinite regression, reduction, "objectification," locality) will find proven answers, or guides toward answers, in agreement with views previously expressed by many authors.[16-32]

5.1. Quantum Mechanical Descriptions Involving Dynamical Observables

5.1.1. Delimitators, Object-Entities

Consider the Hilbert space $\{|\psi\rangle\} = E_\psi$ of the *normalized* state vectors of a studied microsystem S.

Lemma 1. *Each state vector $|\psi\rangle \in E_\psi$ can be regarded as a mathematical representation of a strictly potential object-entity $\eta_\Delta = \eta_\psi$ introduced by a purely physical delimitator $\Delta = \Delta_\psi$ consisting of an operation P_ψ of state preparation.*

Proof. Inside the $[\Delta, \eta_\Delta, \Diamond, D]$-syntaxis the production of any object-entity η_Δ is symbolized with the help of a delimitator Δ acting on "reality" (R), namely, by writing

$$\Delta R \rightarrow \eta_\Delta$$

On the other hand, the objects studied by the quantum mechanical formalism are called "states of microsystems" and are represented by normalized state vectors $|\psi\rangle \in E_\psi$ introduced by a corresponding operator of state preparation P_ψ that acts on some previously existing initial state $|\psi_i\rangle$. So we write

$$P_\psi\, |\psi_i\rangle \rightarrow |\psi\rangle \qquad (15)$$

($\rightarrow$: produces). By comparison of these two expressions, we get

$$[\Delta = \Delta_\psi] \Leftrightarrow P_\psi, \qquad R \Leftrightarrow |\psi_i\rangle, \qquad [\eta_\Delta = \eta_\psi] \Leftrightarrow |\psi\rangle \qquad (16)$$

($\Leftrightarrow$: corresponds to; Δ_ψ the delimitator of the entity $\eta_\Delta = \eta_\psi$; η_ψ: the physical entity labeled by $|\psi\rangle$, i.e., the state of S with state vector $|\psi\rangle$). ∎

It follows from Lemma 1 that a *superposition* $|\psi_{12}\rangle = \lambda_1 |\psi_1\rangle + \lambda_2 |\psi_2|\rangle$ of any two *normalized* state vectors $(|\psi_1\rangle, |\psi_2\rangle)$ belonging to E_ψ, being *also* a normalized state vector from E_ψ, plays also the role of an *object*-entity introduced by a delimitator represented by a corresponding operator $P_{\psi 12}$.

Generalization. The condition (6) from the definition of relative existences had led us to regard any delimitator Δ, inasmuch as it is efficient at all, as acting on some "place in R" that is not "orthogonal to Δ." This suggests that the symbol $|\psi_i\rangle$ can be replaced by a more general label $|\psi_i\rangle'$ denoting any "place" in R that is "nonorthogonal" in the sense of (6) to the operator P_ψ. So we introduce the generalized notations

$$P_\psi\, |\psi_i\rangle' \rightarrow |\psi\rangle \qquad (15')$$

$$P_\psi \Leftrightarrow \Delta_\psi, \qquad |\psi\rangle \Leftrightarrow \eta_\psi, \qquad |\psi_i\rangle' \Leftrightarrow R \qquad (16')$$

which can be applied to the case also of "creations" of the studied microsystem S *itself* by a "reaction" involving elementary particles, out of some initial state $|\psi_i\rangle'$ of *another* type of systems S' different from S (which entails the creation out of $|\psi_i\rangle'$ of also some state $|\psi\rangle$ of S, along with S itself).

5.1.2. Views

The previous identification is trivial. But the following one is less trivial.

It is currently assumed that the eigenvectors of the quantum mechanical observables are descriptors of the same nature as the normalized state vectors. By confrontation with the $[\Delta, \eta_\Delta, \Diamond, D]$-syntax it will appear that such a belief is erroneous.

Lemma 2. *According to the $[\Delta, \eta_\Delta, \Diamond, D]$-syntax, a quantum mechanical observable Ω introduces a conceptual delimitator Δ_Ω—expressed mathematically by the corresponding equation $\Omega\, |u_\alpha\rangle = \omega_\alpha\, |u_\alpha\rangle$—that delimits a family of views $\Diamond_\Omega$ each one of which consists of a pair of two correlated aspect-views, a corpuscular transfer aspect-view of Ω, $\Diamond_{\Omega_c}$, and a wave aspect-view of Ω, $\Diamond_{\Omega_w}$.*

Proof. Consider a quantum mechanical dynamical observable Ω (a self-adjoint operator whose eigenvectors form a basis in E_ψ) and the corresponding equation $\Omega\, |u_\alpha\rangle = \omega_\alpha\, |u_\alpha\rangle$, $\alpha \in \alpha$, α an index set in general continuous. This equation determines simultaneously a class of mutually distinct real eigenvalues $\{\omega_\alpha\}$ and (supposing a nondegenerate situation) a class $\{|u_\alpha\rangle\}$ of one-to-one corresponding eigenvectors that are mutually orthogonal but in general not normalized.

Examine first the ensemble of eigenvalues $\{\omega_\alpha\}$. In general this is a continuous ensemble, and even if in particular it is discrete it still is in general infinite, while inside the $[\Delta, \eta_\Delta, \Diamond, D]$-syntax all the definitions and principles involve a finite number of qualifications. But (see Section 2) each quantum mechanical eigenvalue ω_α is theoretically connected via a connection function $f_\Omega(V_\alpha) = \omega_\alpha$, with a directly observable *localized* mark, a "needle-position value" V_α produced on an Ω-apparatus by an interaction between this apparatus with a replica of the studied state with state vector $|\psi\rangle$. Now a class $\{V_\alpha\}$ of such needle-position values can be *operationally* defined only *relative* to some origin 0_ω of the observable values V_α corresponding to the eigenvalues ω_α, and to some *unit* u_ω for them. The pair $(0_\omega, u_\omega)$ selects out of $\{V_\alpha\}$ a corresponding *discrete* subset $\{V_j\}_d \subset \{V_\alpha\}$ of needle-position values, $j \in J$, J a discrete set. Furthermore any given real investigation introduces only some *finite* subset $\{V_j\}_f \subset \{V_j\}_d$. So, in each definite investigation only a finite ensemble $\{V_j\}_f$ of needle-position values V_j comes in. By the connection $\omega_j = f_\Omega(V_j)$ this subset $\{V_j\}_f$ corresponds to a finite subset $\{\omega_j\}_f \subset \{\omega_\alpha\}$ of eigenvalues ω_j that is *relative* to the pair $(u_\omega, 0_\omega)$. These remarks together with Theorem 1, Lemma 1, the general $[\Delta, \eta_\Delta, \Diamond, D]$-definition of an aspect-view $\Diamond_g$, and the particular $[\Delta, \eta_\Delta, \Diamond, D]$-definition of a transfer-aspect-view, entail that each definite investigation concerning the quantum mechanical observable Ω brings in

- a factual *transfer*-aspect-view involving an aspect of which the values in the sense of (1) are the marks V_j transferred on the utilized apparatus for Ω measurements;
- a corresponding formal aspect-view involving an aspect of which the values in the sense of (1) are the numbers $\omega_j = f_\Omega(V_j)$ calculated from the equation $\Omega\,|u_j\rangle = \omega_j\,|u_j\rangle$.

Now the *localized* character of the makrs V_j is typically a "corpuscular-like" character. Therefore the first view specified above will be denoted $\Diamond_{\Omega_{Fc}}$ (*F*: factual, *c*: corpuscular) and will be called the *factual corpuscular aspect-view* of Ω introducing the aspect Ω_{Fc} with values V_j. The second view will be symbolized by $\Diamond_{\Omega_c}$ (*c*: corpuscular) and will be called the *corpuscular aspect-view* of Ω introducing the aspect Ω_c with values ω_j. And in agreement with the $[\Delta, \eta_\Delta, \Diamond, D]$-definition of a structure (1), we write

$$\Omega_{Fc} \supset \bigvee_j V_j, \quad j \geqslant 1, \quad j \in J,$$
$$V_j \wedge V_{j'} = \varnothing, \quad \forall (j \neq j'), \quad (j, j') \in J \qquad (17_1)$$

$$\Omega_c \supset \bigvee_j \omega_j, \quad j \geqslant 1, \quad j \in J,$$
$$\omega_j \wedge \omega_{j'} = \varnothing, \quad \forall (j \neq j'), \quad (j, j') \in J \qquad (17_2)$$

where J is a *discrete* and *finite* but *arbitrarily rich* index set *relative* to a given choice $(u_\omega, 0_\omega)$.

Consider now also the ensemble $\{|u_j\rangle\}$ of the eigenkets of Ω corresponding to the eigenvalues ω_j from (17) introduced by a definite choice $(u_\omega, 0_\omega)$. This is a discrete and finite subset $\{|u_j\rangle\}_f \subset \{|u_\alpha\rangle\}$. Now, the general $[\Delta, \eta_\Delta, \Diamond, D]$-definition of an aspect-view $\Diamond_g$ permits us to regard the discrete and finite set $\{|u_j\rangle\}_f$ as tied with a conceptual and formal, *non*transferred *wave aspect-view* $\Diamond_{\Omega_w}$ (*w*: wave) introducing an aspect Ω_w of which the *values* are the eigenkets $|u_j\rangle$ from $\{|u_j\rangle\}_f$: indeed according to the HD formalism *any* two different eigenkets from $\{|u_\alpha\rangle\}$ are orthogonal, so we have inside $\{|u_j\rangle\}_f$

$$\Omega_w \supset \bigvee_j |u_j\rangle, \quad j \geqslant 1, \quad j \in J,$$
$$|u_j\rangle \wedge |u_{j'}\rangle = \varnothing, \quad \forall (j \neq j'), \quad (j, j') \in J \qquad (18)$$

with J the *discrete* and *finite* but arbitrarily rich index set relative to the choice $(u_\omega, 0_\omega)$, which is again a $[\Delta, \eta_\Delta, \Diamond, D]$-structure (1).

So each choice $(u_\omega, 0_\omega)$ determines a formal two-aspects view

$$[\Diamond_\Omega = \Diamond_{\Omega_c} \vee \Diamond_{\Omega_w}, (u_\omega, 0_\omega)] \qquad (19)$$

This view will be called *the formal dynamical quantum mechanical view determined by the observable Ω and the choice $(u_\omega, 0_\omega)$*. But there exists an infinite family of possible choices $(u_\omega, 0_\omega)$, and to each one of these there corresponds a given quantum mechanical dynamical view (19). So the observable Ω can be regarded to act like a conceptual delimitator Δ_Ω which, by the formal procedure of delimitation consisting of the solution of the equation $\Omega\,|u_j\rangle = \omega_j\,|u_j\rangle$, determines an infinite family $\{\Diamond_\Omega = \Diamond_{\Omega_c} \vee \Diamond_{\Omega_w}\}$ of dynamical quantum mechanical views. So, finally, we can write

$$\Omega\,|u_\alpha\rangle = \omega_\alpha\,|u_\alpha\rangle \Leftrightarrow \Delta_\Omega : \Delta_\Omega R \to \{\Diamond_\Omega = \Diamond_{\Omega_c} \vee \Diamond_{\Omega_w}\}, \forall (u_\omega, 0_\omega) \;\blacksquare \qquad (19')$$

The factual view (17_1) *is devoid of mathematical HD representation.*

We point out that the treatment (17)–(19) commanded by the $[\Delta, \eta_\Delta, \Diamond, D]$-conditions of finiteness might have significant consequences concerning the well-known problem of the rigorous measurability of this or that self-adjoint dynamical operator. Furthermore it can be related with nonstandard analysis.

Remark

Lemma 1 establishes that each normalized state vector $|\psi\rangle \in E_\psi$ can be regarded as a mathematical representation of an $[\Delta, \eta_\Delta, \Diamond, D]$-*object*-entity $\eta_\Delta = \eta_\psi$, while

Lemma 2 shows that nonnormalized eigenvectors $|u_j\rangle$ *of a dynamical observable* Ω *play the* $[\Delta, \eta_\Delta, \Diamond, D]$-*role of values* [*in the sense of* (1)] *of the corresponding wave aspect-views* $\Diamond_{\Omega}$ *from* (18), (19), (19′).

This establishes a *fundamental* $[\Delta, \eta_\Delta, \Diamond, D]$-*distinction between normalized state vectors and in general nonnormalized eigenvectors of a dynamical observable*, which confirms conclusions otherwise reached before[1] (by difficult and long analyses).

5.1.3. An Eigenvector of a Discrete Dynamical Observable: Object-Entity and Aspect-View Value

Consider a quantum mechanical dynamical observable with discrete spectrum, for instance bound-state Hamiltonian, kinetic momentum, spin.

Lemma 3. *A normalized eigenvector* $|u_j\rangle$ *of a quantum mechanical dynamical observable* Ω *with discrete spectrum can play—both—the descriptional role of object-entity* (16) $\eta_\Delta = \eta_{|uj\rangle}$ *and the descriptional role of an aspect-view value from a view* (18)–(19′) *corresponding to* Ω.

Proof. Obvious, from Lemmas 1 and 2 and their proofs. ▮

According to the principle of separation PS, in any *given* quantum mechanical description these two roles ought to be explicitly distinguished. But in the current use of quantum mechanics they are *not*. These violations of PS, because of the central physical importance of the eigenvectors of dynamical observables and of the seminal part that they have played in the genesis of quantum mechanics, have induced endless confusions.

- They have strongly favored the *complete* identification (Dirac) of normalized state vectors corresponding to preparable states, with eigenstates of an (any) observable. Correlatively:
- they have hindered also a clear perception of the distinction between operations of state preparation and measurement operations;
- they have washed out the distinction between a principle of superposition concerning exclusively normalized state vectors corresponding to preparable states, and Born's principle of spectral decomposition of a normalized state vector, on the basis of eigenvectors of any observable.

All this knot has particularly paralyzing effects when one tries to understand the descriptional status of the quantum theory, and—quite especially—when one tries to make sense of the quantum mechanical representation of a measurement process: The way in which human mind researches "significances" involves the distinctions codified in the $[\Delta, \eta_\Delta, \Diamond, D]$-syntax.

5.1.4. Dynamical Quantum Mechanical Epistemic Referentials

According to the $[\Delta, \eta_\Delta, \Diamond, D]$-syntax any pairing of a delimitator and a view defines an *a priori* possible epistemic referential. So a dynamical quantum mechanical epistemic referential can be defined as any pair

$$(\Delta_\psi, \Diamond_\Omega) = (P_\psi, \Diamond_{\Omega} \vee \Diamond_{\Omega}), \qquad \forall\, |\psi\rangle, \forall \Omega \tag{20}$$

5.1.5. Descriptions $D(\Delta_\psi, |\psi\rangle, \Diamond_\Omega)$ and Relative Existence for the Pairs $(|\psi\rangle, \Diamond_\Omega)$

Consider a pair $(|\psi\rangle, \Omega)$.

Lemma 4. *The quantum mechanical spectral decomposition* $|\psi\rangle = \sum_\alpha |u_\alpha\rangle\langle u_\alpha|\,|\psi\rangle$ *of* $|\psi\rangle$ *on the basis of eigenvectors* $\{|u_\alpha\rangle\}$ *introduced by the dynamical observable* Ω *possesses the* $[\Delta, \eta_\Delta, \Diamond, D]$-*status of the family of all the relative descriptions* $\{D(\Delta_\psi, |\psi\rangle, \Diamond_\Omega)\}$ *of the object-entity* $\eta_\psi = |\psi\rangle$ *via the family of all the wave-aspect views* (18) *introduced by* Ω.

Proof. Lemma 2 identifies an eigenvector $|u_j\rangle$ of Ω with a value of a wave aspect-view (18) introduced by Ω. But in the $[\Delta, \eta_\Delta, \Diamond, D]$-definition (2) of an aspect view each value of the considered aspect emerges as an outcome of an operation of examination of the studied entity by a corresponding operator of examination. This permits one to regard the quantum mechanical operator of projection $P_j = |u_j\rangle\langle u_j|$ onto the eigenvector $|u_j\rangle$ as *a mathematical representation of the operation of examination that produces the value* $|u_j\rangle$ *of the wave-aspect-view* Ω_w *of* Ω. It also permits one to regard the quantum mechanical projector $P_{\{u\alpha\}} = \sum_\alpha P_\alpha = \sum_\alpha |u_\alpha\rangle\langle u_\alpha|$ onto the whole basis of eigenvectors $\{|u_\alpha\rangle\}$ of Ω as a synthetic mathematical representation of the family of *all* the operations of examination by all the wave aspect-views (18), $\forall (u_\alpha, 0_\alpha)$, involved in the infinite family (19′) of views (19) introduced by Ω. Furthermore, according to quantum mechanics we have

$$P_j\,|\psi\rangle = |u_j\rangle\langle u_j|\psi\rangle = \langle u_j|\psi\rangle\,|u_j\rangle,$$
$$P_{\{u\alpha\}}\,|\psi = \textstyle\sum_\alpha |u_\alpha\rangle\langle u_\alpha|\psi\rangle = \sum_\alpha\langle u_\alpha|\psi\rangle\,|u_\alpha\rangle$$

According now to the $[\Delta, \eta_\Delta, \diamond, D]$-syntax, for any *one* wave aspect-view value $|u_j\rangle$ involved in a view (18) we have $\diamond_{u_j}\,|\psi\rangle \rightarrow D(\Delta_\psi, |\psi\rangle, \diamond_{u_j})$ and for a whole wave aspect-view Ω_w we have $\diamond_{\Omega_w}\,|\psi\rangle \rightarrow D(\Delta_\psi, |\psi\rangle, \diamond_{\Omega_w})$. So finally the following correspondences obtain. For one index $\alpha = j$ we can write

$$[P_j\,|\psi\rangle = \langle u_j|\psi\rangle\,|u_j\rangle] \Leftrightarrow D(\Delta_\psi, |\psi\rangle, \diamond_{u_j}) \tag{21}$$

For one subset $\{|u_j\rangle\}_f \subset \{|u_\alpha\rangle\}$ corresponding to one whole wave-aspect view $\diamond_{\Omega_w}$ (18) (with the notation $P_{\{uj\}} = \sum_j \langle u_j|\psi\rangle\,|u_j\rangle$) we have

$$[P_{\{uj\}}\,|\psi\rangle = \textstyle\sum_j \langle u_j|\psi\rangle\,|u_j\rangle] \Leftrightarrow D(\Delta_\psi, |\psi\rangle, \diamond_{\Omega_w}) \tag{22}$$

For the whole basis $\{|u_\alpha\rangle\}$, i.e., for all the subsets $\{|u_j\rangle\}_f \subset \{|u_\alpha\rangle\}$ corresponding to the whole infinite family (19′) of wave aspect-views, we have

$$[P_{\{u\alpha\}}\,|\psi\rangle = \textstyle\sum_\alpha \langle u_\alpha|\psi\rangle\,|u_\alpha\rangle] \Leftrightarrow \{D(\Delta_\psi, |\psi\rangle, \diamond_{\Omega_w})\} \quad ▮ \tag{23}$$

Notice that in the descriptions (21), (22), (23), each nonnumerical qualification $|u_j\rangle$ or $|u_\alpha\rangle$ is tied with a numerical qualification $\langle u_j\,|\psi\rangle$ or $\langle u_\alpha\,|\psi\rangle$. The physical significance of these numerical qualifications is defined in the proof of the Theorem 1′.

Remark

Lemma 4 [Eq. (22)] introduces the *spectral decomposition* of a state vector on the basis of an observable, as a *relative description*, whereas Lemma 1 entails that a *superposition* of two or more *normalized* state vectors plays the $[\Delta, \eta_\Delta, \diamond, D]$-role of an *object-entity*. This establishes a fundamental distinction between spectral decompositions and superpositions of normalized state vectors.

But a state vector $|\psi\rangle$ being given , if we desire to study it via a given wave-aspect-view $\diamond_{\Omega_w}$, how do we know that the chosen object-entity $|\psi\rangle$ and the chosen view $\diamond_{\Omega_w}$ do mutually exist in the sense of (8)?

Lemma 5. *The quantum mechanical closure condition for the basis introduced by any dynamical observable Ω—which defines the concept of "basis"—ensures the $[\Delta, \eta_\Delta, \diamond, D]$-mutual existence* (8) *for any object-entity $|\psi\rangle$ and any wave-aspect view* (18): *this amounts to the quantum mechanical* principle *of spectral decomposition* (*decomposability*).

Proof. By the very definition of the concept of a quantum mechanical observable, the eigenkets of any observable form a "basis" with respect to the Hilbert space of the normalized state vectors $\{|\psi\rangle\} = E_\psi$. This means by definition that for any observable Ω with eigenvectors $\{|u_\alpha\rangle\}$ the closure relation

$$\forall\Omega, \qquad \textstyle\sum_\alpha |u_\alpha\rangle\langle u_\alpha| = \mathbf{1} \tag{24}$$

does hold ($\mathbf{1}$ is the identity operator on E_ψ). The closure relation (24) ensures that any state vector $|\psi\rangle \in E_\psi$ can be rewritten as $\sum_\alpha \langle u_\alpha|\psi\rangle\,|u_\alpha\rangle$: *It is the unique quantum mechanical prerequisite for the possibility of the* (*any*) *descriptions* (23), *so also* (22). Inside the $[\Delta, \eta_\Delta, \diamond, D]$-syntax now, the unique prerequisite for the possibility of the description of an object-entity η_Δ with respect to a view $\diamond$ is the condition (8) of mutual relative existence. So with the writings (22)–(23) and Lemmas 1 and 2, the assertion of the closure (24) for *any* pair $(|\psi\rangle, \Omega)$—i.e. the principle of spectral decomposability—is readily recognized to ensure the $[\Delta, \eta_\Delta, \diamond, D]$-condition (8) of mutual relative existence for any $\eta_\Delta = |\psi\rangle$ and *any* wave-aspect view (18) of Ω. ▮

Lemma 6. *The mutual existence* (8) *for any $\eta_\Delta = |\psi\rangle$ and any wave-aspect view holds also for any pair* $(|\psi\rangle, \diamond_{\Omega})$ *formed with any complete view* (19), *its corpuscular-aspect view included.*

Proof. According to the quantum mechanical equation $\Omega\,|u_\alpha\rangle = \omega_\alpha\,|u_\alpha\rangle$, in the absence of degeneracy any value $|u_j\rangle$ of a wave-aspect view (18) of Ω corresponds to an eigenvalue ω_j of Ω. This, by definition, extends the mutual existence (8) ensured by (24) for any wave-aspect-view (18), to also any pair $(|\psi\rangle, \diamond_{\Omega_c} \vee \diamond_{\Omega_w})$ formed with a complete view (19). ▮

5.1.6. Quantum Mechanical Dynamical Descriptions

Lemma 6 entails as a $[\Delta, \eta_\Delta, \diamond, D]$-consequence that the relative description $D(\Delta_\psi, |\psi\rangle, \diamond_{\Omega})$ of any object-entity $|\psi\rangle \in E_\psi$ via any view (19), is possible. According to Theorem 1 any such description is a transferred probabilistic description belonging to a branch of a quantum mechanical probability tree. But what, more specifically, are the characteristics of such a description?

Theorem 1′. *A quantum mechanical predictive description $D(\Delta_\psi, |\psi\rangle, \diamond_{\Omega})$ is a transferred probabilistic description expressed in* *mathematical HD terms where the posited factual probability law is calculated with the help of the numerical qualifications* $\langle u_j | \psi \rangle$ *from the formal relative description* (22) *corresponding to the acting view* $\langle\Omega\rangle$.

Proof. A view $\langle\Omega\rangle = \langle\Omega_c\rangle \vee \langle\Omega_w\rangle$ from (19) is a two-aspects view. The $[\triangle, \eta_\triangle, \Diamond, D]$-definitions 2 and 3 of an aspect view and a view entail that on any one given level of description there holds the following "additivity of examinations":

$$(\langle 1\rangle \vee \langle 2\rangle)\eta_\triangle = \langle 1\rangle\eta_\triangle \vee \langle 2\rangle\eta_\triangle \tag{25}$$

(on the contrary, in a *meta*description the views from the previous descriptions do in general "interact" by cross aspects, which suppresses the additivity of examinations). So in our case, with $\langle\Omega\rangle\eta_\triangle \rightarrow D(\triangle, \eta_\triangle, \langle\Omega\rangle)$, we have

$$\begin{aligned} D(\triangle, \eta_\triangle, \langle\Omega\rangle) &= D(\triangle_\psi, |\psi\rangle, \langle\Omega_c\rangle \vee \langle\Omega_w\rangle) \\ &= D((\triangle_\psi, |\psi\rangle, \langle\Omega_c\rangle) \vee D((\triangle_\psi, |\psi\rangle, \langle\Omega_w\rangle) \end{aligned} \tag{26}$$

The contents of the two terms from the right side of (26) can be identified as follows. According to Theorem 1, *globally* $D(\triangle_\psi, |\psi\rangle, \langle\Omega_c\rangle \vee \langle\Omega_w\rangle)$ is a transferred *probabilistic* relative description of type (14) or (14′) where—explicitly—the acting view is *exclusively* the *corpuscular* aspect-view $\langle\Omega_c\rangle$ with aspect Ω_c and aspect-values ω_j (see Figs. 1 and 4 and their captions). This corpuscular aspect-view is involved in *only* the first term from (26). We have shown (Section 2) that a formal quantum mechanical probability chain $(\psi, \Omega, \{\omega_j\}) \rightsquigarrow [\{\omega_j\}, \tau, \pi(\psi, \Omega)]$ corresponds to a factual probability chain $(P_\psi, M_\Omega, \{V_j\}) \rightsquigarrow [\{V_j\}, \tau_F, \pi(P_\psi, M_\Omega)]$ via the connecting relations $\omega_j = f_\Omega(V_j)$ and $\pi(\psi, \omega_j) = \pi(P_\psi, M_\Omega, V_j) = |\langle u_j | \psi \rangle|^2$. According to the general relativized reconstructions (14), (14′) of a probability chain and to Lemma 2, Eq. (17_1), this factual probability chain must be rewritten as

$$\begin{aligned} &D(P_\psi, \eta_\psi, \langle\Omega_{Fc}\rangle) \\ &\quad \Leftrightarrow [\{(\triangle_\psi \eta_{\psi i} \rightarrow \eta_\psi, \langle\Omega_{Fc}\rangle\eta_\psi \rightarrow V_j]_k, k = 1, 2, ..., K\}, \{V_j, j \in J\}] \\ &\quad \rightarrow [\{V_j\}, \tau_F, \pi(P_\psi, M_\Omega)] \end{aligned} \tag{26$_1$}$$

where $\eta_{\psi i}$, η_ψ are the factual object-states labeled respectively by the state vectors $|\psi_i\rangle$, $|\psi\rangle$; $\langle\Omega_{Fc}\rangle$ is the factual corpuscular-aspect view (17_1) with values V_j; $[\langle\Omega_{Fc}\rangle\eta_\psi \rightarrow V_j] \Leftrightarrow M_\Omega$ is one *individual* process of examination corresponding to one *individual* measurement evolution M_Ω; τ_F is the total factual algebra on $\{V_j\}$; $\pi(P_\psi, M_\Omega)$ is the probability law "induced" (com-

patible with) the statistical distribution factually found on $(\{V_j\}, \tau_F)$. This relativized reexpression of type (14) of the transferred probabilistic description from the first term (26), though each element in it is explicit and symbolized, is not a *mathematical* representation. Its mathematical translation in HD language is given by the connecting relations $\omega_j = f_\Omega(V_j)$ and $\pi(\psi, \omega_j) = \pi(P_\psi, M_\Omega, V_j) = |\langle u_j | \psi \rangle|^2$ and the definitions (16):

$$\begin{aligned} &D(\triangle_\psi, |\psi\rangle, \langle\Omega_c\rangle) \\ &\quad \Leftrightarrow [\{(\triangle_\psi |\psi_i\rangle \rightarrow |\psi\rangle, \langle\Omega_c\rangle |\psi\rangle \rightarrow \omega_j]_k, k = 1, 2, ..., K\}, \{\omega_j, j \in J\}] \\ &\quad \rightarrow [\{\omega_j\}, \tau_\Omega, \pi(\psi, \Omega)] \end{aligned} \tag{26$_2$}$$

(simplified notations (14) with obvious significance). The representation (26_2) is the *first* term from (26).

Now, what probability measure $\pi(\psi, \Omega)$ does quantum mechanics assert inside the probability space from (26_2)? This measure is determined (Section 2) via the law of total probabilities from the elementary probability densities $\pi(\psi, \omega_j)$ and these in their turn are posited to be $\pi(\psi, \omega_j) = |\langle u_j | \psi \rangle|^2$. But the numbers $|\langle u_j | \psi \rangle|$ are produced by precisely the purely formal description (22) that is the *second* term from the right side of (26) (contrary to current belief, these numbers cannot be deduced). ∎

Notice how the $[\triangle, \eta_\triangle, \Diamond, D]$-syntaxis yields an entirely explained and organized perception of the contents of the predictive probabilistic descriptions of the quantum theory. In agreement with Theorem 1, from a purely factual viewpoint the description $D(\triangle, \eta_\triangle, \langle\Omega\rangle)$ is indeed—strictly—just a transferred probabilistic description with the characteristic spacetime structure represented in Figs. 1 and 4; namely, the probabilistic transferred description of $|\psi\rangle$ by $\langle\Omega_c\rangle$ that translates in HD terms *only* the factual transfer processes $\langle\Omega_{Fc}\rangle\eta_\psi \rightarrow V_j$ corresponding exclusively to the corpuscular transfer aspect Ω_c of Ω. But—with the degree of necessity of a veritable calculus—the $[\triangle, \eta_\triangle, \Diamond, D]$-*syntaxis reveals that the description* $D(\triangle, \eta_\triangle, \langle\Omega\rangle)$ *contains furthermore the purely "conceptual" term* $D(\triangle, \eta_\triangle, \langle\Omega_w\rangle)$ which defines the probability measure.

5.2. Dirac's Brakets: A Hierarchy of "Similitude Descriptions"

We shall now show that besides the dynamical descriptions the quantum mechanical formalism involves also another sort of relative descriptions, directly connected with Dirac's ket-bra algebra: a hierarchy of "similitude descriptions" expressed by a veritable calculus of similitudes.

Views of "Relative Similitude." Consider the ensemble $\{|u_\alpha\rangle\}$, $\forall \Omega$, of all the bases of eigenvectors determined by all the quantum mechanical observables Ω. Let us denote this ensemble by E_u. According to Dirac's formalization the ensemble E_ψ of all the normalized kets $|\psi\rangle$ (state vectors) and the ensemble E_u of the (in general) nonnormalized eigenkets $|u_\alpha\rangle$ (eigenvectors) form together a vector space $E = E_\psi + E_u$ of "generalized kets" $|x\rangle$, and each ket $|x\rangle \in E$ possesses inside the dual E^* of E a corresponding bra $\langle x| \in E^*$. Dirac defines any bra from E^* as a functional on the whole space E, to be calculated as $\langle x|x'\rangle = c(x, x') = \int x^*(\mathbf{r})\, x'(\mathbf{r})\, d\mathbf{r}$, $\langle x|x\rangle = 1$, $\forall (|x\rangle, |x'\rangle) \in E$. Does such a functional possess some definite $[\Delta, \eta_\Delta, \diamond, D]$ "significance"?

To get preliminary hints, consider a bra $\langle u_\alpha|$ corresponding to a *non*-normalized ket $|u_\alpha\rangle$ from E_u belonging to the basis of a dynamical observable Ω with continuous spectrum. If applied to a normalized ket $|\psi\rangle \in E_\psi$) this bra produces the number $\langle u_\alpha|\psi\rangle$. This number occurs in the expansion (23) $P_{\{u\alpha\}}|\psi\rangle = \sum_\alpha \langle u_\alpha|\psi\rangle |u_\alpha\rangle$ of $|\psi\rangle$ on the basis $\{|u_\alpha\rangle\}$ of Ω. It also occurs in "conceptual" relative *descriptions* (22) of $|\psi\rangle$ via a wave-aspect view Ω_w (18) determined by Ω. Inside such a relative description the braket $\langle u_\alpha|\psi\rangle$ yields for $|\psi\rangle$, considered as an *object*-entity η_ψ, a "numerical *qualification*" associated with the "value" $|u_\alpha\rangle$ of the wave-aspect view Ω_w [in the sense of (1) and (18)]. We suggest that:

> The term $\langle u_\alpha|\psi\rangle\, |u_\alpha\rangle$ can be consistently read as: "$|\psi\rangle$ is *like* $|u_\alpha\rangle$ to a *degree* measured (directly or indirectly) by the absolute value $|\langle u_\alpha|\psi\rangle|$ of the complex number $\langle u_\alpha|\psi\rangle$."

Indeed a numerical measure of a similitude has to be real; furthermore, since the norm of any $|\psi\rangle \in E_\psi$ is 1, in this case we have always $0 \leqslant |\langle u_\alpha|\psi\rangle| \leqslant 1$, which fits satisfactorily with the proposed interpretation. This interpretation amounts to:

> a qualifica*tion* of $|\psi\rangle$ regarded as an object-entity, where the qualifica*tor* (the operator of examination) is the bra $\langle u_\alpha|$ that corresponds to the ket $|u_\alpha\rangle$, the ket $|u_\alpha\rangle$ itself holds the role of a "model" or *sample* of a *quality* (a possible species of wave-like phenomenon), and the number $|\langle u_\alpha|\psi\rangle|$ plays the role of a numerical measure of the degree of some sort of similitude between the object-entity $|\psi\rangle$ and the sample of a quality $|u_\alpha\rangle$ taken as a standard.

The same remarks hold, *mutatis mutandis*, when the roles of $|u_\alpha\rangle$ and $|\psi\rangle$ are inverted: The bra $\langle\psi|$ produces the complex number $\langle\psi|u_\alpha\rangle$ that admits of an interpretation analogous to that of $\langle u_\alpha|\psi\rangle$ but where the roles (object-entity, qualificator) are inverted. Such an inversion does not change the absolute value involved, which alone can be regarded as a

numeric… some sort of similitude: so the state
vector |ψ… ⟩ of a wavelike phenomenon to the
same de… is like the state vector $|\psi\rangle$: Dirac's
ket-bra … east—is insensitive to the distinction
between … methodologically required inside the
$[\Delta, \eta_\Delta,$ … principle of separation (PS): *it seems to*
extract … *mutual" similitude that is an invariant of*
the inver… *ity, view*).
Con… $|\phi\rangle$ involving two normalized kets
$|\psi\rangle \in E_\psi$ … e value of this number, again, can be
regarded … e of similitude of $|\phi\rangle \in E_\psi$ considered
as an ob… $\langle\psi|$ corresponding to the ket $|\psi\rangle \in E_\psi$,
regarded … n the roles of $|\psi\rangle$ and $|\phi\rangle$ are inverted
the same … und. This time, however, the numerical
qualifica… the braket involves the *set* of *all* the
possible … the quantum theory the kets $|\ \rangle \in E_\psi$
are pools … ny dynamical operator Ω, and also for
non"dyna… spect to the formerly examined case of
similitu… lized ket from E_ψ and a nonnormalized
ket from … are in the presence of a certain
*meta*simil… with kets and bras seems to imply a
hierarchy … uishes between nonnormalized eigenkets
and norm…

Fina… $\langle v_\gamma|u_\alpha\rangle$ involving two *non*normalized
eigenke… $v_\gamma\rangle) \in E_u$, $(|u_\alpha\rangle, |v_\gamma\rangle) \notin E_\psi$, with $|u_\alpha\rangle$
belongin… servable A and $|v_\gamma\rangle$ to the basis of some
other ob… mber $\langle v_\gamma|u_\alpha\rangle$ belongs to the set of the
transform… $v_\gamma|u_\alpha\rangle$ from the basis of A' to that of A.
Though … annot be an object-entity in a relative
descripti… not normalized, it cannot be associated
with pro… , correlatively, it is not producible by a
physical … ψ), nothing hinders $|v_\gamma\rangle$ from playing the
role of a… cts of relative descriptions. Though $|v_\gamma\rangle$
is itself … 1), belonging to wave aspect-views (18)
determin… it *can—as such*—be (meta)qualified by
the bra … he term $\langle u_\alpha|v_\gamma\rangle\, |u_\alpha\rangle$ from the quantum
mechanic… n $|v_\gamma\rangle = \sum_{\alpha'} \langle u_{\alpha'}|v_\gamma\rangle\, |u_{\alpha'}\rangle$ can be read
as follow… *it qualifies a normalized ket* $|\psi\rangle \in E_\psi$ *as*
$|u_\alpha\rangle$ *do*… by the absolute value $|\langle u_\alpha|v_\gamma\rangle|$ of the
complex … is comparable to the qualification of a
qualifica… the sense of (1) of some aspect—say the
value "r… via a value of another aspect, say the

value "red" belonging to a "color-view", which would yield an assertion of the form "this 'round' (circular surface) is like my color-view 'red' to the degree n, with n a real number" (such an assertion could indicate what fraction of the considered circular surface is red). So the number $|\langle u_\alpha | v_\gamma \rangle|$ seems to be interpretable as a measure of a sort of "semantic proximity" between two qualificators. However we exclude for the moment this case from the following (its character of *meta*description requires *explicit* reference to the kets $|\psi\rangle \in E_\psi$ qualified on a first level of description):

Theorem 2. *According to the* $[\Delta, \eta_\Delta, \Diamond, D]$*-syntaxis,*

(a) *any bra* $\langle x| \in E^*$ *can be considered to determine a family of aspect-views of similitude with the corresponding ket* $|x\rangle \in E$;

(b) *correlatively, any braket* $\langle x|x'\rangle$ *involving at least one normalized ket* $|\psi\rangle \in E_\psi$ *can be regarded as a family of relative descriptions of some sort of "mutual" similitude between the two kets* $|x\rangle \in E$ *and* $|x'\rangle \in E$, *invariant to the inversion of the roles of object-entity and view;*

(c) *a relative description of mutual similitude that involves two normalized kets is a metadescription with respect to any relative description of mutual similitude that involves only one of these normalized kets and a nonnormalized ket.*

Proof. The HD formalism permits one to represent any bra $\langle x| \in E^*$ as the operator $(P_x/|x\rangle)$ of projection onto $|x\rangle \in E$ divided by $|x\rangle$: $\langle x| \Leftrightarrow (P_x/|x\rangle)$. Consider the interval $[0, 1]$ and choose a "unit" u that is a divisor of this interval. The juxtapositions of u in $[0, 1]$ starting from 0 and ending on 1 determine a finite set of points defining a finite set $\{r_f\}$ of real numbers, $f \in F(u)$, $F(u)$ a finite but arbitrarily rich index set corresponding to the choice of the unit u. We have $\{r_f\} \subset [0, 1]$.

Compare now with the $[\Delta, \eta_\Delta, \Diamond, D]$-definition of an aspect-view. It appears that the bra $\langle x| \Leftrightarrow (P_x/|x\rangle)$ *associated* with the finite set $\{r_f\}$ determined by a given choice of a unit u, can be regarded as a (formal) *aspect-view of similitude with the ket* $|x\rangle$ (symbol $\Diamond_{sx}$) introducing an *aspect* $s(|x\rangle)$ *of similitude with the ket* $|x\rangle \in E$ (*s: similitude*) with structure (1). We can then write

$$[\langle x| \Leftrightarrow ((P_x/|x\rangle), u)] \Leftrightarrow \Diamond_{sx}$$
$$s(|x\rangle) \supset \vee_f r_f, \quad f \geq 1; \quad f \in F(u), \quad \{r_f\} \subset [0, 1] \tag{27}$$
$$r_f \wedge r_{f'} = \varnothing, \quad \forall (f \neq f')], \quad 0 \leq |r_f| \leq 1$$

But for every bra $\langle x| \in E^*$ there exists a whole (infinite) family of possible choices u. So the point (a) is established.

Consider now a braket $\langle x|x'\rangle$ where $|x'\rangle$ is a normalized ket $|\psi\rangle \in E_\psi$. Imagine the examination $\Diamond_{sx}\, |x'\rangle = (P_x/|x\rangle)\, |x'\rangle$ of the ket $|x'\rangle$—regarded as an object-entity delimited by a corresponding conceptual delimitator $\Delta(|x'\rangle)$—by a given aspect-view of similitude $\Diamond_{sx}$ (27). This examination can yield only a number r_f, $f \in F(u)$, $r_f \in (\{r_f\} \subset [0, 1])$. But Dirac's definitions entail that the absolute value of any braket $\langle x|x'\rangle$ involving at least one normalized ket satisfies the condition $r_f(x, x') 0 \leq |\langle x|x'\rangle| \leq 1$. So we can *define* the result of the examination $\Diamond_{sx}\, |x'\rangle = (P_x/|x\rangle)\, |x'\rangle$ as the first number from $r_f(x, x') \in \{r_f\}$ that is equal or superior to $|\langle x|x'\rangle|$ (a definition of precisely this nature comes in whenever a physical measurement involving a definite choice of some unit is performed). This definition now entails that the object-entity ket $|x'\rangle$ exists in the sense of (7) with respect to any aspect-view of similitude (27) determined by the bra $\langle x|$. So the ket $|x'\rangle \in E$ can obtain a relative description in any epistemic referential $(\Delta(|x'\rangle), \Diamond_{sx})$. And because $0 \leq r_f(x, x') \leq 1$, the number $r_f(x, x')$—*in agreement with usual language*—can be regarded as a convenient numerical expression of the "degree of similitude" of the object-ket $|x'\rangle$ with the ket $|x\rangle$ to which corresponds the bra $\langle x| \Leftrightarrow (P_x/|x\rangle)$. According to the $[\Delta, \eta_\Delta, \Diamond, D]$-syntaxis and to (27), we can then write for any given choice u and any pair of kets $(|x\rangle, |x'\rangle)$ involving at least one normalized ket $|\psi\rangle \in E_\psi$

$$\Delta(|x'\rangle) E_\psi \to |x'\rangle, \ \langle x|x'\rangle \to [D(\Delta(|x'\rangle), |x'\rangle, \langle x|) = r_f(x, x')] \tag{28}$$

(with $E_\psi \Leftrightarrow R$; $[\langle x| \Leftrightarrow (P_x/|x\rangle), u)] \Leftrightarrow \Diamond$). When, conversely, $|x'\rangle$ is treated as the generator of views (27) while $|x\rangle$ is treated as object-entity, accordingly to the principle of separation (PS) a *new* description arises, $D(\Delta(|x\rangle), |x\rangle, \langle x'|) \neq D(\Delta(|x'\rangle), |x'\rangle, \langle x|)$. But both introduce the same number $r_f(x, x') = r_f(x', x)$. So a description (28) is *invariant* to the inversion of the $[\Delta, \eta_\Delta, \Diamond, D]$-roles of object-entity and view. Therefore, we call it a *description of "mutual" similitude between* $|x\rangle$ and $|x'\rangle$. Since there exists a whole (infinite) family of possible choices u, the point (b) is established.

Finally, consider a definite description (28). Imagine first that this description involves a *non*normalized eigenket $|u_\alpha\rangle \in E_u$. Then the number $r_f(x, x')$ from this description can be tied with only one braket, namely $\langle u_\alpha|\psi\rangle$. Imagine now that, the normalized ket $|\psi\rangle$ from the former description (28) being maintained, $|u_\alpha\rangle \in E_u$ is replaced by some also normalized ket $|\phi\rangle \in E_\psi$, which yields a new description (28). What is the relation between the new number $r_f(x, x') = |\langle\phi|\psi\rangle| = |\langle\psi|\phi\rangle|$ produced by this new description and the number $|\langle u_\alpha|\psi\rangle| = |\langle\psi|u_\alpha\rangle|$ produced by the preceding one? The answer appears when both $|\psi\rangle$ and $|\phi\rangle$ are represented by spectral decomposition (23) in the basis $\{|u_\alpha\rangle\}$ to which belongs $|u_\alpha\rangle$: inside the new description (28) the number $r_f(x, x') = |\langle\phi|\psi\rangle| = |\langle\psi|\phi\rangle|$ refers globally to *all* the measures of mutual similitude obtained when both expansions $|\psi\rangle = \sum_{\alpha'} \langle u_{\alpha'}|\psi\rangle\, |u_{\alpha'}\rangle$ and $|\phi\rangle = \sum_{\alpha''} \langle u_{\alpha''}|\phi\rangle\, |u_{\alpha''}\rangle$ are introduced in the braket $\langle\phi|\psi\rangle$ (or $\langle\psi|\phi\rangle$). This suffices for showing that the new description (28) is a metadescription with respect to the former one, which establishes the point (c). ∎

Remark

According to Spencer Brown[34] the "calculus of differences"—already in its only qualitative stage—is the *most basic* epistemic mover. Therefore, it seems very remarkable indeed that

> *Quantum mechanics incorporates a calculus of mutual similitudes* (*or mutual differences*)!

This calculus is represented by a very general sort of algebra that probably *can be conveniently generalized or adapted to represent numerically other—or any—operations of comparison relatively to this or that aspect, leading to corresponding descriptions of mutual similitudes.* The $[\triangle, \eta_\triangle, \diamond, D]$-syntaxis permits one to recognize this fact, which is precious in the endeavor toward a mathematical epistemology.

5.3. The Flattening Confusion between the Aspects "Norm" and "Similitude" and Consequences

Similitude Versus Norm. To stay in agreement with usual language, the concept of mutual similitude requires a *finite* maximal valuation when something is compared with *itself.* Furthermore, in a satisfactory mathematization one desires this finite maximal valuation to be invariantly the *same* whatever is *what* is compared to itself. Therefore Dirac's definition of any braket $\langle x|x'\rangle$ as $\langle x|x'\rangle = \int x^*(\mathbf{r})\, x'(\mathbf{r})\, d\mathbf{r}$ and $(\langle x|x\rangle) = 1$, $\forall\, |x\rangle \in E$ is satisfactory—and even necessary—*insofar* as it is related with a concept of mutual similitude between the ket $|x'\rangle$ and the ket $|x\rangle$ corresponding to the bra $\langle x|$.

But consider now the (formal) aspect "norm" (or length) of a ket. By the HD definitions, any ket $|x\rangle \in E$ does exist in the sense of (7) with respect to this aspect also. The values in the sense of (1) of *this* aspect *can*, without any conceptual inconvenience, be represented numerically, for *any* ket $|\;\rangle \in E$, by the *unrestricted* scalar product $\langle\;|\;\rangle$, finite or *divergent*,

according as the considered ket $|\;\rangle$ is, respectively, a normalized ket from E_ψ or a nonnormalized eigenket from only E_u. Indeed, the *length* of a *non*-normalized *eigen*ket belonging exclusively to E_u (not to the intersection of E_u with E_ψ) is a qualification of that eigenket which, inside quantum mechanics, is *devoid of physical counterpart*. Only the form of the function involved by a nonnormalized eigenket belonging exclusively to E_u points toward a specifiable designatum: a sample, a model of wavelike pattern. But its length—contrary to that of a ket from E_ψ or an eigenket from $E_u \cap E_\psi$—has no defined physical significance in quantum mechanics. A nonnormalizable eigenket acts *exclusively* as a value in the sense of (1) of the wave aspect-views (18) determined by the observable to the basis of which that eigenket belongs (cf. also Ref. 1, pp. 1433, 1442–1446). So with respect to the aspect norm there is *no need* of Dirac's treatment[33] (p. 58 top):

> "With an orthogonal representation, the natural thing to do is to normalize the basic vectors rather than to leave their length arbitrary, and so introduce a further simplification in the representation. However, it is possible to normalize them only if the parameters that label them all take on discrete values. If any of these parameters are continuous variables...the basic vectors are of infinite length. Some other *procedure is then needed to fix the numerical factors by which the basic vectors may be multiplied. To get a convenient method of handling this question a new mathematical notation is required....*"

(our italics). Then Dirac defines his δ-function and "arranges" to have $\langle\xi'|\xi''\rangle = \delta(\xi' - \xi'')$, ($\xi'$, ξ'': the *eigenvalues* corresponding to $\langle\xi|$ and $\langle\xi'|$, respectively) [Ref. 8, p. 63, Eq. (21)]. This "procedure," useful with respect to the aspect of mutual similitude, is superfluous with respect to the aspect of norm.

> *The* $[\triangle, \eta_\triangle, \diamond, D]$*-syntaxis removes a noxious confusion between the value of the length of a ket* $|\;\rangle$ *and the value of its self-similitude.*

Generalized Kets, Superposition of State Vectors, Spectral Decomposition of a State Vector. Dirac's absence of distinction between normalized kets and nonnormalized eigenkets, connected with his absence of distinction between mutual similitude and norm, is furthermore tied structurally with absense of distinction between superposition of several normalized state vectors and spectral decomposition of one normalized state vector on the basis of an observable. Dirac writes[33] (p. 12):

> "The procedure of expressing a state as a result of superposition of a number of other states is a mathematical procedure that is always

permissible, independent of any reference to physical conditions, like the procedure of resolving a wave into Fourier components."

In consequence of Lemmas 1–6 and Theorems 1′ and 2, Dirac's treatment of all the linear compositions of "generalized" kets—*indistinctly*—as "superpositions" *in the purely mathematical sense*, appears inside the $[\Delta, \eta_\Delta, \Diamond, D]$-syntaxis as a huge knot of confusions where are flatteningly mixed up delimitators Δ and views $\Diamond$ as well as the phase of delimitation $\Delta R \to \eta_\Delta$ and that of examination $\Diamond \eta_\Delta$, and, correlatively, object-entities and relative descriptions. The $[\Delta, \eta_\Delta, \Diamond, D]$-*semantic* expressed by the $[\Delta, \eta_\Delta, \Diamond, D]$-syntaxis is left out from Dirac's algebra where only the algorithmic and numerical features are worked out. In physics such a sort of mathematical "generality" hinders the understanding of the phenomena represented by the algorithms.

5.4. The Big Epistemic Problems

5.4.1. Universality

> "...we conclude that although it can describe *anything*, a quantum description cannot include *everything*. Whenever we place the interface between the quantum world and our experimental records, this interface is either an abstract probability rule with no physical description, or it must have *two incompatible descriptions*, one quantized and one classical, and it is the "translation" between them which is probabilistic."—A. Peres and H. Zurek, *Am. J. Phys.* **50**, (1982).

Theorem 3. *The quantum mechanical dynamical descriptions can in principle be* performed *for any physical subsystem of the Universe, being in this sense universal. But they cannot* include *intrinsic metaconceptualizations of physical systems, nor*, a fortiori, *intrinsic models for them ("classical" descriptions). On the contrary any intrinsic metaconceptualization of a physical system includes implicitly the corresponding quantum mechanical descriptions; and the extracted intrinsic models presuppose them.*

Proof. A metadescription includes the previously achieved descriptions that it concerns, but not *vice versa* (the $[\Delta, \eta_\Delta, \Diamond, D]$-definition of a metadescription). Any intrinsic metaconceptualization is a metadescription of some transferred description, so—like also any intrinsic model extracted from it—it presupposes a previously achieved transferred description, while a transferred description presupposes no previously

achieved description (the $[\Delta, \eta_\Delta, \Diamond, D]$-definition of a transferred description, an intrinsic metaconceptualization, an intrinsic model). These are general $[\Delta, \eta_\Delta, \Diamond, D]$-assertions. According to Theorems 1 and 1′ the quantum mechanical descriptions are transferred (probabilistic) descriptions, while what is called a "classical" description refers—by implicit current consensus—to intrinsic models in the sense of the $[\Delta, \eta_\Delta, \Diamond, D]$-definition of these. So the preceding general assertions do apply to the relation between quantum mechanical descriptions and classical ones. ▮

The "boundary" between quantum mechanical and classical descriptions concerns directly *how* one describes, not *what* one describes, nor with what degree of *accuracy*. It concerns the very *structure* of the description, not its content or "precision." It follows that *no sort of numerical considerations (passages to the limit, sufficiently small actions, wavelengths, etc.) can fully account for this boundary.*(26)

The restriction to subsystems of the Universe corresponds to

Theorem 4. *The concept of a transferred description of the Universe is—both—factually impossible and self-contradictory.*

Proof. By the general $[\Delta, \eta_\Delta, \Diamond, D]$-definitions: (a) the object of an intrinsic metaconceptualization is a previously achieved transferred description of an entity η_Δ *delimited by a purely physical delimitator* Δ; (b) the description consists exclusively of observable marks produced by the process of transferred examination $\Diamond_T \eta_\Delta$ on an "apparatus" which is posited as an object able to *interact* with η_Δ but *spatially exterior* to it outside the durations of interactions. A purely physical delimitator able to delimit the Universe as a whole is a factual impossibility for any human observer, while the existence of an apparatus in the sense specified above is incompatible with the Universe as a whole in the role of the object-entity η_Δ. ▮

5.4.2. Measurement: Reduction, Indefinite Regression

Theorem 5. *The "reduction problem" is a false problem that is effaced by an explicit knowledge of the spacetime structure of the quantum mechanical dynamical descriptions and of the* $[\Delta, \eta_\Delta, \Diamond, D]$-*definition of a transferred probabilistic description.*

Proof. In a quantum mechanical dynamical description [Theorem 1′, Eqs. (26), (26_1), (26_2)], the state vector $|\psi\rangle$ labels only *globally* the object-entity η_ψ delimited by the identically reproducible operation of state preparation P_ψ. Namely, it is a synthetic mathematical HD construct elaborated *exclusively* as a representation of the ensemble $\{\pi(\psi, \Omega), \forall\Omega\}$ of all the *probability measures* from the probability tree *of the operation of state preparation* P_ψ *that precedes all the elementary outcomes* V_j of the individual elementary quantum mechanical chain experiments $P_\psi - M_\Omega - V_j$ from that probability tree (cf. Lemma 1, Eq. (22), Theorem 1′, as well as the proof from Ref. 1, pp. 1408–1410). As *such* the descriptor $|\psi\rangle$ *cannot* be "changed" by an individual outcome V_j of an individual elementary quantum mechanical chain experiment $P_\psi - M_\Omega - V_j$. This impossibility holds *notwithstanding* the fact that in general each individual outcome V_j destroys the state produced by the operation of state preparation P_ψ:

— These destructions that *end* the *elementary* chain experiments $P_\psi - M_\Omega - V_j$ cannot affect the *predictive* role assigned to the descriptor $|\psi\rangle$, which is *imparted by—exclusively—the operation of preparation* P_ψ *realized beforehand.*

— An individual event from the random phenomenon that brings forth the universe of elementary events from a probability tree cannot change the probability measures from that tree, even if it contributes to the definition of one of these.

So it is erroneous to assert that the state vector $|\psi\rangle$ gets "reduced" when an eigenvalue $\omega_j = f_\Omega(V_j)$ is registered. ▌

Notice that *the elementary chain experiments* $P_\psi - M_\Omega - V_j$ *from the random phenomenon that produces a quantum mechanical probability tree are devöid of any HD mathematical representation.* This favors an unclear perception of the difference between the two levels of conceptualization where are placed on the one hand the elementary quantum mechanical chain experiments $P_\psi - M_\Omega - V_j$ and on the other hand the descriptor $|\psi\rangle$. The confusion is furthermore increased by the character of an as yet strictly nonqualified, a-cognitive *monolith* of potentialities possessed by the object-entity delimited by an operation of state preparation P_ψ. Very elusively, such an object-entity is at the same time "individual" and "multiple".

By definition the object-entity of the Schrödinger equation is *exclusively* "$|\psi\rangle$," i.e., the set of all the probability measures $\{\pi(\psi, \Omega), \forall\Omega\}$. This equation simply has not been constructed in order to represent also the elementary quantum mechanical chain experiments that produce the probability spaces where the probability measures $\{\pi(\psi, \Omega), \forall\Omega\}$ are posited.

As to the state-destructive action of the individual outcomes V_j, according to the $[\Delta, \eta_\Delta, \diamond, D]$-syntax each *one* of these (j fixed) can be regarded as labeling a new operation of state preparation (a new physical delimitator) which, if identically reiterated, generates a new own probability tree with its own new state-vector. According to the $[\Delta, \eta_\Delta, \diamond, D]$-syntax there is nothing paradoxical in this. Any modality for delimiting an object-entity out of the continuum of reality is, by the definition 1, a possible delimitator, and this, if it is purely physical, can found a transferred description.

Theorem 6. *What is called the "problem" of infinite regression (the Wigner's friend "problem") in fact is not a problem but just a general characteristic of any transferred description, whether individual or probabilistic.*

Proof. Obvious. Consider the $[\Delta, \eta_\Delta, \diamond, D]$-definition of a transferred description. It involves an object-entity and an "apparatus" on which are registered observable transferred marks entailed by the interaction with the object entity. If another object entity is considered, consisting of the former one plus the corresponding apparatus, and for this new entity an equally *transferred* (meta)description is imagined, this, by definition, will introduce a new apparatus, and so on indefinitely. The chain, furthermore, will have to replace progressively each *intrinsic* model, presupposed more or less implicitly (and effectively available) for the "classical" apparatuses, by a poorer description, an only transferred description: another sort of regression. ▌

5.4.3. "Objectification"

In very interesting recent works[17–19] Busch, Lahti, and Mittelstaedt (BPM) have proven that the quantum mechanical formalism is incompatible with

— a "hypothesis of weak objectification" ("it is possible to assign the property a_j of the observable A to the system S in state φ such that the value a_j pertains objectively to the system but this value is subjectively unknown to the observer");

— a "hypothesis of strong objectification" ("it is possible to attribute an eigenstate φ^{a_j} of A to a system S in state φ such that S is actually in the state φ^{a_j} but this state is unknown to the observer who knows only its probability").

It seems interesting to notice that the $[\Delta, \eta_\Delta, \diamond, D]$-approach quite essentially imposes the same conclusion (throughout this work we have preferred to use the term "intrinsic(ally)" as opposed to "subjective(ly)," instead of the term "objective(ly)," because a "transferred" qualification is *at least* as objective as an intrinsic one, in the sense of possibility of *consensus*, so we continue to use it here):

Theorem 7. *A quantum mechanical dynamical description leaves the involved object-entity strictly non qualified intrinsically.*

Proof. By the general $[\Delta, \eta_\Delta, \diamond, D]$-definition of a transferred description and in consequence of Theorems 1, 1′ and 3:

— (a) An eigenvalue a_j of the (dynamical) observable A is a value in the sense of (1) of a formal *transfer* corpuscular-aspect *view* defined by A [Theorem 1, Theorem 1′, and Lemma 2 Eq. (17)]. The number $a_j = f_A(V_j)$ translates into mathematical HD terms a factual observable feature V_j *of a macroscopic device* D_A for "measurements of A," the feature V_j being a value in the sense of (1) of a factual *transfer* corpuscular-aspect view corresponding to the formal transfer corpuscular-aspect view of A [the general $[\Delta, \eta_\Delta, \diamond, D]$-definition of a transferred description and Theorem 1′, Eq. (26_1)]. So "weak objectification" is not asserted (is contradicted) by a dynamical quantum mechanical description.

— (b) An eigenstate φ^{uj} of A is a value in the sense of (1) of a *formal* wave-aspect *view* defined by A [Lemma 2, Eq. (18)]. The eigenstate φ^{uj} of A is utilized in a dynamical quantum mechanical description involving A, *exclusively for the calculation of the probability density* $\pi(\psi, a_j) = |\langle \varphi^{uj} | \psi \rangle|^2$ (Theorem 1′); being not normalized in general, it cannot even be conceived of, in *general*, as possessing the significance of a physically realizable object-entity-state, though in particular this can happen (Lemmas 1 and 3). So in general "strong objectification" is not asserted by a dynamical quantum mechanical description.

Finally:

— (c) Suppose that the transferred qualifications a_j, φ^{uj} from a dynamical quantum mechanical description were also intrinsic qualifications of the involved object-entity. Then the considered description would also contain an intrinsic metaconceptualization of the involved object-entity. This would contradict Theorem 3. ▮

"Actualized" reasoning[35] is incompatible with the quantum mechanical dynamical descriptions.

5.4.4. Remark on Locality, Probabilities, Realism

In the language established in this work, we can make the following remarks.

Bell's theorem of nonlocality[36] concerns the probability tree of an operation of preparation corresponding to "a two-systems state vector $|\psi_{12}\rangle$" (which involves a transfer view (19) with a tensor-product mathematical HD representation). The inequalities from Bell's theorem have been recognized by Pitowsky[12] and Beltrametti and Maczinsky[13] to express the condition that the hidden variables utilized for an intrinsic metaconceptualization of this quantum mechanical probability tree shall belong to only one probability space (shall form a "classical polytope"). From an *exclusively probabilistic* viewpoint this condition—*by definition*—is necessary and sufficient for a "classification" of the involved quantum mechanical transferred description, since what is called a "classical (probabilistic) polytope" amounts to just this condition.

But, as other authors also have perceived,[24,25,37] the inequalities themselves, while they decide concerning the probabilistic structure, *do strictly not concern the "problem of realism"* (how the physical reality "is intrinsically," i.e. how it can be represented by an intrinsic model). This problem is tied exclusively with the *supplementary* condition that the probability measure from the unique "classicising" probability space required by the inequalities shall furthermore satisfy Bell's "locality" (independence) relations, and *this* condition has *nothing* to do with *probabilities*. It concerns only the intrinsic model of the *individual* phenomena involved, namely the elementary quantum mechanical chain experiments $P_{\psi 12} - M_{\Omega 12} - V_{j12}$ from the probability tree of the experiment, which are devoid of any mathematical HD representation. These, in their turn, involve durations, that is, *individual* relative times, concerning which quantum mechanics tells strictly nothing and furthermore certainly implies *non*true assumptions, since the formalism is Newtonian.

> The intrinsic metaconceptualization of the *individual* quantum mechanical chain experiments $P_{\psi 12} - M_{\Omega 12} - V_{j12}$, with their individual relative times, by taking into consideration relativistic requirements adapted to *micro*phenomena[15] as well as the quantum mechanical principle of superposition[1] (pp. 1429–1446), seems the unique route toward a satisfactory explicitly "realist" microphysics.

And we agree with F. Selleri and V. L. Lepore[22,23] that so far nothing forbids the exclusion of an intrinsic modelization compatible with relativity.[26,29,30,32]

6. CONCLUSION

We have started from quantum mechanics. We have identified the *spacetime* structure of the probabilistic organization of this theory. This has shown the fundamental importance, in quantum mechanics, of the *operations* by which the studied object-entities (states of microsystems) are introduced (state preparations) and are qualified (measurement operations): these two sorts of basic operations, *their* spacetime structure, determine and reveal the specificities and the novelties of the quantum mechanical formalism.

Recognition of this fact led us to develop a general $[\Delta, \eta_\Delta, \diamond, D]$-syntax "of relativized conceptualization" where any description is explicitly and systematically referred to the two basic sorts of epistemic operations by which the observer, unavoidably involved, introduces the object-entity and examines it. This syntax incorporates, purified by maximal generalization, the essential features of the quantum mechanical representations. Inside the typology of relativized descriptions generated by the $[\Delta, \eta_\Delta, \diamond, D]$-syntax the specific descriptional type of the quantum mechanical predictive descriptions acquired a precise definition. This definition brought forth the remarkable fact that quantum mechanics has incorporated—for the first time in the history of thought—an explicit representation of a particular instance of a first phase of conceptualization, that of transferred description, which—*universally*—lies at the bottom of *any* description whatsoever. An explicit *mathematical* representation realized directly for the most complex sort of transferred descriptions, the probabilistic ones.

Furthermore the *reflexive* back-examination of the quantum mechanical formalism inside the general $[\Delta, \eta_\Delta, \diamond, D]$-syntax that stems from it, permitted us to identify in the quantum mechanical formalism various other basic descriptional significances, *as well as the mathematical expression of each one of these.* We have also found basic insufficiencies, false problems, and certain "solutions" or guides to such. Globally, we hope to have shown this, clarifying and possibly fertile new insights have been obtained.

But the most important consequence might lie in the future. It becomes conceivable now to attempt a *mathematical* reconstruction of the *general* syntax of relativized conceptualization, and thus a mathematical epistemology, by convenient generalizations of the quantum mechanical mathematical expressions of the basic descriptional significances identified in the formalism. Indeed—in agreement with all the $[\Delta, \eta_\Delta, \diamond, D]$-requirements—one can try to represent any object-entity by a "ket" from a "vector space of kets," while any view is represented with the help of a convenient "bra." The corresponding generalized braket algebra could be

constructed to represent a general hierarchical calculus of relativized similitudes, and to generate representations of any chain of hierarchically connected relative descriptions from the typology defined by the general $[\Delta, \eta_\Delta, \diamond, D]$-syntax. There might, however, arise a necessity of generalization in order to transgress, when adequate, the restrictions involved by the principle of superposition, imposed by the vector-space representations.[1]

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