Spacetime Quantum Probabilities, Relativized Descriptions, and Popperian Propensities. Part I: Spacetime Quantum Probabilities

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Spacetime Quantum Probabilities, Relativized Descriptions, and Popperian Propensities. Part I: Spacetime Quantum Probabilities

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An integrated view concerning the probabilistic organization of quantum mechanics is obtained by systematic confrontation of the Kolmogorov formulation of the abstract theory of probabilities, with the quantum mechanical representation and *its factual counterparts. Because these factual counterparts possess a peculiar spacetime structure stemming from the operations by which the observer produces the studied states (operations of state preparation) and the qualifications of these (operations of measurement), the approach brings forth "probability trees," complex constructs with treelike spacetime support.*

Though it is strictly entailed by confrontation with the abstract theory of probabilities as it now stands, the construct of a quantum mechanical probability tree transgresses *this theory. It indicates the possibility of an extended abstract theory of probabilities including explicit representations of the cognitive operations involved in the probabilistic descriptions. So quantum mechanics appears to be neither a "normal" probabilistic theory nor an "abnormal" one, but a pioneering particular realization of a* future *extended abstract theory of probabilities.*

The consequences of the integrated perception of the probabilistic organization of quantum mechanics are developed constructively. The current identifications of spectral decompositions, with superpositions of states, are removed. Then: (a) Inside the frontiers of the purely operational-observational orthodox formalism, operators of state preparation and the calculus with these are defined consistently with the definition and the calculus of quantum mechanical operators representing measurable dynamical quantities. This permits to grasp the physical meaning of superselection rules. Furthermore, a complement to the quantum theory of measurements is obtained. These prolongations of the orthodox formalism bring forth a "probabilistic incompleteness" of the quantum theory. (b) Beyond quantum mechanics as it now stands, a model is outlined that removes this probabilistic incompleteness, "the [particle + medium] individual model," microscopic *by certain aspects and* cosmic *by others.*

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Globally, the approach draws attention upon the possibility and the interest of a general representation of the descriptions of any kind founded upon the explicit specification of the epistemic operations—with their spacetime features—by which the observer, who always is involved, produces the objects to be qualified and the qualifications of these.

1. INTRODUCTION TO PART I

Quantum mechanics yields probabilistic predictions concerning physical events. Nevertheless, since already more than 60 years, the probabilistic status of quantum mechanics constitutes an unsolved problem. It is currently asserted that quantum mechanics is not a "normal" probabilistic theory, because the various probability spaces defined by it cannot be embedded into a unique probability space, while in all the other probabilistic physical theories this is possible. Still more radically, certain mathematicians hold that, notwithstanding the fact that it introduces probability measures, quantum mechanics simply is not a probability theory.

In what follows it will be shown that quantum mechanics is neither a "normal" nor an "abnormal" realization of the abstract theory of probabilities, but a pioneering (particular and implicit) materialization of a deep-rooted possible future extension of the abstract theory of probabilities as it now stands, incorporating explicit representations for the cognitive operations (with their spacetime structure) involved in a probabilistic description.

The concept of a quantum mechanical probability tree, which is the central construct of the space-time probabilistic organization of quantum mechanics, has been already defined and utilized by us in previous works.(1–5) But we reiterate here its deduction because it yields the basis necessary for all the further developments.

I am happy to have the opportunity to publish this work in the first issue of *Foundations of Physics* dedicated to Sir Karl Popper's 90th birthday.

In the first place, Karl Popper has probably been the very first one in the whole world who has *globally* perceived the structure, a very complex structure indeed, that is introduced by any probabilistic conceptualization, and which, curiously, still remains more or less hidden to the mathematicians, physicists, and philosophers. In Part I only the special materialization of this structure that is involved in the formalism of quantum mechanics will appear. But in Part II, in a subsequent issue of *Foundations of Physics* dedicated to Sir Karl Popper, inside a relativized representation of the descriptions of any sort, will emerge a quite general representation of the probabilistic descriptions, relativized to the epistemic actions by which the observer—necessarily involved—produces the objects to be qualified and the qualifications of these. This relativized representation of the probabilistic descriptions brings forth a significance of the concept of probability measure which can be regarded as a confirmation and a formalized development of the Popperian "propensity" interpretation of the probabilities.(6,7)

In the second place, the two parts of this work considered as a whole will strongly confirm Sir Karl Popper's contention that quantum mechanics, notwithstanding the striking novelty of its formalism, is much less essentially singular, much more "normal" than it is thought to be. However, it will appear that this is so *not* because it is possible to banish the observer from quantum mechanics.(7) On the contrary, this is so because, for the first time in the history of the representations of reality, the quantum theory has captured and formalized a fundamental feature that marks *universally* the *initial* stage of any chain of conceptualization whatever, rendering *this* phase *nonremovably* **dependent** *on the observer (on the "epistemic referential" chosen by him and on the spacetime features of the corresponding epistemic processes)*. This, it will appear, does not in the least hinder objectivity, but brings into evidence all the *relativities* of objectivity.

2. THE QUANTUM MECHANICAL PROBABILITY TREES

2.1. The Abstract Theory of Probabilities, Physical Probabilistic Theories, Quantum Mechanics

In Kolmogorov's formulation of the abstract theory of probabilities any probability measure π is defined inside a probability space $[U, \tau, \pi]$, where $U = \{e_i\}$ ($i \in I$, I an index set) is a universe of elementary events e_i, τ is an algebra of events chosen on U, and π is a probability measure posed on τ. Furthermore, the universe U is conceived to be produced by a random phenomenon. But quite currently this supposed random phenomenon is neither defined nor only symbolized. Throughout what follows this lacuna will be compensated as follows.

Let us denote a random phenomenon by (P, U), where P is an "identically" reproducible procedure, each one realization of which brings forth one elementary event $e_i \in U$, in general variable from one realization of P to another one (notwithstanding the supposed identity of the reiterations), whereby the whole universe U is generated. In order to express explicitly that each probability space is tied with some random phenomenon, we shall always consider a complete "probability *chain*" where the probability space is preceded by the symbolization of the corresponding random phenomenon:

$$(P, \mathrm{U}) \rightsquigarrow [\mathrm{U}, \tau, \pi] \tag{1}$$

The abstract theory of probabilities does not describe specified phenomena; it only introduces symbols and defines the calculi with these characterizing any probabilistic conceptualization of phenomena of any nature. As soon as some specified domain of reality undergoes a probabilistic conceptualization, an interpretation of the abstract theory is obtained. Inside this interpretation, unavoidably, some probability chains are supposed, but where, now, the constituting symbols point, more or less explicitly, toward entities from the described domain of reality. So *a particular semantics* comes in. But very often, when physical problems are treated probabilistically, only the probability measures are defined explicitly and are symbolized. The elementary events and the algebra of events are usually indicated by words only, while quite currently the random phenomenon which produces them remains entirely implicit. However, by reference to the abstract theory of probabilities, it is obvious that without a universe of elementary events, without an algebra of events chosen on this universe, a probability measure simply is not defined. It does not conceptually exist. A probability measure alone is not a concept, it is a rag of a concept. Furthermore, by definition, in the absence of any random phenomenon, a universe of elementary events cannot emerge, hence no probability space either: The probability chains (1) are *indivisible molds* imposed by the abstract theory of probabilities. So what are the particular probability chains specific of quantum mechanics? What is the specific semantics toward which point the quantum mechanical probability chains?

2.2. The Quantum Mechanical Representation of the Probabilistic Aspects from the Theory

For the sake of simplicity, throughout what follows we consider exclusively the basic case of only "one microsystem," whatever definition one associates to this concept. Examination of this basic case will suffice for conveying the essence of our view.

2.2.1. The Formal Quantum Mechanical Probability Chains. Consider a pair $(|\psi\rangle, \mathrm{A})$ where $|\psi\rangle = |\psi(t)\rangle$ is the state vector assigned at the time

t to the considered microsystem S, and A is a Hermitian operator representing a dynamical observable—in the mathematical sense—defined for S. For each such pair the quantum mechanical formalism defines a family of probability densities $\pi(\psi, a_j)$, $j \in J$ (J an index set) for the emergence of an eigenvalue a_j of the observable A when a measurement of A is performed on S in the state $|\psi\rangle$. Namely, it is postulated that the specified probability density can be calculated by use of the formula (for simplicity we suppose a nondegenerate situation)

$$\forall j \in J, \qquad \pi(\psi, a_j) = |\langle u_j | \psi \rangle|^2 \tag{2}$$

where $|u_j\rangle$ is the eigenvector corresponding to the considered eigenvalue a_j, determined, like a_j, by the equation $\mathrm{A}\,|u_j\rangle = a_j\,|u_j\rangle$ for eigenvectors and eigenvalues of A. Usually the algorithm (2) for the computation of probability measures is postulated without any explicit specification of the probability space where the measure (2) is incorporated, nor, *a fortiori*, of the random phenomenon from which this space stems. But it is obvious that the space which contains the measure (2) can be represented by the writing

$$[a, \tau_{\mathrm{A}}, \pi(\psi, \mathrm{A})] \tag{3}$$

where the universe of elementary events $a = \{a_j, j \in J\}$ (J an index set) is the spectrum of the observable A, τ_{A} is the total algebra of events on a, and $\pi(\psi, \mathrm{A})$ is the probability density measure on τ_{A} determined, via the law of total probabilities, by the elementary probability density (2). So the whole probability *chain* corresponding to a space (3) can be represented by the writing

$$[(|\psi\rangle, \mathrm{A}) \rightsquigarrow [a, \tau_{\mathrm{A}}, \pi(\psi, \mathrm{A})] \tag{1'}$$

This is the researched integrated representation of the formal quantum mechanical probability chains, achieved with the help of the quantum mechanical descriptors.

2.2.2. The Factual Quantum Mechanical Probability Chains. The formal chains (1′) are only a coded representation of other, *factual* quantum mechanical probability chains. Let us identify now these factual chains.

The Factual Quantum Mechanical Probability Spaces. We postpone for the moment the specification of the factual random phenomenon corresponding to the symbol $(|\psi\rangle, \mathrm{A})$ from the chain (1′) and we consider first only the space (3) $[a, \tau_{\mathrm{A}}, \pi(\psi, \mathrm{A})]$ involved by this chain. The corresponding factual space can be immediately specified as follows:

$$[V_A(D_A, t_2), \tau_A, \pi(\psi, M_A)] \tag{3'}$$

where A designates an observable and numerically valued physical aspect *of a macroscopic device* D_A able to generate certain materializations of the numerical values to be assigned to the quantum mechanical observable (in the mathematical sense) A, namely "needle positions" of D_A; $\mathrm{V}_A(\mathrm{D}_A, t_A)$ is the universe of all the possible values V_j of the physical aspect A of D_A, a universe brought forth by "one" realization of what is *globally* called a "measurement process" of the observable A, consisting by definition of a very big number of *reiterations* of a registration of a value V_j, operated each time by starting from the state of S symbolized by the state-vector $|\psi\rangle$ *newly prepared* and each such registration covering some spatial domain d_A and beginning at a time t when the state vector of S is $|\psi\rangle$ and then *lasting* for some nonnull time interval $(t_A - t) > 0$ (let us denote this measurement process by $\mathrm{M}_A(\psi, \mathrm{D}_A)$); τ_A is the total algebra on the universe $\mathrm{V}_A(\mathrm{D}_A, t_A)$; $\pi(\psi, \mathrm{M}_A)$ is the density of the probability measure put on τ_A, depending on the state labeled by the state-vector $|\psi\rangle$ and on the measurement process M_A performed on this state.

The probability measure $\pi(\psi, \mathrm{M}_A)$ on the algebra τ_A from the probability space (3′) is determined, via the law of total probabilities, by the probability density $\pi(\psi, \mathrm{M}_A, V_j)$ postulated on the universe $\mathrm{V}_A(\mathrm{D}_A, t_A) = \{V_j, j \in J\}$ of elementary events from this space.

The Factual Quantum Mechanical Random Phenomena. What is the factual random phenomenon that brings forth the universe of elementary events $\mathrm{V}_A(\mathrm{D}_A, t_A) = \{V_j, j \in J\}$ from a factual quantum mechanical probability space (3′)? It seems that up to now nobody has tried to specify explicitly thus random phenomenon, not even the rare authors who have developed explicit researches concerning the probabilistic organization of quantum mechanics (Mackey,[8] Gudder,[9] Suppes,[10] etc.). However, as soon as it is researched, the definition can be easily constructed. It is then found to possess a very complex structure that brings in a sequence of three partial procedures covering three distinct spacetime domains:

— The first partial procedure is the preparation operation $\mathrm{P}(\psi_0)$ which, at its final moment t_0 (supposed to be definable), introduces an initial state of S represented by the state vector $|\psi(t_0)\rangle = |\psi_0\rangle$; this operation covers some nonnull spacetime domain $[\Delta r \times \Delta t_0]_{\mathrm{P}}$.

Prière de lire partout « operation of state generation » au lieu de « operation of preparation » (expression qui permet des confusions avec la « préparation pour mesure » d'un microétat déjà existant) et de remplace $P(\psi_0)$ par $G(\psi_0)$ ou $P(\psi)$ par $G(\psi)$.

— The second partial procedure, which does not necessarily exist, is a process $\mathrm{E}(H, t_0, t)$ of evolution of the initial state of S, leading at the time t to the state with state-vector $\psi(t)\rangle = |\psi\rangle$. When it does exist, this evolution (formally described by the writing $|\psi\rangle = \mathrm{T}(H, t_0, t)\,|\psi_0\rangle$ where $\mathrm{T}(H, t_0, t)$ is the acting propagator) covers some new spacetime interval $[\Delta r \times \Delta t]_{\mathrm{E}}$, where $\Delta t = t - t$.

— The third partial procedure is the measurement operation $M_A(\psi, \mathrm{D}_A)$ from the definition of the observable space (3′), performed on the state of S symbolized by the state vector $\psi\rangle$.

As soon as the time $t \geqslant t_0$ is fixed, the succession

$$P = [\mathrm{P}(\psi_0), \mathrm{E}(H, t_0, t), \mathrm{M}_A(\psi, \mathrm{D}_A)] \tag{4}$$

constitutes "*one* identically reproducible procedure P," each reiteration of P reestablishing the origin of times t_0. Note that the succession of only the first two partial procedures from (4) can be regarded as a preparation operation $\mathrm{P}(\psi)$ producing the *studied* state represented by the state vector $|\psi\rangle = \mathrm{T}(H, t_0, t))\,|\psi_0\rangle$. So we can also write

$$P = [\mathrm{P}(\psi), \mathrm{M}_A(\psi, \mathrm{D}_A)] \tag{4'}$$

where the initial operation $\mathrm{P}(\psi_0)$ and the evolution symbolized by $\mathrm{E}(H, t_0, t)$ become implicit.

Each realization of the procedure P brings forth *one*, V_j, among all the various possible elementary events from the universe of elementary events $U = \mathrm{V}_A$. Thus we are finally in the presence of a random phenomenon (P, U) in the *standard* sense of the term, namely

$$(P, \mathrm{U}) = ([\mathrm{P}(\psi_0), \mathrm{E}(H, t_0, t), \mathrm{M}_A(\psi, \mathrm{D}_A)], \mathrm{V}_A(\mathrm{D}_A, t_A)) \tag{5}$$

or

$$(P, \mathrm{U}) = ([\mathrm{P}(\psi), \mathrm{M}_A(\psi, \mathrm{D}_A)], \mathrm{V}_A(\mathrm{D}_A, t_A)) \tag{5'}$$

The Factual Quantum Mechanical Probability Chains. So the factual quantum mechanical probability chains can be written as follows:

$$([\mathrm{P}(\psi), \mathrm{M}_A(\psi, \mathrm{D}_A)], \mathrm{V}_A(\mathrm{D}_A, t_A)) \rightsquigarrow [\mathrm{V}_A(\mathrm{D}_A, t_A), \tau_A, \pi(\psi, \mathrm{M}_A)] \tag{1''}$$

The expressions (3′) to (1″) indicate now explicitly and exhaustively the specific semantic contents of the quantum mechanical probability chains.

2.2.3. The Connection between the Factual and the Formal Quantum Mechanical Probability Spaces. How can we translate a factual observable quantum mechanical probability space into the corresponding formal space, so as to be able to apply to it the quantum mechanical algorithms?

In quantum mechanics each eigenvalue $a_j \in a$ is posited to be calculable as a function $f_A(V_j)$ of the observed factual value $V_j \in \mathrm{V}_A(\mathrm{D}_A, t_2)$ which is labeled by the same index $j \in J$:

$$a_j = f_A(V_j) \tag{6}$$

Furthermore, each observable elementary probability density $\pi(\psi, \mathrm{M}_A, V_j)$ is posed to be numerically equal to the corresponding formal elementary probability density, i.e., for any $|\psi\rangle$ and any $j \in J$, it is postulated that (degenerate cases being excluded)

$$\pi(\psi, \mathrm{M}_A, V_j) = \pi(\psi, a_j) = |\langle u_j | \psi \rangle|^2 \tag{7}$$

where $|u_j\rangle$ is the eigenvector of the observable A corresponding to the eigenvalue $a_j = f_{\mathrm{A}}(V_j)$. (Notice that thereby a_j can be regarded as a random variable on the factual space (3′), a space that is not defined inside the formalism). In *this* sense, the formal probability density (2) is a "predictional law," verifiable with the help of the relative frequencies of emergence of the observed values V_j, at the limit of large numbers.

The equations (6) and (7) form the key of the code which translates the factual observable quantum mechanical probability space (3′) into the formal space (3). Any quantum mechanical prediction belongs to some formal probability space (3) corresponding to a factual space (3′).

2.2.4. The Processual Roots of the Quantum Mechanical Elementary Events in the Sense of Probabilities. The expression (5) of a factual quantum mechanical random phenomenon involves reiterations of a chain of operations and processes:

[(preparation operation $\mathrm{P}(\psi_0)$)-(evolution process E)-(measurement operation M_A)-(registration of a needle position V_j of the utilized device D_A)] (eqmce)

((eqmce): elementary quantum mechanical chain experiment): *These are the processual roots of the quantum mechanical elementary events in the sense of probabilities.* An elementary quantum mechanical chain experiment possesses a remarkable unobservable *depth* wherefrom emerges into the observable only the extremity V_j, $j \in J$, that contributes to the construction of the factual observable universe of elementary events $\mathrm{V}_A(\mathrm{D}_A, t_A) =$

$\{V_j, j \in J\}$. Each observable quantum mechanical "event" (*non*elementary) from an algebra τ_{A} from a factual quantum mechanical probability space (3′) contains inside its semantic substratum all the unobservable chains of operations and processes forming the elementary quantum mechanical chain experiments that end up with the registration of a needle position V_j contained in that factual observable quantum mechanical event. So *any* quantum mechanical prediction concerns either an elementary quantum mechanical chain experiment, or a union of such experiments. *The elementary quantum mechanical chain experiments (eqmce) yield the "fibers" out of which is made the factual substance of the quantum theory.*

2.2.5. Partial Conclusion. We are now endowed with an explicit knowledge of the relations between, on the one hand, the basic abstract concepts of the probabilistic conceptualization (identically reproducible procedure P, universe of elementary events U, algebra of events τ, probability measure π), and, on the other hand, the quantum mechanical formal descriptors, state vectors $|\psi\rangle$, observables A, eigenvectors $|u_j\rangle$, and eigenvalues a_j. It appears that quantum mechanics contains definite realizations of each basic concept from the abstract theory of probabilities. So, in *this* sense, it can be asserted that quantum mechanics is *not* an "abnormal" probabilistic theory. Furthermore, we have also explicated the specific semantical content assigned by the quantum mechanical description to the basic abstract probabilistic concepts. Now, do these first results entail that quantum mechanics is a "normal" probabilistic theory?

2.3. The Probability Trees of State Preparations

We arrive now at the crucial point of this section, where *new* consequences of the preceding analysis will manifest themselves.

We have shown that any quantum mechanical prediction concerns one or several elementary quantum mechanical chain experiments. We shall now show that the ensemble of all the elementary quantum mechanical chain experiments falls apart into *classes of meta-structures* possessing a treelike spacetime organization.

Let us *fix* a preparation $\mathrm{P}(\psi_0)$, a time interval $\Delta t = t - t_0$, and a Hamiltonian H. That is, let us fix the transform $|\psi\rangle = T(t_0, t, H)\,|\psi_0\rangle$ of $|\psi_0\rangle$. Consider now the ensemble of all the probability chains (5) or (5′) corresponding to the fixed pair $(\mathrm{P}(\psi_0), |\psi\rangle)$ *and* to *all* the distinct dynamical observables A, B, C, D... defined in quantum mechanics: The chains from this ensemble constitute together a certain *unity*, because of their common provenance $(\mathrm{P}(\psi_0), |\psi\rangle)$. What is the spacetime structure of this unity?

For all the chains from the considered unity, the spacetime support of the operation of sate preparation $P(\psi_0)$ and of the Schrödinger evolution $T(H, t_0, t)\,|\psi_0\rangle] = |\psi\rangle$, $t \geq t_0$, of the prepared state, which follows this operation, is, by construction, the *same*, a common spacetime trunk. If in particular $|\psi\rangle \equiv |\psi_0\rangle$, i.e., if $t = t_0$, then the trunk is reduced to the operation of state preparation alone.

Consider now the spacetime supports of the measurement processes M_A involved in this unity. The ensemble of these processes *splits* into subensembles M_X, M_Y,... of mutually "compatible" processes of "measurement evolution" corresponding to mutually commuting observables.

Contrary to many very confusing considerations concerning "successive measurements of compatible observables" (versus the projection postulate) that can be currently found in the textbooks of quantum mechanics, let us stress this; Each *one* measurement evolution from the subensemble M_X is such that each one registration of a value V_j of the "needle position" of the macroscopic device D_X associated with M_X permits one to calculate, from the *unique* datum V_j, via a set of *various* theoretical connecting definitions (6) $a_j = f_A(V_j)$, $b_j = f_B(V_j)$,..., *all* the *different* eigenvalues a_j, b_j,... labeled by the *same* index j, for, respectively, all the observables A, B,... measurable by a process belonging to the class M_X. This entails that *for all the commuting observables corresponding to one same class M_X, the process of registration of a value of the "needle position" of the device D_X can be* one *common process covering one common spacetime support* (no succession whatever is necessary).

Which is *not* possible for two noncommuting observables belonging to two distinct classes M_X and M_Y.

This is what is commonly designated as "Bohr complementary," nothing else.

Now, this entails that, globally, the ensemble of all the factual probability chains (1″) corresponding to a fixed pair $(P(\psi_0), |\psi\rangle)$ constitutes a unity, a meta-construct, *with a branching, treelike spacetime structure.* Let us symbolize this treelike structure by $\mathcal{T}(\psi_0, \psi)$ and let us call it a "quantum mechanical probability tree" (in short, a probability tree). (Since all the probability trees involving the same studied state vector $|\psi\rangle$ introduce the same branch structure, carrying on top the same probability spaces, in contexts where the distinction between the state vector of the initially prepared state and that of the studied state is not relevant we shall assume that $|\psi\rangle \equiv |\psi_0\rangle$ and the abbreviated symbol $\mathcal{T}(\psi)$ can be used).

So the pairs $(P(\psi_0), |\psi\rangle)$ define, on the ensemble of all the quantum mechanical probability chains, a *partition* in probability trees. *A fortiori*,

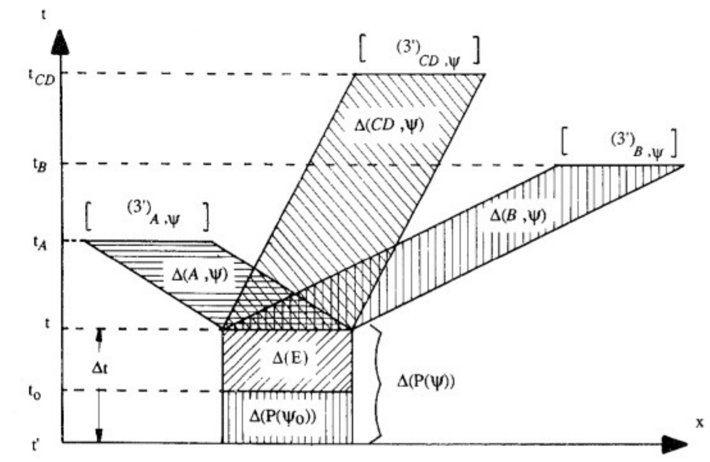

Fig. 1. A quantum mechanical probability tree $\mathcal{T}(P(\psi_0), |\psi\rangle)$

they define such a partition also on the ensemble of all the elementary quantum mechanical chain experiments (eqmce) out of which the quantum mechanical probability chains are made.

Figure 1 provides a simplified example of a probability tree of a state preparation, with only four observables, and making use of somewhat abbreviated notations: *A*, *B*, *C*, *D* are physical observable aspects ("needle-positions" of macroscopic devices) corresponding to the quantum mechanical observables A, B, C, D, respectively. The measurement $M_{C,D}$ corresponds to two commuting observables C, D: (the commutator of C and D is zero, $[C, D] = 0$), while M_A, M_B correspond to two noncommuting observables A, B with: $[A, B] \neq 0$. The notations $(3')_A$, $(3')_B$ and $(3')_{C,D}$ indicate the observational spaces (3′) corresponding, respectively, to the measurement processes M_A, M_B, and M_{CD} performed on the state represented by $|\psi\rangle = T(t_0, t, H)\,|\psi_0\rangle$. Each one of the spaces (3′) emerges at some specific time t_A, t_B, t_{CD}. The commuting observables C, D generate together one common branch producing an observable space (3′) more detailedly characterized, namely with respect to both observables involved. $\Delta(P(\psi_0))$, $\Delta(E)$, $\Delta(P(\psi))$ indicate respectively the spacetime domains covered by the process of: preparation $P(\psi_0)$ of the state with state vector $|\psi_0\rangle$; evolution $E(t_0, t, H)$ represented by $T(t_0, t, H)\,|\psi_0\rangle$ $|\psi\rangle$; or, globally, preparation $P(\psi) = [P(\psi_0), E(t_0, t, H)]$ of the state with state vector $|\psi\rangle$. $\Delta(A, \psi)$, $\Delta(B, \psi)$, $\Delta(CD, \psi)$ indicated respectively the spacetime domains covered by the measurement evolutions M_A, M_B, M_{CD}.

A quantum mechanical probability tree is a remarkably comprehensive metastructure of probability chains. Most of the fundamental algorithms of the quantum mechanical calculus which combine *one* normed state vector, with the dynamical operators representing the quantum mechanical observables, can be defined *inside* — any — *one* tree $\mathscr{T}(\mathrm{P}(\psi_0), |\psi\rangle)$:

— the mean value of an observable A, in a state with state vector $|\psi\rangle$, namely

$$\langle\psi|\,\mathrm{A}\,|\psi\rangle,\ \forall\,|\psi\rangle,\ \forall \mathrm{A}$$

— the uncertainty theorem, for any pair of observables,

$$\langle\psi|\,(\Delta\mathrm{A})^2\,|\psi\rangle\langle\psi|\,(\Delta\mathrm{B})^2\,|\psi\rangle \geqslant |\langle\psi|\,(i/2)(\mathrm{AB}-\mathrm{BA})\,|\psi\rangle| = (1/2)(h/2\pi),\quad \forall\,|\psi\rangle,\ \forall \mathrm{A, B}$$

— the principle of spectral decomposition (expansion postulate)

$$|\psi\rangle = \sum_j c(\psi, a_j)\,|u_j\rangle,\ \forall\,|\psi\rangle,\ \forall \mathrm{A}{:}\ \mathrm{A}\,|u_j\rangle = a_j\,|u_j\rangle,$$

($c(\psi, a_j)$: the expansion coefficients)

which permits one to calculate the probability density $\pi(|\psi\rangle, a_j)$ via the probability postulate

$$\pi(\psi, a_j) = |\langle u_j \mid \psi\rangle|^2 = |c(\psi, a_j)|^2$$

— and, finally, the whole quantum mechanical "transformation theory" from the basis of an observable A, to that of an observable B

$$c(\psi, b_k) = \sum_j \alpha_{kj} c(\psi, a_j),$$

$$\forall \mathrm{A, B}{:}\ \mathrm{A}\,|u_j\rangle = a_j\,|u_j\rangle, \text{ and } \mathrm{B}\,|v_k\rangle = b_k\,|v_k\rangle,\ \forall j \in J,\ \forall k \in K$$

(where J, K, are the index sets for the eigenvalues of A, B, respectively, and $\alpha_{kj} = \langle v_k \mid u_j\rangle$ are the transformation coefficients).

But as soon as either the principle of superposition or the orthodox quantum mechanical representation of successive measurements come into play, the corresponding quantum mechanical algorithms cease to be embeddable into one single probability tree: there the embeddability into one tree hits a limit. *Several trees have to be combined.* So a still higher degree of complexity than that of only one probability tree is formed and acts inside the organization implicitly reached by the probabilistic conceptualization hidden inside the quantum mechanical formalism. The quantum mechanical formalism contains implicit calculi with *whole probability trees.*

2.4. Integrated View

The probabilistic organization of the quantum theory, when it is perceived in a globalized, integrated way, is found to consist of the ensemble of all the probability chains of the type (1′)–(1″) partitioned in subensembles of probability chains possessing a treelike spacetime structure, each one of these corresponding to a pair $(\mathrm{P}(\psi_0), |\psi\rangle)$ of an operation of state preparation and a studied state. This partition is a partition also of the ensemble of the factual elementary quantum mechanical chain experiments which constitute the individual fibers of the quantum mechanical probability chains (1′), (1″). So furthermore it is a partition also of the factual observable *extremities* of these elementary quantum mechanical chain experiments, namely the registered needle positions $V_j \in \mathrm{V}_A$ of the utilized devices D_A. The factual registered "values" $V_j \in \mathrm{V}_A$, which seemed to float freely on the surface of the observable (like nenuphar flowers seem to float freely on the reflecting surface of a lake) *expose* now their fixtures to stalks of operations and processes rooted into trunks of initial operations of state preparation.

How did we obtain this integrated perception of the probabilistic organization of quantum mechanics? We have performed just an attentive analysis of the connections between Kolmogorov's standard fundamental probabilistic concepts (identically reproducible procedure, universe of elementary events, an algebra of events on this universe, a probability measure on this algebra), the main descriptors of the quantum mechanical formalism (state-vectors, operators, eigenfunctions), and the factual counterparts of the quantum mechanical writings. This, *because of the spacetime characteristics of the factual counterparts of the quantum mechanical writings*, brought forth, with a sort of inner necessity, the probabilistic meta-construct with treelike spacetime support described above. But this metaconstruct of distinct probability chains, though it has been produced by systematic confrontation with the standard probabilistic concepts, *transcends* the abstract theory of probabilities as it now stands: So far the most complex basic probabilistic structure explicitly defined in the theory of probabilities is *one* probability *space*. Not even the notion of *one* probability *chain* is explicitly defined as a monolithic construct. *A fortiori*, the concept of a probability tree, which connects several irreducibly distinct probability chains, is not defined in the present theory of probabilities.

Are these novelties probabilistic "anomalies"? Inasmuch as they are rooted into the present abstract theory of probabilities, it seems more adequate to regard them as germs of a possible extension of this theory.

3. TOWARD AN EXTENDED THEORY OF PROBABILITIES: PROBABILISTIC META- AND META-METADEPENDENCE

We shall now show that the concept of quantum mechanical probability tree indicates the definibility, probably in quite general abstract terms, of two new sorts of "probabilistic dependences," placed—with respect to the Kolmogorov definition—on two hierarchically connected higher descriptional levels, a metalevel where a probabilistic connection between distinct probability chains appears, and a meta-metalevel where distinct whole trees appear to be probabilistically related. These metaprobabilistic qualifications appear to be intimately related with a radical distinction between operations of state preparation and operations of measurement, and, correlatively, with the differences and relations between the principle of superposition and the principle of spectral decomposition.

These results, while they clarify and deepen fundamental features of the quantum theory, point toward the necessity and the possibility of a deepened abstract theory of probabilities: a theory of probabilities that will incorporate explicitly the cognitive physical operations—with their spacetime characteristics—by which, at the most fundamental level of the action of extraction of knowledge, the observer produces the objects to be qualified and obtains the very first qualifications of these objects.

3.1. Probabilistic Meta-dependence via a Common Potentiality

The fact that the quantum mechanical usage of probability measures exceeds the "classical" theory of probabilities has already been perceived long ago by several important authors (Mackey,[8] Gudder,[9] Suppes,[10] Mittelstaedt,[11] Van Fraassen and Hooker,[12] etc.). But this transgression is usually mentioned in negative terms: "nonembeddability" into a unique probability space, of the quantum mechanical measures corresponding to noncommuting observables, which is an "anomaly" that "hinders" a classical definition of a conditional probability for two incompatible events, etc.). Recently L. Cohen[13] has, on the contrary, shown by very interesting

calculational arguments that quantum mechanics suggests a possible extension of the standard theory of probabilities. The concept of probability tree permits one to strongly develop this constructive perception.

The quantum mechanical transformation theory ($c(\psi, b_k) = \sum_j \alpha_{kj} c(\psi, a_j)$, $\forall$A, B: A $|u_j\rangle = a_j\,|u_j\rangle$, B $|v_k\rangle = b_k\,|v_k\rangle$, $\forall j \in J$, $\forall k \in K$, J, K index sets, A, B two noncommuting observables, $\alpha_{kj} = \langle v_k \mid u_j\rangle$ the transformation coefficients) permits one to entirely determine, from the knowledge of the probability measure $\pi(\psi, a_j)$ from one branch of a probability tree, any other probability measure $\pi(\psi, b_k)$ belonging to another branch of that same tree. Indeed the equalities $|(c(\psi, b_k)|^2 = |\sum_j \alpha_{kj} c(\psi, a_j)|^2$, $\forall j \in J$, $\forall k \in K$, are equivalent to the specification of a functional relation

$$\pi(\psi, b_k) = F_{\mathrm{QM}}[\pi(\psi, a_j)]$$

between the probability measures corresponding to the noncommuting observables A and B. But the standard concept of functional relation between two probability measures does not singularize the particular sort of probabilistic connection between two probability measures introduced by the quantum theory. *Nor does it permit one to recover it fully*, as L. Cohen has shown ([Ref. 13, pp. 991–93). As it is stressed by the index QM, we are in the presence of a specifically quantum mechanical functional relation. What status can we assert for it?

According to the present theory of probabilities the concept of "probabilistic dependence" is by definition confined inside *one* probability space where it concerns *isolated* pairs of *events*. Two events are tied by a "probabilistic dependence" if knowledge of one of these events "influences" the expectations concerning the other one. So the relation $\pi(\psi, b_k) = F_{\mathrm{QM}}[\pi(\psi, a_j)]$ of mutual determination of the probability measures from a quantum mechanical probability tree can naturally be regarded as a "maximal probabilistic metadependence":

— "maximal" because it consists in mutual determination

— "probabilistic" because, though this determination is a certainty about "influence," nevertheless it concerns probabilistic constructs;

— "metadependence" because it concerns, not pairs of events from one space, but *globally* pairs of probability measures on entire algebras of events, which, with respect to events, are metaentities.

Now, if this view and language are accepted, what has just been named the probabilistic metadependence defined by the quantum mechanical transformation theory appears as *reflecting the studied state with state* *vector* $|\psi\rangle$ *from the common trunk of the tree*. This state that stems from a preparation operation $P(\psi_0)$ and then might have evolved accordingly to some law $|T(t_0, t, H)\, |\psi_0\rangle = |\psi\rangle$, but that has never yet been *observed*, has to be conceived of, in consequence of this lack of previous qualifications, merely as a monolith of still nondifferentiated observational *potentialities* that sets a genetic unity beneath the various incompatible measurement processes of *actualization* of that or that particular observational potentiality, leading to that or that *actualized* observable space (3′). Though in quite different contexts, Bohm,[14] de Broglie,[15] Primas,[16] as well as other authors, have also explicitly stressed the multiple potential meaning of the quantum mechanical concept of state. Here we reexpress it as follows.

The probability tree of a studied state with state vector $|\psi\rangle$ *is a complex unity which, with respect to the observable manifestations of a microsystem, possesses a "potential-actualization-actualized character."*

The quantum mechanical functional relations F_{QM} between the probability measures from irreducibly distinct observable spaces—considered as wholes—belonging to a same probability tree, reflect the genetic unity of these spaces via the common observational potentialities captured inside the state from the trunk of the tree. The quantum mechanical transformation theory involves new probabilistic features that are neither probabilistic "anomalies" nor mere numerical algorithms. They are a mathematical description of particular realizations of probabilistic metaproperties, brought forth by a growth of the probabilistic thinking that happened inside the process of conceptualization of the microphenomena: a growth that draws attention upon the necessity, at the most basic level of description where no previously elaborated conceptualization is presupposed, to represent and to study the cognitive *operations* by which the observer—who necessarily exists and acts—*produces* the objects to be qualified and the processes of qualification of these. Indeed, these operations *themselves* possess physical characteristics, in particular spacetime supports, that entail nontrivial consequences on the probabilistic descriptions constructed with their help.

3.2. Probabilistic Metaproblems

The probabilistic metaproperties mentioned above, as soon as they are perceived, involve certain probabilistic metaproblems that will have to be explicitly considered, together with the reduction problem and the other

problems raised by the probabilistic character of the quantum theory. An example is this.

The *passage* from the monolith of observational potentialities labeled by a state vector $|\psi\rangle$, to that or that process of actualization M_A of that or that particular observational potentialities contained in $|\psi\rangle$, involves from the part of the observer *an act of free choice* of one among all the possible measurement processes M_A. In this sense the nature of this passage is not "purely physical"; it is a phenomenon that depends also on the observer's "free will."

The quantum mechanical transformation theory involves an intervention of the observer's mind already before *the final act by which a registered eigenvalue is perceived by the human observer.*

This introduces an anterior supplement to the so amply discussed reduction problem (Wigner's Friend, etc.). Indeed:

What is the relative frequency

$$\pi(M_A/\psi)$$

of the occurrence of a given measurement process M_A once the state $|\psi\rangle$ to be studied has been prepared? Obviously this relative frequency has to be radically distinguished from the "conditional" relative frequency

$$[\pi(\psi, a_j)/A]) \equiv \pi(\psi, a_j)]$$

of the occurrence of a given eigenvalue a_j of the observable A, *once* the choice of measuring upon $|\psi\rangle$ the class of observables compatible with A *has been made*. It seems likely that the relative frequency of choice of a measurement process M_A, so $\pi(M_A/\psi)$, such as it emerges spontaneously, because it involves the observer's free will, simply cannot be defined at all in a way that is stable, "identically" reproducible, so as to permit the assertion of a probability "law" in the sense of the theorem called the law of big numbers. So this relative frequency cannot be inserted into the structure called a probability space. The concept of a probability space is very restrictive; it involves severe conditions of "identical" reproducibility. The space $[U, \tau]$ where U is a universe of elementary events each of which emerges as the result of {first: [a realization of an operation $P(\psi) = [P(\psi_0), T(H, t_0, t)\, |\psi_0\rangle = |\psi\rangle]$ which prepares a state $|\psi\rangle$ to be studied] *and* afterwards: [a choice of a measurement process M_A]} and τ is an algebra of events on the universe U, might simply not be a "probabilizable" space, *if* the relative frequency of the choice of a measurement process M_A is considered such as it emerges naturally in the mind of that or that observer, i.e. (in the absence of any conventional constraint). Now, if this nonprobabilizable character is admitted, then, from the viewpoint of probabilities, it introduces a solution of continuity between the numerical estimations from the different observable branch spaces of a tree, a kind of noncomparability of the probability measures from these spaces. This seems to contradict the quantum mechanical algorithms: As long as we do not possess some definition for the conditional probabilities $\pi(M_A/\psi)$ and $\pi(M_B/\psi)$, with $[A, B] \neq 0$, *what content* can we assign to the assertion that the measures $\pi(\psi, A) = \{\pi(\psi, a_j) = |c(\psi, a_j)|^2\}$ and $\pi(\psi, B) = \{\pi(\psi, b_k) = |c(\psi, b_k)|^2\}$ are connected in agreement with the quantum mechanical transformation relations $c(\psi, b_k) = \sum_j \alpha_{kj} c(\psi, a_j)$? As long as we do not assert some definite ratio for the relative frequencies of emergence of the two measurements M_A and M_B, how is it possible to make such precise numerical assertions involving the relation between the relative frequency of an event b_k and the relative frequencies of the events a_j?

Lubkin,[17] without detailing all the dimensions of the conceptual situation, has nevertheless perceived the necessity of some conventional substitute for the otherwise unrealizable definition of a conditional probability measure $\pi(M_A/\psi)$. And indeed the quantum mechanical formalism certainly does involve such a conventional substitute: quite probably, a decision of equipartition, $\pi(M_A/\psi) = \text{constant}$, $\forall A$ (and $\forall |\psi\rangle$), in order to "smooth out" the unpredictable effects of the free choices of a measurement process, thus offering expression exclusively to the "objective" factors. (A decision of equipartition can be stated in terms of "certainty," which, up to normalization, amounts to the same: Once $|\psi\rangle$ has been created, suppose first that the measurement M_A follows *certainly* and calculate the expansion coefficients $c(\psi, a_j)$, so the individual probabilities $\pi(\psi, a_j) = |c(\psi, a_j)|^2$; then suppose that M_B follows *certainly* and calculate the coefficients $c(\psi, b_k)$, so the individual probabilities $\pi(|\psi\rangle, b_k) = |c(\psi, b_k)|^2$; *then* verify the assertion $\pi(\psi, b_k) = |c(\psi, b_k)|^2 = |\sum_j \alpha_{kj} c(\psi, a_j)|^2$ involved by the quantum mechanical transformation theory.)

But Van Fraassen and Hooker,[12] quite curiously, have formulated a purely mathematical argument for the "impossibility" of a conditional probability measure $\pi(M_A/\psi)$, again without stating explicitly the epistemological dimensions of the problem (this impossibility, however, is necessarily relative to some presuppositions, and these might not possess an unavoidable character).

Anyhow, the preceding example shows that the quantum mechanical transformation theory, while it suggests a possible extension of the concept of probabilistic dependence, involves correlatively certain specific

probabilistic problems that would have to be dealt with inside such an extension.

3.3. The Germ of a Concept of Probabilistic Meta-Metadependence

3.3.1. State Preparations versus Measurements. The absence of an integrated perception of the probabilistic organization which underlies the formalism of the quantum theory not only hinders a clear understanding of the novelties and of the problems involved by the theory, but furthermore it entails insufficiencies inside the theory itself. The most important among these stem from the tendency to confound the operations of state preparation, with measurements, that is, to mix up temporal orders which, quite essentially, do act.

Absence of Mathematical Representation For the Operations of State Preparation. In quantum mechanics as it now stands, the degree of definition of the operations of state preparation is much lower than that of the measurement operations. Correlatively, the mutual characterization of operations of state preparation and of measurement operations is very imperfect.

The measurement operations are quite explicitly represented by Hermitian linear differential operators and by a well-defined calculus with these. The compatibility or incompatibility of two measurements has been recognized and formally described, and consequences have been drawn systematically from this. On the contrary, as far as we know, no clear-cut and unanimously practised definition does as yet exist for the concept of state preparation. A fortiori, the operations of state preparation are not endowed with a mathematical representation clearly assigned to them. They are not even systematically symbolized:

Quantum mechanics as it now stands does not specify a calculus with, specifically, operations of state preparation, distinguished from the calculus with measurement operations and related with it.

The source of this situation can be associated with the Copenhagen formulation of the postulates of quantum mechanics which *interlaces* the concept of state preparation with that of measurement. Indeed, in the Copenhagen formulation of the postulates of quantum mechanics, certain operations of state preparation *are* defined. These consist of a measurement evolution M_A for an eigenvalue registration corresponding to some observable A the final phase of registration included such an evolution being postulated (the projection postulate) to leave the studied system in a state describable by the "normalized" eigenvector $|u_j\rangle$ of A corresponding to the registered eigenvalue a_j. (For simplicity, we do not singularize here the case of an evolution corresponding to a complete set of commuting observables, as it is usually done). Now, is this definition conceived to designate only a subensemble of the ensemble of all the possible operations of state preparation, or is it conceived to exhaust this ensemble, so, to entirely absorb the concept of state preparation into that of measurement? When one reads the various papers that have been written on this subject, and quite particularly the current textbooks, the answer is far from being clear. Anyhow, a general distinct *definition* of what is to be called an operation of state preparation, in contradistinction to what has to be called a measurement operation, is uniformly absent. The *term* "preparation," nevertheless, is uniformly present.

However, notice the following:

Mere contemplation of the figure representing a probability tree makes it obvious that, by the very definition of the words, an operation of

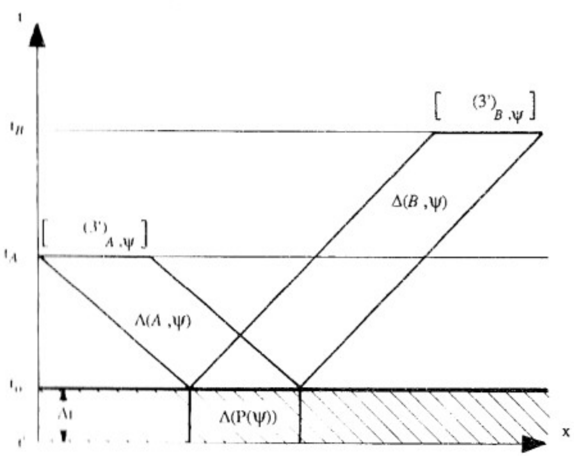

Fig. 2. The processes $P(\psi)$ which by reiteration bring forth the studied state vector $|\psi\rangle$ are contained in a spacetime domain $\Delta(P(\psi))$ of which the temporal extension is $\Delta t = t_0 - t'$. The processes of measurement evolution M_A, M_B, etc. which by reiteration produce, respectively, the observable probability spaces $[3'_A, \psi]$, $[3'_B, \psi]$, etc., are contained in spacetime domains $\Delta(A, \psi)$, $\Delta(B, \psi)$, etc. of which the temporal extensions $t_A - t_0$, $t_B - t_0$, etc., are *posterior* to t_0.

state preparation $P(\psi_0)$ is a *primary* operation of generation of an *object* for subsequent examinations, while the operations of measurement are *secondary* operations of *qualification* of this object (Fig. 2). It jumps at one's eyes that the two concepts of state preparation and of measurement concern two essentially *different phases* of the development of *any* quantum mechanical elementary chain experiment, i.e., of the emergence of any quantum mechanical elementary event in the sense of probabilities. Two phases placed at two different temporal levels of a tree ($\Delta t = t_0 - t'$ for the operation of state preparation and $\Delta t = t_A - t_0$ for an operation of measurement of an observable A) possessing essentially different cognitive roles and which are *both* necessarily present inside any process of emergence of a quantum mechanical elementary event in the sense of probabilities. Any "measurement," by the very definition of the concept, presupposes necessarily some previously produced state, thus some "operation of state preparation," deliberate or spontaneous, natural. While an operation of state preparation presupposes nothing, it is by *convention* the *first* operation that is considered, the origin, the zero of the considered chain of phenomena leading to one quantum mechanical elementary event in the sense of probabilities, $a_j = f_A(V_j)$.

— Furthermore, in order to be able to specify a definite eigenvector $|u_j\rangle$ of an observable A as being the "normalized state" in which the system is left, it is indeed necessary to obtain a final information that singularizes a definite eigenvalue a_j, thus labeling observably the asserted state. But if, for this aim, an interaction is produced that involves the state to be labeled itself, then, in general, this state is destroyed. So, as is well known, some substitute must be found ("nondemolishing" procedure) that preserves the state to be labeled (as in the case of the Stern–Gerlach method for spin 1/2, or as in the case of indirect identifications, by the registration of photonic spectra, of electronic states bounded inside atoms). However, such substitutes *cannot* be found for any sort of measurement. Therefore the category of measurements that can act as preparations of known states *is very limited.*

— Correlatively, in a certain sense, the eigenvectors of most quantum mechanical observables (of all the observables with continuum spectrum, position, momentum, all the Hamiltonians corresponding to nonbounded evolutions) simply do not accept, *stricto sensu*, normalization, so they cannot *rigorously* play the role of a state vector also. This is currently called "the problem of normalization" and is "solved" by proving that any eigenvector can be arbitrarily well approximated by a corresponding eigendifferential that is a normalized state vector. But it seems very awkward indeed *to found a postulate on approximations.*

— Finally, when one considers the ensemble of *all* the conceivable quantum mechanical states, in particular the free states, it seems quite clear that—simply by *lack of any connection* with some measurement evolution—an infinity of states of which nothing whatsoever hinders the *preparability* and which are currently considered in the quantum theory, do *not* accept, neither rigorously *nor even grossly*, a description in terms of a definite eigenvector of some observable. (For instance, produce, on the left side of a screen with three somewhat extended holes in it, an arbitray but definite electronic state; what is the state on the right side of the screen?). The elimination of all these states would be a huge amputation of the theory. This, no doubt, is why most authors consider that in general a preparation operation is not also a measurement operation.

In these conditions it seems natural to try to establish a clear-cut definition of the specificities of the operations of state preparation with respect to the operations of measurement.

With this aim in mind, we consider preliminarily the following question. Are there reasons, perhaps, that *oblige* one to work with a definition of the operations of state preparation that is tied with the eigenvectors of the quantum mechanical observables, notwithstanding the fact that such a definition is doubly flawed, in the first place by a character of approximation that seems unacceptable in basic assumptions and in the second place by obvious restrictions of the factual possibilities? This question will be decomposed into two other ones:

— Is it perhaps impossible to associate a mathematical representation to a state that is physically preparable but is not tied with some eigenvector? The answer will be found to be negative.

Then, for what other reasons did the physicists from the Copenhagen school introduce the group of concepts constituted of the projection postulate and the definition of preparations by measurements? They certainly were not naive thinkers, so they must have felt some compelling motivation that has to be identified and explicitly dealt with. A possible explanation will appear later, in Sec. 5.

The Result of Any Operation of State Preparation Can be Represented by a State Vector. The quantum mechanical formalism seems to involve a possibility which has never before been incorporated into the formal construction of a physical theory. Namely, the possibility to *initially* "define" an entity to be studied by an exclusively physical operation, in a strictly conceptual way, a purely factual way, quite independently of both the knowledge of the result produced by this operation and the operations performable on this result by which subsequently the result will be qualifiable. The formalism of quantum mechanics permits one to envisage to individualize a state of a microsystem, to form it out of the continuum of "reality," *to capture it as a monolith of physically determined but as yet entirely unknown observational potentialities*, and to keep it available for any future actualizing examinations, without strictly *nothing* presupposing about "how" this state would appear under that or that subsequent examination. Indeed precisely this is what happens each time an operation of state preparation is performed that is defined by operational instructions entirely independent of any measurement operation: Initially, the state produced by this operation is devoid of any expressed definition *of its own* (mathematical, or not), *distinct* from the definition of the operation of its preparation. For instance, consider again the electronic state imagined above, produced by passage of an arbitrary but definite previous state through a screen with three somewhat extended holes in it; or imagine some definite interactions which generate still nonidentified elementary particles (thus *a fortiori* still unknown states of these). Now, as announced, the question is whether it is possible to construct a mathematical representation connectable with such a state.

Let us just label *a priori* by the signs $|\psi_0\rangle$ and $P(\psi_0)$, respectively, the considered state and the operation of preparation that defines it. Since the operation of preparation labeled $P(\psi_0)$ is instructionally defined, *it is known how to reproduce indefinitely the state labeled by the symbol* $|\psi_0\rangle$, notwithstanding the fact that this state is still devoid of any description *of its own* (not of its operation of preparation). So nothing whatsoever prevents one from establishing experimentally probability distributions $\pi(\psi_0, A) = \{\pi(\psi_0, a_j) = |c(\psi_0, a_j)|^2\}$ corresponding to this still non-described state for as many quantum mechanical observables A as one desires (an approach similar to that for the identification of unknown nuclear potentials via experimental estimations of cross sections). This means that, in the expansion $|\psi_0\rangle = \sum_j c(\psi_0, a_j)\,|u_j\rangle$ of the unknown, researched state-vector $|\psi_0\rangle$, on the basis of eigenvectors $\{|u_j\rangle\}$ corresponding to any given quantum mechanical observable A, the *real* factors $|c(\psi_0, a_j)|$ from the complex expansion coefficients $c(\psi_0, a_j) = |c(\psi_0, a_j)|\,e^{i\alpha(j)}$ can be determined experimentally.

What about the imaginary factors? Consider an observable A. Write the corresponding expansion in the more explicit form

$$|\psi_0\rangle = \sum_j e^{i\alpha(j)}\,|c(\psi_0, a_j)|\,|u_j\rangle \tag{8}$$

where $|\psi_0\rangle$ and the imaginary factors $\{e^{i\alpha(j)}\}$ are not known, while the ensemble of real numbers $\{|c(\psi_0, a_j)|\}$ has been established experimentally. Let us make an *arbitrary* choice of an ensemble of factors $\{e^{i\alpha(j)}\}$. The expansion (8) is then defined. Thus it associates now to the symbol $|\psi_0\rangle$ a first mathematical representation where the numbers $c(\psi_0, a_j) = |c(\psi_0, a_j)|\, e^{i\alpha(j)}$ accept—consistently with the mutual orthogonality of the eigenvectors of A—the definition $c(\psi_0, a_j) = \langle u_j \mid \psi_0\rangle$. This representation, by construction, is consistent with the experimental data $|c(\psi_0, a_j)| = \sqrt{\pi(\psi_0, a_j)}$ concerning the observable A. Furthermore, it permits one now to determine *consistently* the imaginary factors $\{e^{i\beta(k)}\}$ corresponding to any *other* observable B that does not commute with A. Indeed, the expansion for an arbitrary observable B, $[\mathrm{B}, \mathrm{A}] \neq 0$, can be written as

$$|\psi_0\rangle = \sum_k e^{i\beta(k)}\, |c(\psi_0, b_k)|\, |v_k\rangle \tag{9}$$

where

$$c(\psi_0, b_k) = e^{i\beta(k)}\, |c(\psi_0, b_k)|$$

By the standard rules of the quantum mechanical calculus, we have then, for *each* fixed k,

$$\langle v_k \mid \psi_0\rangle = c(\psi_0, b_k) = e^{i\beta(k)}\, |c(\psi_0, b_k)| = \sum_j \tau_{kj}(\mathrm{A}, \mathrm{B})\, c(\psi_0, a_j), \quad j = 1, 2,... \tag{10}$$

where the $\tau_{kj}(\mathrm{A}, \mathrm{B}) = \langle v_k|\, |u_j\rangle$, $k = 1, 2,...,$ $j = 1, 2,...,$ are the coefficients of transformation from the basis of eigenvectors of A to that of B. Thus, for each fixed k (any one), the quantum theory of transformations yields a separate condition

$$e^{i\beta(k)} = \langle v_k \mid \psi_0\rangle \Big/ |c(\psi_0, b_k)| = \Big[\sum_j \tau_{kj}(\mathrm{A}, \mathrm{B})\, c(\psi_0, a_j)\Big] \Big/ |c(\psi_0, b_k)|, \quad k = 1, 2,... \tag{11}$$

that determines the complex factor $e^{i\beta(k)}$ corresponding to that k, consistently with the previous arbitrary choice of the ensemble of imaginary factors $\{e^{i\alpha(j)}\}$ and with the experimental datum $|c(\psi_0, b_k)|$. Thus we can determine the ensemble of factors $\{e^{i\beta(k)}\}$ for the observable B in a way that is consistent by construction with

— all the experimental data $\{|c(\psi_0, b_k)| = \sqrt{\pi(\psi_0, b_k)}\}$ concerning the observable B; all the experimental data $\{|c(\psi_0, a_j)| = \sqrt{\pi(\psi_0, a_j)}\}$ concerning the observable A;

— *and* the initial arbitrary choice made in (1) for the factors $\{e^{i\alpha(j)}\}$). Since this is achieved by the use of the conditions $c(\psi_0, b_k) = \sum_j \tau_{kj}(\mathrm{A}, \mathrm{B})\, c(\psi_0, a_j)$, $k = 1, 2,...,$ imposed by the quantum theory of transformations, what is called in this context the interference of probabilities, $|c(\psi_0, b_k)|^2 = |\sum_j \tau_{kj}(\mathrm{A}, \mathrm{B})\, c(\psi_0, a_j)|^2$, is *ensured.* This brings into evidence that:

The information contained in the quantum mechanical representation of a state vector $|\psi_0\rangle$ does not exceed the constraints stemming from the observable data brought forth by measurements (the real factors $|c(\psi_0, a_j)|$ from the expansion coefficients $c(\psi_0, a_j) = e^{i\alpha(j)}\, |c(\psi_0, a_j)|$), on the one hand, and on the other hand from the conditions imposed by the quantum mechanical theory of transformations (the imaginary factors $e^{i\alpha(j)}$) which determine the "interferences of probabilities connected with the passage from the basis of an observable A, *to that of an observable* B, $[\mathrm{B}, \mathrm{A}] \neq 0$."

Any mathematical representation associated to the initial label $|\psi_0\rangle$ that satisfies the two mentioned constraints is as good as any other one. These constraints correspond to *a whole family* of convenient state vectors $|\psi_0\rangle$ (state functions ψ_0) of which a member can be found in the way indicated above.

Now, *once* the process of construction of a mathematical representation, by a state vector, of the designatum of the *a priori* introduced symbol $|\psi_0\rangle$ is *closed,* from that stage *on,* it can be admitted by induction that each time that the same operation $\mathrm{P}(\psi_0)$ of state preparation will be performed, its result will admit mathematical representation by the same state vector $|\psi_0\rangle$ already constructed.

We conclude that any physical operation that can be performed on a microsystem and is specifiable by a definite set of instructions permitting one to reiterate this same operation an arbitrary number of times, produces a state of the microsystem that *can* be represented mathematically, and thus can be studied inside the quantum theory. (This corresponds to what is called a "pure" state. Obviously, the meta-case of what is called a "mixture" can be treated in a similar way.)

The quantum theory involves the possibility of a sort of self-organizing descriptional dynamics that starts inside pure factuality, *with a-conceptual, strictly operational (instructional) definitions (determinations) of the studied states, involving no conceptualized qualifications whatever. Mathematical descriptions and predictional qualifications—all contextual—*can *be associated* retroactively *to these instructional definitions.*

This is probably one of the most original and important conceptual innovations introduced (implicitly) by the quantum theory:

It marks the extreme limit of operationalism.

Quantum mechanics, only implicitly but quite essentially and for the first time in the history of thought, *incorporates this limit into a formalization.*

But when, in particular, an operation of preparation is defined by a measurement evolution, this innovation remains nonutilized and thus *hidden*: While in the history of a state of which the operation of preparation is not tied with some measurement evolution there *necessarily* exists an initial stage when this state was devoid of a known mathematical representation, when, at most, it was only *a priori* labeled by a symbol, in the particular case of the states prepared by a measurement evolution such an initial stage *is lacking*. The state vector is known *in advance* to be a definite normed eigendifferential that approximates arbitrarily well a corresponding definite eigenvector. This entails that:

A state prepared by a measurement evolution emerges endowed with certain predecided (thus known) observational qualifications.

For instance, if it is known that the result of the preparation of a pure state can be represented by some given eigenvector $|u_j\rangle$ of a dynamical observable A, then, *ipso facto*, it is also known that successive (nondemolishing, etc.) A-measurements operated on this state would produce reiterated registrations of the corresponding eigenvalue a_j of A, which is a predetermined observational qualification of the prepared state. So the fact that in general an operation of state preparation *can* be freed of *any* dependence on predetermined qualifications remains inapparent.

It might seem that this possibility is irrelevant. But in fact it constitutes a loss of seminal potentialities of description: We have shown in other works[2–4],[22] that it is very fertile to introduce *independent* representations for the operation by which is produced the object of a description, and the operation by which this object is examined and qualified.

It is strikingly paradoxical that precisely the Copenhagen school, the champion of operationalism, has introduced a definition of the state preparations that—inasmuch as it is not transgressed by some generalization—hides the remarkable fact that the quantum theory permits one to reach and to represent explicitly the extreme limit of operationalism. The reason that motivated this definition will be interesting to identify. This will be done in Secs. 4 and 5. As for now, let us examine just below how the distinction—or not—between preparations and measurements, is related with the distinction—or not—between superpositions of state vectors and spectral decompositions of a state vector.

3.3.2. Superpositions of Several States Versus Spectral Decompositions of One State. The feeble mutual individualization of state preparations

and measurements, tied with a fluctuating and feeble distinction between state vector *of a microsystem* and eigenvector *of an observable*, entails an insufficient distinction also between the principle of superposition (discussed mainly by Dirac) and Born's principle of spectral decomposition (the expansion postulate). Indeed, though these two principles have been introduced independently of one another, the spectral decompositions of a state vector on the basis of eigenvectors determined by an observable A are quite currently designated as "superpositions of eigen*states* of A." The two concepts tend to merge into one another inside the moulds of a relaxed language. However:

A spectral decomposition $|\psi\rangle = \sum_j c(\psi, a_j)\,|u_j\rangle$ possesses the following characteristics.

— It is a representation that is relative, by definition, to some observable A.

— The *expansion* coefficients $c(\psi, a_j)$ are necessarily *complex* numbers (if they were not, the "interference of probabilities" via transformation to another representation, an essential feature of the formalism, would disappear).

— They are in general *time-dependent* in the Schrödinger representation.

— The summed *eigen*vectors $|u_j\rangle$ of A, in general an *infinity*, even a *continuous* infinity, are *all* involved, by definition.

— They are *independent of time.*

— They are in general *not normed*, and furthermore not normalizable *strico sensu.*

— They are *mutually ortogonal by definition*, $\langle u_k | u_j\rangle = 0$, $\forall (k \neq j)$.

— Concerning "interference of probabilities":

* In consequence of the mutual orthogonality of the summed terms, the scalar products $\langle u_j | \psi\rangle$ with individual eigenvectors $|u_j\rangle$ yield one-term results so that for the individual probabilities $\pi(\psi, a_j)$ we have the one-term expressions

$$\pi(\psi, a_j) = |\langle u_j | \psi\rangle^2 = |c(\psi, a_j)|^2 \tag{7}$$

which shows the **absence** *of "interference of the probabilities" inside the representation with respect to the* **one** *observable A itself to which the considered expansion is relative.*

* While by *passage* to *another* basis corresponding to another observable $B \neq A$ that does not commute with A, an "interference of probabilities" does appear:

$$\pi(\psi, b_k) = |c(\psi_0, b_k)|^2 = \left|\sum_j \tau_{kj}(A, B)\, c(\psi_0, a_j)\right|^2$$

$$= \sum_j |\tau_{kj}(A, B)|^2\, |c(\psi_0, a_j)|^2$$

+ [interference terms involving all the pairs of products

$$\tau_{kj}(A, B)\, c(\psi_0, a_j),\ \tau_{jk}(A, B)\, c(\psi_0, a_k)]$$

This is an *abstract* sort of interference which is *relative to a* **pair** *of noncommuting observables* (A, B) and which, though it entails certain consequences (as well as many false interpretations, for instance in Bohm[14] pp. 384–386), is devoid of a *directly* observable counterpart: The square roots $c(\psi_0, a_j)$ of all the values of the probabilities $\pi(\psi, a_j)$ of the elementary events a_j emerging when a measurement of the observable A is performed on a state with state vector $|\psi\rangle$ "interfere" abstractly, numerically, in the value of each probability $\pi(\psi, b_k)$ of an elementary event b_k that *might* emerge *if* a measurement of the other observable B that does not commute with A *were* performed on that same state. In what follows this sort of *abstract* interference by transformation from a representation A to another representation B that never coexists with A, will be called "*interference relative to incompatible observables.*"

On the contrary, *a superposition of states* $|\psi_{abc...}\rangle = \lambda_a|\psi_a\rangle + \lambda_b|\psi_b\rangle + \lambda_c|\psi_c\rangle + \ldots$ possesses the following as if opposed characteristics.

— It is a representation *not* tied with some particular observable.

— The coefficients *of linear combination* λ_a, λ_b, λ_a... *can* relevantly be chosen to be *real* numbers. Nothing in the formalism interdicts that.

— They are *time-independent*.

— It is permitted to superpose an *arbitrary* number—usually a *small* number—of state vectors $|\psi_a\rangle$, $|\psi_b\rangle$,...

— They are in general *time-dependent* in the Schrödinger representation.

— They are *always normed*.

— *In general they are* **not** *mutually orthogonal*. (However, when in particular they are orthogonal, the scalar products $\langle\psi_a|\psi_{abc...}\rangle$, $\langle\psi_b|\psi_{abc...}\rangle$, ...acquire one-term expressions $\langle\psi_a|\psi_{abc...}\rangle = \lambda_a$, $\langle\psi_b|\psi_{abc...}\rangle = \lambda_b$, etc., analogously to what happens in the case of a spectral decomposition for the products $\langle u_j|\psi\rangle$. But notice that in this case, in contradistinction to the products $\langle u_j|\psi\rangle = c(\psi_0, a_j) = \sqrt{\pi(\psi, a_j)}$, the values of the "corresponding" products $\langle\psi_k|\psi_{abc...}\rangle = \lambda_k$, $k = a, b, c,...$ do not possess a probabilistic significance).

— Concerning "interference of probabilities":

* The scalar products $\langle u_j|\psi_{abc...}\rangle$ with individual eigenvectors $|u_j\rangle$ from the basis of an observable A do *not* have a one-term expression; they have a multi-term expression $\langle u_j|\psi_{abc...}\rangle = \sum_k \lambda_k \langle u_j|\psi_k\rangle$, $k = a, b, c,...$. So when the square modulus is calculated in order to estimate the corresponding probability $\pi(\psi_{abc,...}, a_j)$, an "interference of probabilities" appears *no matter* whether yes or not the superposed terms $|\psi_a\rangle$, $|\psi_b\rangle$, $|\psi_c\rangle$... are orthogonal (insofar as these terms are not themselves elements $|u_j\rangle$ from the basis of A, which can happen either in the case of a discrete spectrum, or approximately). For instance, for a superposition state vector with only two terms, the elementary predictional probabilities concerning the elementary outcomes a_j for an observable A acquire the well-known "interference form"

$$\pi(\psi_{ab}, a_j) = |\langle u_j|\psi_{ab}\rangle|^2 = |\lambda_a\langle u_j|\psi_a\rangle + \lambda_b\langle u_j|\psi_b\rangle|^2$$
$$= |\lambda_a|^2\, |\langle u_j|\psi_a\rangle|^2 + |\lambda_b|^2\, \langle u_j|\psi_b\rangle|^2 + 2\,\mathrm{Re}\{(\lambda_a)(\lambda_b)^* \times \langle u_j|\psi_a\rangle\langle u_j|\psi_b\rangle^*\}] \qquad (12)$$

This is a sort of interference of probabilities where *the quantum mechanical theory of transformation from the basis of one observable to the basis of another observable is not involved*, an interference that emerges "directly" *with respect to the summed state vectors, for any* **one** *observable*. So we shall call it *"interference relative to the superposed state vectors,"* and we shall distinguish it radically from the interference relative to incompatible observables.

* When one considers, for a superposition state vector, the interference relative to the superposed state vectors that concerns the *position* observable $A = X$, then—if the spatial supports of the superposed state vectors are not disjoint—the corresponding form of the type (12)

$$\pi(\psi_{ab}, x_j) = |\langle\delta(x - x_j)\,|\psi_{ab}\rangle|^2 = |\lambda_a\langle\delta(x - x_j)\,|\psi_a\rangle + \lambda_b\langle\delta(x - x_j)\,|\psi_b\rangle|^2$$
$$= |\lambda_a|^2\, |\psi_a(x_j)|^2 + |\lambda_b|^2\, |\psi_b(x_j)|^2 + 2\,\mathrm{Re}\{(\lambda_a)(\lambda_b)^*\, \psi_a(x_j)\, \psi_b(x_j)^*\}] \qquad (12')$$

is associated with the so amply discussed *interference patterns in the physical space*, directly *observable* on the domains where the spatial supports of the summed state vectors overlap. In this sense the interference relative to the superposed state vectors, in contradistinction to the interference relative to incompatible observables, is not an abstract interference. The possibility of such observable interference patterns disappears only if the spatial supports of the superposed state vectors are all mutually disjoint, in which case (12′) acquires the degenerate noninterferent (but still multi-term) form

$$\pi(\psi_{ab}, x_j) = |\lambda_a|^2\,|\psi_a(x_j)|^2 + |\lambda_b|^2\,|\psi_b(x_j)|^2 \qquad (12'')$$

The mutual specificities emphasized above do not in the least manifest an identity between spectral decompositions of one state vector and superpositions of several state vectors. Quite the contrary, they manifest a sort of *opposition*. In particular, they reveal *a splitting of the central concept of interference of probabilities*.

Now, where do the observable effects tied with superposition state vectors stem from? In what follows we show that they are essentially related with a "multiple structure" of the operation of state preparation that produces the state corresponding to the considered superposition state vector.

Consider for simplicity a two-term superposition state vector

$$|\psi_{ab}\rangle = \lambda_a\,|\psi_a\rangle + \lambda_b\,|\psi_b\rangle \qquad \text{(S)}$$

tied with a state preparation $P(\psi_{ab})$. If the state $|\psi_a\rangle$ of S is considered *separately*, it stems from some operation of state preparation $P(\psi_a)$. If the state $|\psi_b\rangle$ of S is considered separately, it stems from some operation of state preparation $P(\psi_b)$. If now the superposition state vector $|\psi_{ab}\rangle$ of S is considered, it stems from some operation of state preparation $P(\psi_{ab})$, *again* only *"one"* operation of state preparation since it produces the "one" *pure* state $|\psi_{ab}\rangle$ that entails its own specific quantum mechanical predictions. However, the operation of state preparation $P(\psi_{ab})$ somehow is conceived to "depend" on the two other operations $P(\psi_a)$ and $P(\psi_b)$ that are tied with the two state vectors $|\psi_a\rangle$ and $|\psi_b\rangle$ that *would* have been produced by these operations, respectively, *if* they would have been realized *separately*. Implicitly but quite essentially, these other two preparation operations are supposed to be

— mutually distinguishable

— realizable separately

— *combinable* so as to constitute *together* "one" other operation, distinct from both $P(\psi_a)$ and $P(\psi_b)$ and realizable on *one* previous initial state of the studied system, associated with an initial state vector $|\psi_i\rangle$ of that system.

So—quite systematically—in the case of a superposition of state vectors we can write symbolically

$$P(\psi_{ab}) = f(P(\psi_a), P(\psi_b)) \qquad (f\text{: some function})$$

In its last essence the principle of superposition is a statement, not directly about state vectors, but, more fundamentally, about a—past—operation of state preparation.

But these two different operations of state preparation $P(\psi_a)$ and $P(\psi_b)$ have *not* been realized separately. They have been realized only "together," "inside" the global procedure $P(\psi_{ab})$. So the states represented by the corresponding state vectors $|\psi_a\rangle$ and $|\psi_b\rangle$ also, which *could have been* produced separately via the *separate* realizations of the operations $P(\psi_a)$ and $P(\psi_b)$—*which entails that they are normed*—have *not* been realized individually via $P(\psi_{ab})$. They are only *conceived of* separately, in relation with the *one* state vector $|\psi_{ab}\rangle$ corresponding to the one realized global operation of state preparation $P(\psi_{ab})$ (realized either by the observer or by some "natural" substitute of the observer, as in the case of atomic states of an electron). They are conceived of **and** explicitly *represented* in the mathematical expression (S) of $|\psi_{ab}\rangle$ where they play the role of *elements of reference* in the calculation of any individual probability $\pi(\psi_{ab}, a_j)$: as can be read on the relation (12), $\pi(\psi_{ab}, a_j)$ is a function of $\pi(\psi_a, a_j)$ and $\pi(\psi_b, a_j)$. In particular, when one considers the position observable $A = X$ and the corresponding presence probabilities, this reference concerns patterns of impacts observable in the physical space. The algorithm (12) applied to the calculation of an individual presence probability $\pi(\psi_{ab}, x_j)$ as a function of the individual probabilities $\pi(\psi_a, x_j)$ and $\pi(\psi_b, x_j)$ permits, via (12′), *a quantitative comparison* between

— the observable pattern of position registrations corresponding to the realized state represented by the descriptor $|\psi_{ab}\rangle$

— the patterns that would be produced by each one of the states represented by the descriptors $|\psi_a\rangle$, $|\psi_b\rangle$ if these states acted (or effectively do act) separately on the device for the registration of eigenvalues of the position observable.

What is designated by the term "interference of probabilities" *as applied to observable patterns of position registration*, is precisely the *difference* brought forth by this comparison between the two patterns corresponding separately to $|\psi_a\rangle$ and $|\psi_b\rangle$, and the pattern corresponding to $|\psi_{ab}\rangle$: One sees how *such patterns are essentially tied with the "multiple" structure* $P(\psi_{ab})=f(P(\psi_a), P(\psi_b))$, *(f: some function) of the involved operation of state preparation.*

And notice that, remarkably, overlapping of the spatial supports of the superposed state vectors at least somewhere in space*time* (if time is left to increase indefinitely) is somehow related with a "multiple" structure $\mathrm{P}(\psi_{ab})=f(\mathrm{P}(\psi_a), \mathrm{P}(\psi_b))$ (f: some function) of the operation of state preparation. Here comes somehow into play (HOW?) the fact that in a superposition of states the combined state vectors are time-dependent, while the coefficients of linear combination are not. In this context, L. Cohen's criterion[18] for identifying the "decompositions" of a state vector into "*meaningful* parts" by estimations of standard deviations of "currents" (p. 1470) might appear to be very relevant inside the deepened development of the de Broglie–Bohm model that is introduced in the last section of this work. (However—imperatively—one should speak then of superpositions of independently preparable state vectors, not of "decompositions in parts.")

We summarize in general terms.

In a superposition representation (S), the unique *physically realized operation of state preparation is that one symbolized by the notation* $P(\psi_{ab\ldots})$, *of which the* unique *result is that one symbolized by the global notation* $|\psi_{ab\ldots}\rangle$. *This*—past—*operation of state preparation* $P(\psi_{ab\ldots})$ *somehow involved, "contained," two or more other operations of state preparation,* $P(\psi_a)$, $P(\psi_b)$,..., *mutually "distinct" and which* can *be realized* separately. *The state vectors* $|\psi_a\rangle$, $|\psi_b\rangle$,..., *corresponding to the states that* would *have emerged* if $P(\psi_a)$, $P(\psi_b)$,..., *would have been accomplished separately, are explicitly specified inside the formal expression of the state vector* $|\psi_{ab\ldots}\rangle$ *corresponding to the unique physically realized state produced by* $P(\psi_{ab\ldots})$. *There they play the role of* elements of reference *incorporated into the mathematical representation: It is with respect to them that there emerges a concept of interference of probabilities that is tied with patterns of position registrations directly observable in the physical space.*

This is in strong contrast with what is involved by the expression (D) of a spectral decomposition. There the representation does not designate observable effects of a particular type of structure of the past operation of state preparation of the studied state vector. What is represented in a spectral decomposition of a studied state vector $|\psi\rangle$ is the observable effects of a *future* operation of measurement of an observable A performed on $|\psi\rangle$ (Fig. 2). The representation is given in terms of the *projections* of the considered state vector $|\psi\rangle$, onto all the abstract eigenvectors $|u_j\rangle$, $\forall j \in J$, of the considered observable A. Such an eigenvector $|u_j\rangle$ according

to its very definition by the equation for eigenvectors and eigenvalues of A—is not in general a descriptor tied with a state producible by some specific operation of state preparation. It is only part of the mathematical representation of *a framework for the qualification of quantum mechanical states, a framework introduced by the observable A.* Namely, the eigenvectors $|u_j\rangle$, $\forall j \in J$, define a family, specific of this observable A, of "directions of qualification," of "*semantic* directions" (unidimensional, in the absence of degeneracy) each of which is associated with an observable eigenvalue of A. In general these semantic directions are only tangent to the Hilbert space that contains the state vectors $|\psi\rangle$; they are *exterior* to this space. They are images of elements endowed with a primary definition only inside the dual of the Hilbert space of the system. By a function (involved in a linear functional on the space of the states) the eigenvector $|u_j\rangle$ corresponding to an eigenvalue a_j of an observable A *qualifies* some feature (which one exactly?) of the same global factual situation that is also qualified by the eigenvalue a_j. As to the eigenvalue a_j itself, it qualifies the individual observable outcomes V_j, with $f_{\mathrm{A}}(V_j)=a_j$, of the elementary quantum mechanical chain experiments which, in their turn, via the corresponding probabilistic metaqualifications $\pi(\psi, a_j)$, qualify globally what is called a "quantum mechanical state" and is represented by a state vector $|\psi\rangle$. We sum up:

A spectral decomposition $|\psi\rangle=\sum_j c(\psi, a_j)\,|u_j\rangle$ *is referred to a* future *operation of measurement, upon the studied—already prepared—state vector* $|\psi\rangle$, *of an observable A. Each eigenvector* $|u_j\rangle$ *of A is a descriptor of a particular* qualification *from a whole* framework for qualification *introduced by A, a framework that is defined on the whole space of the state vectors.*

Though a descriptor $|u_j\rangle$ *is utilized for calculating the probability of an outcome* $f_A(V_j)=a_j$ *for any given state vector, there is* nothing *probabilistic in this descriptor* itself. *The descriptor* $|u_j\rangle$ *is tied with* one *eigenvalue* a_j *(in a nondegenerate situation), so it points toward an essentially* individual *predication. There is* **no reason** *whatever to require* normability *for the mathematical descriptor* $|u_j\rangle$ *(like for the state vectors* $|\psi\rangle$ *which, by definition, generate probability measures). Quite the contrary, this would simply be grossly inadequate from a semantic point of view.*

Correlatively, the spectral decomposition with respect to one observable A—by itself—entails no interference of probabilities, neither observable nor abstract. An interference of probabilities tied with spectral decompositions arises only by transformation from the basis of one observable A to the basis of another observable B that does not commute with A.

Since the eigenvectors are descriptors with individual meaning, the "problem" of normalization of the eigenvectors of observables with continuous spectrum is a false problem. Thus the "resolution" by the construction of state vectors yielding approximated normed representations of eigenvectors is a resolution without a corresponding problem—just noxious mathematically generated semantic fog that masks under a veil of superficial uniformity a radical solution of continuity, in the space of the concepts, between eigenvectors and state vectors. Even the standard theory of probabilities rejects (implicitly of course) the confusion between eigenvectors and state vectors. This, for instance, is illustrated by very interesting remarks by L. Cohen [Ref. 13, pp. 991–992, Eqs. (54)–(59)]. In order to understand fully the veritable problem involved in the quantum mechanical description of the measurements, in order to formulate it in more analyzed terms and to form a veritable answer to it, the conceptual difference between the designata of the eigenvectors and those of state vectors has to be recognized as essential, to be specified, and to be set at the bottom.

In short, the code is in essence this for distinguishing between the factual conterparts of, on the one hand, the superposition writings and, on the other hand, the spectral decompositions:

— A linear combination of an *arbitrary* number of (in general) *time-depending* and mutually *non*orthogonal *"state"* vectors *of a system S*, all *normed*, that is not relative to some observable and that can, in particular, be relevantly written with *real* coefficients, can be regarded as: the formal expression of the result of *one* operation of state preparation somehow "depending" on (referrable to) other (two or more) operations of state preparation, individually realizable but not individually realized, and which are such that if they were individually realized, would produce the states corresponding to the linearly combined state vectors.

— A linear expansion of *one normed "state"* vector, on the basis of *all* the mutually *orthogonal* and (in general) infinitely numerous and *non-normalizable "eigen"* vectors *of an observable A*, with *complex* and *time-dependent* expansion coefficients, can be considered as: a formal expression of the qualification of the physical state represented by that state vector, inside the framework for qualification of *any* quantum mechanical state introduced by the observable A; namely, a probabilistic qualification of the state by the probability densities $|c(\psi, a_j)|^2 = |\langle u_j \mid \psi \rangle|^2$ of the observable outcomes $f_A(V_j) = a_j$ of the quantum mechanical elementary chain experiments performed with that state and with the measurement evolutions M_A for A.

In particular, it can happen that the spectrum of the considered observable A be discrete (Hamiltonian of a bounded state or a kinetic momentum). This entails then an identification of each eigenvector, with a state vector of a preparable state (which involves then also a definite finite norm for the eigenvectors, as well as mutual orthogonality, and independence of time for the ensemble (a discrete infinity) of these "eigenstate vectors"). Nevertheless, even in these particular situations which introduce for each eigenstatevector a cumulation of two distinct roles, the conceptual difference still quite fully subsists between the designatum of a superposition of several eigenstate vectors on the one hand (way of preparing the superposition state vector), and on the other hand the designatum of a decomposition of a state vector along the whole infinity of eigenstate vectors from the basis of eigenvectors of the considered observable (way of qualifying that state vector). And the existence of this difference continues to be even formally disclosed by the subsistence of the possible relevance, or not, of real coefficients.

So the code explicated above always avoids confusion between superpositions and spectral decompositions. (The removal of this confusion might clarify the significance of conceptually rather obscure perturbation methods used for the calculation of the spectrum of energy of quantified systems, etc.) But resort to the code ceases to be necessary as soon as one is in possession of the concept of a probability tree. Again, by the simple contemplation of the figure that represents a tree, it becomes *obvious* that the superpositions concern the *primary* operation $P(\psi_0)$ of generation of an *object* for subsequent examinations, while the decompositions concern the *secondary* operations M_A of *qualification* of this object (Figs. 2 and 3A (p. 1440)). Again it jumps at one's eyes that these two concepts concern essentially *different phases* of the genesis of the quantum mechanical events, placed at two different temporal levels of a tree, imbedded in different spacetime domains and *possessing essentially different cognitive roles.*

3.3.3. The Germ of a Calculus with Whole Probability Trees (Probabilistic Meta-Metadependence). The quasi-confounded treatment of superpositions and of spectral decompositions hides the important fact that, in a certain sense, a superposition of states—but not also a spectral decomposition—involves a germ of *a calculus with several probability trees, globally considered.*

Consider a state vector $|\psi\rangle$ wich is instructionally defined by the specification of only one preparation procedure $P(\psi)$. Then the probability measures from the observational spaces (3′) of the corresponding probability tree are completely specified by reference to the only *one* state vector $|\psi\rangle$ tied with the unique operation of state preparation $P(\psi)$ (for simplicity we suppose measurements directly on the prepared state $|\psi\rangle$, i.e., we consider the particular case $t - t_0 = 0$, $|\psi\rangle \equiv |\psi_0\rangle$). For example, in $(3')_A$ the measure $\pi(\psi, a_j)$ is calculable on the basis of the postulate (2),

$\pi(\psi, a_j) = |\langle u_j | \psi \rangle|^2$, by making use of exclusively the state vector $|\psi\rangle$. But the situation changes if we consider a superposition state vector

$$|\psi_{ab}\rangle = \lambda_a |\psi_a\rangle + \lambda_b |\psi_b\rangle$$

(as, for example, in the case of a Young interference). Then—physically—the corresponding preparation $P(\psi_{ab})$ still introduces only one state $|\psi_{ab}\rangle$, so only one probability tree. Nevertheless, as has already been stressed, the probability measures from the observable spaces (3′) of this unique tree are now calculated by reference to, also, the two state vectors $|\psi_a\rangle$ and $|\psi_b\rangle$ from the mathematical expression of $|\psi_{ab}\rangle$. This happens algorithmically, via the combination of

— the additive quantum mechanical representation of the state $|\psi_{ab}\rangle$ by a superposition writing

— the spectral decomposition writings.

— the probability postulate (2).

Indeed, when accordingly to (S) and (2) the measure $\pi(\psi_{ab}, a_j)$ will have to be calculated by the use of the relation (12): $\pi(\psi_{ab}, a_j) = |\lambda_a|^2 |\langle u_j | \psi_a\rangle|^2 + |\lambda_b|^2 \langle u_j | \psi_b\rangle|^2 + 2\,\mathrm{Re}\{(\lambda_a)(\lambda_b)^* \langle u_j | \psi_a\rangle \langle u_j | \psi_b\rangle^*\}]$ (or in particular (12′): $\pi(\psi_{ab}, x_j) = |\lambda_a|^2 |\psi_a(x_j)|^2 + |\lambda_b|^2 |\psi_b(x_j)|^2 + 2\,\mathrm{Re}\{(\lambda_a)(\lambda_b)^* \psi_a(x_j)\, \psi_b(x_j)^*\}]$), *three* probability trees are—globally—brought into play, namely: The unique tree $\mathscr{T}(\psi_{ab})$ physically generated by the unique physically realized preparation $P(\psi_{ab})$, and furthermore the two trees $\mathscr{T}(\psi_a)$ and $\mathscr{T}(\psi_b)$ corresponding to the two preparations $P(\psi_a)$ and $P(\psi_b)$ on which the preparation $P(\psi_{ab}) = f(P(\psi_a), P(\psi_b))$ "depends"—considered separately—which have not been realized individually, but, being reflected in the writings by the specification of their possible individual results $|\psi_a\rangle$ and $|\psi_b\rangle$, act there as a conceptual reference. In fact what is brought into play is *a structure of three mutually consistent rules of "formal composition,"* namely the rule of composition of:

— The reference preparation operation $P(\psi_a)$ with the reference preparation operation $P(\psi_b)$:

Some definition *of the function* $f(P(\psi_a), P(\psi_b)) = P(\psi_{ab})$ *and of its physical counterpart are supposed to "exist": this supposition in fact constitutes the fundamental principle of superposition. However, this basic definition is not spelled out inside quantum mechanics as it now stands.*

— The reference state vector $|\psi_a\rangle$ corresponding to the preparation operation $P(\psi_a)$, with the reference state vector $|\psi_b\rangle$ corresponding to the preparation operation $P(\psi_b)$ (the additive rule (S)).

— The corresponding reference observable probability measure $|\lambda_a \langle u_j | \psi_a\rangle|^2$, with the reference observable probability measure $|\lambda_b \langle u_j | \psi_b\rangle|^2$ (the quantum mechanical algorithm $(2 + S) = (12)$).

Globally, what comes here in implicitly is a complex algorithm of formal composition of the two only conceived reference-probability trees $\mathscr{T}(\psi_a)$ and $\mathscr{T}(\psi_b)$, such as to yield

by a sort of "probabilistic dependence" defined between entire trees

precisely the result postulated by the relation (2) for the probability measures from the unique tree $\mathscr{T}(\psi_{ab})$ which is physically realized. *Such an algorithm amounts to endowing the mathematical representation assigned to each level of the unique physically realized tree (operation of preparation, prepared state vector, observable probability space) with an incorporated* reference *to the corresponding level of the two other, only conceived trees.*

Obviously, such a representation, endowed with such a reference, transgresses essentially the concept of one probability tree; it involves certain meta-qualifications with respect to the qualifications which can be expressed inside the nonreferred representation of one single tree. We are here in the presence of a probabilistic meta-metadependence with respect to the present standard concept of probabilistic dependence (since the quantum theory of transformations involves already—inside a unique tree—a sort of probabilistic metadependence with respect to the probabilistic dependence in the sense of the theory of probabilities as it now stands). Only if this probabilistic meta-metadependence, globally considered, is taken into consideration also, does it become possible to try to encompass the whole significance of the quantum mechanical principle of superposition.

Thus, inside quantum mechanics as it now stands, the germ of certain algorithms can be discerned corresponding to an implicit calculus with entire probability trees. This happens each time that superposition states are represented. (This happens also each time that successive measurements are represented. But then the conceptual insertion is different: Instead of the principle of superposition, the projection postulate acts at the bottom, identifying the operations of preparation in the general sense, with the particular category of preparations by measurement evolutions. This distorts and flattens the conceptual space involved.) However, with the implicit and incomplete quantum mechanical calculus with entire probability trees we penetrate into this confused frontier zone—which always does exist—where the representations already elaborated by a theory plunge into the still unconceptualized. *The basic lacuna is that the operations of state preparation* *are devoid of mathematical representation.* This is a lacuna of which the consequences mark the intelligibility of the whole orthodox formalism.

4. COMPLEMENTS TO THE ORTHODOX FORMALISM: OPERATORS OF STATE PREPARATION, MEASUREMENT PROPAGATOR; A THEOREM RELATING PREPARATIONS AND MEASUREMENTS

The explicit and integrated perception of the probabilistic organization of quantum mechanics and of the spacetime aspects involved has permitted us in Sec. 3 to discern, inside quantum mechanics, probabilistic features not yet described in the standard theory of probabilities: probabilistic meta-dependences and meta-metadependences. These point toward a possible extension of the abstract theory of probabilities. But in connection with the mentioned features also certain insufficiencies of the quantum theory itself have been perceived. In what follows, in order to diminish these insufficiencies, we produce three constructive prolongations of the orthodox formalism.

4.1. Operators of State Preparation and Their Calculus

What operators and what calculus with these can be defined in order to represent mathematically the physical operations of state preparation in a way that is consistent with the orthodox formalism as it now stands?

Suppose that $G(\psi)$ (G: generator) is an operator that represents mathematically the operation of state preparation $P(\psi)$. For consistency with the linear formalism of quantum mechanics, let us require $G(\psi)$ to be a *linear* operator. Then, to represent mathematically the preparation of the states with state vectors $|\psi_a\rangle$, $|\psi_b\rangle$, $|\psi_{ab}\rangle=\lambda_a|\psi_a\rangle+\lambda_b|\psi_b\rangle$, we have to write, respectively, for any choice of some initial state vector $|\psi_i\rangle$:

$$G(\psi_a)\,|\psi_i\rangle=|\psi_a\rangle,\qquad G(\psi_b)\,|\psi_i\rangle=|\psi_b\rangle,\qquad G(\psi_{ab})\,|\psi_i\rangle=|\psi_{ab}\rangle$$

(read: $G(\psi_a)$ acting on some—any—previously existing state with state vector $|\psi_i\rangle$ (known or *not*), generates out of it the state with state vector $|\psi_a\rangle$, etc.). The unknown functional relation "f" from the representation $P(\psi_{ab})=f(P(\psi_a), P(\psi_b))$ concerning the three *factual* operations denoted $P(\psi_{ab})$, $P(\psi_a)$, $P(\psi_b)$ involved in the preparation of a superposition state vector $|\psi_{ab}\rangle=\lambda_a|\psi_a\rangle+\lambda_b|\psi_b\rangle$, will somehow translate into a formal

relation "g," $G(\psi_{ab})=g[G(\psi_a), G(\psi_b)]$. To find the translation we write down the conditions, in agreement with the linearity required for the $G(\psi)$:

$$\begin{aligned}G(\psi_{ab})\,|\psi_i\rangle&=|\psi_{ab}\rangle=\lambda_a|\psi_a\rangle+\lambda_b|\psi_b\rangle=\lambda_a G(\psi_a)\,|\psi_i\rangle+\lambda_b G(\psi_b)\,|\psi_i\rangle\\&=[\lambda_a G(\psi_a)+\lambda_b G(\psi_b)]\,|\psi_i\rangle\end{aligned}\tag{13}$$

The function g that connects the operators $G(\psi_{ab})$, $G(\psi_a)$, $G(\psi_b)$ is the same linear combination that connects the state vectors $|\psi_{ab}\rangle$, $|\psi_a\rangle$, $|\psi_b\rangle$. So, in general terms,

$$g[G(\psi_a), G(\psi_b), G(\psi_c),\ldots]=G(\psi_{abc\ldots})=\sum_k \lambda_k G(\psi_k),\quad k=a,b,c,\ldots\tag{14}$$

Furthermore, since for the well-known quantum mechanical operator of projection onto $|\psi\rangle$, P_ψ, we have $P_\psi|\psi_i\rangle=|\psi\rangle\langle\psi\mid\psi_i\rangle$, $\forall\,|\psi_i\rangle$; while by definition $G(\psi)\,|\psi_i\rangle=|\psi\rangle$, we can write

$$G(\psi)=(1/\langle\psi\mid\psi_i\rangle)\,P_\psi\tag{15}$$

which we shall call a "normed projector" onto $|\psi\rangle$:

A "normed" projector P_ψ yields an adequate representation for the concept of an "operator $G(\psi)$ of state preparation" such as required by (14).

From (14) and (15) it follows that for a superposition state vector $|\psi_{abc\ldots}\rangle$ we can write

$$G(\psi_{abc\ldots})=(1/\langle\psi_{abc\ldots}\mid\psi_i\rangle)\,P_{\psi abc\ldots}=\sum_k(\lambda_k/\langle\psi_k\mid\psi_i\rangle)\,P_{\psi k},\qquad k=a,b,c,\ldots\tag{16}$$

The operator of preparation of a superposition state vector can be represented mathematically by a linear combination of normed projectors.

This includes automatically the *particular* case of preparation by a measurement evolution M_A posited by the orthodox projection postulate: In that case the state preparation operator becomes indeed $(1/\langle u_j\mid\psi_i\rangle)\,P_{|uj\rangle}$ where $|u_j\rangle$ is the eigenvector of the observable A corresponding to the registered eigenvalue a_j. But it has to be clearly realized that in the formalism as it now stands the projectors P_ψ are *not* utilized with the fundamental role of general formal representatives of the operations of state preparation. The projectors P_ψ are utilized most currently in the algorithms connected with *measurement* operations [density (or statistical) operators].

The definition (15) + (16) has interesting implications concerning the coherence between the semantics to be assigned to the formal feature of commutativity of two linear operators and the nonformalized qualification of "compatibility" drawn from the current language:

For consistency with the linear formalism of quantum mechanics we have required linearity for the operators of state preparation. This entailed the necessity, in (16), of a linear superposition of *distinct*, thus *non*commuting, normed projectors $P_{\psi a}$, $P_{\psi b}$,... that will *all* act on *one* same initial state vector $|\psi_i\rangle$. While two commuting projectors—which reduce in fact to one single projector—cannot generate a superposition state vector because they (it) cannot represent the required *distinct* actions on *one* same initial state vector $|\psi_i\rangle$. In *this* sense:

For the mathematical representation of the process of generation of a superposition state, distinct and noncommuting operators of state preparation are "compatible" operators.

This appears "opposite" to what happens for the mathematical representation of the operations of measurement of dynamical quantities: two *dynamical* operators, as is well known, are considered to be "compatible" when they commute, while if they do not commute they are considered to be "incompatible."

Now, we have emphasized that in the case of the representation of measurement operations, the factual counterpart of the "compatibility" of two—commuting—dynamical operators A and B consists of the possibility of *individual* measurement evolutions M_{AB} for A and B possessing one *common* spacetime support. This is what entails the possibility, from each (one) registered "needle position" V_j that has been the unique factual observable outcome of one given reiteration of a measurement evolution M_{AB}, to calculate a *pair* of two correlated eigenvalues $a_j = f_A(V_j)$, $b_j = f_B(V_j)$ (which is verbally designated as the possibility of a "simultaneous" measurement of the observables A and B). While if A and B do not commute, the individual measurement evolutions M_A for A and M_B for B possess necessarily distinct spacetime supports, which is designated by the assertion that "simultaneous measurements for A and B are not possible" (the factual substance of Bohr complementarity).

In short: When exclusively measurement operators are considered, the two qualifications "commuting" and "compatible" apply to the same subensemble of operators, so that they tend to identify. But when also normed projectors as representatives of operations of state preparation are considered, the domains of application of these two qualifications separate. So a new language emerges which concerns a more complex situation. We shall now establish explicitly this new language. Take into account:

— the usage of language found above and the corresponding designata for the case of measurements,

— the fact that two different projectors do not commute while two commuting projectors identify,

— the fact that the different projectors involved in the preparation of a superposition state represent individual operations that are physically different and, nevertheless, can all act on one same initial—individual—factual situation corresponding to one same initial quantum mechanical state vector $|\psi_i\rangle$,

— systematic distinction between abstract descriptor and its physical designatum

— systematic distinction between

* the *individual* level of description (where are placed the various individual realizations of an operation of state preparation, or of one measurement evolution, or of an elementary chain experiment)

* the metalevel of *probabilistic* description (where is placed by definition the quantum mechanical concept of state vector $|\psi\rangle$ and, correlatively, the concept of "one" (complete) quantum mechanical measurement involving a whole ensemble of elementary chain experiments,

and, finally,

— requirement of one same stable language valid no matter whether measurements or state preparations are described.

The elements listed above entail together the following rather complex distionary.

— "compatibility" or "noncompatibility" of two linear operators (dynamical or not): respectively, the relevance or not of the action of both these operators on *one* **individual** realization of a state of the studied system corresponding to one given quantum mechanical state vector.

— "commutativity" or "noncommutativity" of two linear operators (dynamical or not): respectively, the identity or the disjoint character of the space*time* supports of the individual physical operations represented by these two operators.

— *Multiplicative* composition of the action of two (or more) commuting *dynamical* operators upon one given state vector $|\psi\rangle$: mathematical expression of the *factual* identity of two (or more) processes of *qualification* of any *one* individual realization of a state of a system corresponding to $|\psi\rangle$, via *one* common sort of *individual* measurement evolutions $M_{AB...}$

realizable on one same spacetime support, but of which the—one, common—*factual* observable outcome V_j, once it has emerged, can then be *conceptually* worked out in *various* ways, $a_j = f_A(V_j)$, $b_j = f_B(V_j)$, etc. (which justifies the above somewhat misleading wording "two or more" processes of qualification).

— *Additive* composition of two (or more)—necessarily—noncommuting operators *of state preparation* (normed projectors) upon one given initial state vector $|\psi_i\rangle$: mathematical expression of the generation, out of any *one* (individual) realization of a factual state of the studied system tied with the quantum mechanical state vector $|\psi_i\rangle$, of one realization of a new factual state of the studied system tied with a new quantum mechanical state vector $|\psi_0\rangle$, via the action of two (or more) *factually* different processes of "preparation" possessing disjoint spacetime supports, all these processes being posited to end at a same moment, which is the initial moment t_0 of the newly prepared state vector $|\psi_0\rangle = |\psi(t_0)\rangle$.

With this dictionary, we can now say that:

In the case of the representation of an operation of state preparation $G(\psi abc...) = (1/\langle\psi_{abc...} \mid \psi_i\rangle)\, P_{\psi abc...} = \sum_k (\lambda_k/\langle\psi_k \mid \psi_i\rangle)\, P_{\psi k}$, $k = a, b, c,...$ *that generates a superposition state* $|\psi_{abc...}\rangle = \sum_k(\lambda_k \mid \psi_k\rangle$, *the distinct* noncommuting *normed projectors* $(1/\langle\psi_{abc...} \mid \psi_i\rangle)\, P_{\psi k}$ *that are involved correspond to compatible physical actions of which nevertheless the spacetime supports are* disjoint.

So quantum mechanics permits (could we even say that it requires?) a certain *coherent prolongation* of its formalism and its language, where the operations of state preparation (all of them, not only those consisting of measurement evolutions M_A) are mathematically represented by operators of state preparation $G(\psi)$ that are normed projectors combined accordingly to a specific calculus entailed by the fact that the space of the normed kets $|\psi\rangle$ is a vector space. This calculus with operators of state preparation is distinct from the calculus with dynamical operators, which represent measurements and are tied with the principle of spectral decomposition. This finally *demonstrates* that the formal structure of the quantum theory by no means entails the orthodox flattening identifications between preparations and measurements and between superpositions of several state vectors and spectral decompositions of one state vector: It rejects them in fact, if we go to the bottom.

The definition (15) + (16) of operators of state preparation effaces the lacuna in the rules of combination of two or more probability trees regarded as wholes. So the implicit quantum mechanical calculus with

whole probability trees, expressing a new probabilistic concept of probabilistic meta-metadependence, is now entirely explicated.

But the most important consequence is indicated below.

4.2. The Minimal Model Involved by the Principle of Superposition

In quantum mechanics as it now stands, the mathematical expression of the principle of superposition is referred exclusively to the state vectors. This is misleading. Indeed—fundamentally—the principle of superposition talks about operations of state preparation. And the definition (15) + (16) is equivalent to a deepened reformulation involving now explicitly these operational roots also. This permits progress concerning the physical implications of the principle.

Consider a two-term superposition state vector $|\psi_{ab}\rangle = \lambda_a |\psi_a\rangle + \lambda_b |\psi_b\rangle$. We have shown that in order to represent mathematically the operation of preparation of $|\psi_{ab}\rangle$ we must make use of a normed projector $(1/\langle\psi_{ab} \mid \psi_i\rangle)\, P_{\psi ab} = (\lambda_a/\langle\psi_a|\psi_i\rangle)\, P_{\psi a} + \lambda_b/\langle\psi_b|\psi_i\rangle\, P_{\psi b}$ that is a linear combination of two distinct normed projectors $(1/\langle\psi_a|\psi_i\rangle)\, P_{\psi a}$ and $(1/\langle\psi_b \mid \psi_i\rangle)\, P_{\psi b}$ which act on *one* initial state vector $|\psi_i\rangle$ out of which they generate $|\psi_{ab}\rangle$: $[(\lambda_a/\langle\psi_a|\psi_i\rangle)\, P_{\psi a} + (\lambda_b/\langle\psi_b|\psi_i\rangle)\, P_{\psi b}]\, |\psi_i\rangle = |\psi_{ab}\rangle$. We have also shown that this mathematical representation involves the assumption of "compatibility" of the physical processes described by the two operators $(1/\langle\psi_a\psi_i\rangle)\, P_{\psi a}$ and $(1/\langle\psi_b \mid \psi_i\rangle)\, P_{\psi b}$, in a definite sense which concerns the spacetime features of the mentioned processes. Now, in consequence of the conditions of norm, the two spatial domains $\Delta(|\psi_a|^2, t) = \Delta(a)$ and $\Delta(|\psi_b|^2, t) = \Delta(b)$, where $|\psi_a\rangle$ and $|\psi_b\rangle$, respectively, yield presence probabilities that are not quasi-null, are *finite* with respect to any fixed definition of quasi-nullity. And, since the current formulation of the principle of superposition asserts that the state represented by $|\psi_{ab}\rangle$ can be created for *any* pair $|\psi_a\rangle$ and $|\psi_b\rangle$, we are free to imagine in particular that $|\psi_a(x, t)\rangle$ and $|\psi_b(x, t)\rangle$ are such that, at a given time t (in the observer's referential) the two spatial domains $\Delta(a, t)$ and $\Delta(b, t)$ are disjoint *and* the (purely spatial) distance that separates them is very big, say, of the order of light-years. Nevertheless quantum mechanics still assumes, as it is explicitly expressed by the new writing $[(\lambda_a/\langle\psi_a|\psi_i\rangle)P_{\psi a} + (\lambda_b/\langle\psi_b|\psi_i\rangle)P_{\psi b}]|\psi_i\rangle = |\psi_{ab}\rangle$, that there *does* exist an initial state vector $|\psi_i(x, t')\rangle$, $t' < t$, of the **one** considered "system," such that the two preparation processes represented by the two mathematical writings $(1/\langle\psi_a \mid \psi_i\rangle)\, P_{\psi a}$ and $(1/\langle\psi_b \mid \psi_i\rangle)\, P_{\psi b}$ can both take place "compatibly" on each individual realization of a factual state corresponding to the state vector $|\psi_i(x, t')\rangle$. But this is a *model*.

The principle of superposition associates to the entity called "one" quantum system, a model *according to which an individual factual realization of a state of this entity* can *be such that—whatever be other nonspecified qualifications of it—this state* covers an arbitrarily big spatial domain, *notwithstanding that in some nonspecified sense a quantum system is conceived to be also "microscopic" (it is even often called "one microsystem").*

Horribile dictu, but the orthodox formulation, though it proclaims interdiction of any model, in fact is itself *founded* on a model. And this model, on the one hand, violates the natural slopes of the connection between what we would agree to call a microsystem and the designatum forced upon us by the princile of superposition, and, on the other hand, is not achieved, not worked out. In this sense it is a "minimal" model. Whether it is explicitly declared or not, this minimal model *is* there, encapsulated into the principle of superposition. Camouflaged loosely inside the conceptual volume delimited by its noncommittal absence of full specification, this minimal model fluctuates there implicitly, leading to confusion and perplexity. And it acts on our speaking and on our thinking. It literally invades them in the form of problems and paradoxes that haunt the quantum theory ever since it appeared. "Schrödinger's cat" or more abstractly "the reduction problem," as well as the "locality" problem, are only the most striking distillations and scandalous amplifications of consequences of this hidden unfinished model. Only further specifications could remove the ambiguities that emanate from this model, and perhaps thereby also its queerness. In Sec. 5 of this work we shall sketch out qualitatively such further specifications, while just beneath we continue to stay inside the orthodox theory.

4.3. Measurement Propagator

We have brought forth a radical distinction between, on the one hand, preparations and superpositions of several state vectors and, on the other hand, measurements and spectral decompositions of one state vector. We shall now try—confined inside the orthodox approach—to understand more clearly how these two distinct pairs of concepts are *related.*

Bohm,[14] de Broglie,[15] Margenau and Park[19] (in their study of the "time of flight" method for the measurement of the momentum observable), as well as other authors, have already strongly and variously emphasized that an evolution of the descriptor $|\psi(x, t)\rangle$, if it is "good" for producing "measurement evolutions" M_A of the first kind for an observable A, possesses specific characteristics. Nevertheless, quantum mechanics as it now stands does not introduce an explicit general definition of the operator of evolution

H_A to be tied with the individual measurement evolutions M_A corresponding to a dynamical observable A. It only supposes implicitly that, given a "physically significant" quantum mechanical observable A (as is well known, not any quantum observable is measurable), such an operator H_A *can* be found for A. Below we introduce a condition that ensures some of the characteristics identified by other authors.

Condition CH_A. A quantum mechanical evolution operator H_A can be connected with the *individual* measurement evolutions M_A of the first kind corresponding to a quantum mechanical observable A only if it works like an operator

$$[1/\langle\chi(x, t') \mid \psi(x, t)\rangle]\, P_{\chi(x,t')}$$
$$= \sum_j [|c(\psi, a_j)|\, e^{i\alpha(j)}/\langle\Phi_j(x, t') \mid \psi(x, t)\rangle]\, P_{\Phi j(x,t')}$$

of preparation, out of the studied state vector $|\psi(x, t)\rangle$, of the superposition state vector

$$|\chi(x, t')\rangle = \sum_j |c(\psi, a_j)|\, e^{i\alpha(j)}\, |\Phi_j(x, t')\rangle, \qquad t' > t$$

where

— $|\Phi_j(x, t')\rangle$, for any j, is a normed eigendifferential corresponding to an eigenvector $|u_j(x)\rangle$ of A

— the coefficients of linear combination reproduce the real parts $|c(\psi, a_j)|$ of the expansion coefficients $c(\psi, t, a_j)$ from the spectral decomposition $|\psi(x, t)\rangle = \sum_j c(\psi, t, a_j)\, |u_j(x)\rangle$ of the studied state vector $|\psi(x, t)\rangle$ on the basis of eigenvectors $|u_j(x)\rangle$ of A, the factors $e^{i(\alpha)j}$ being arbitrary (in particular, these factors can reproduce those from the $c(\psi, t, a_j)$, or, alternatively, they can be all set equal to 1, thus introducing a superposition with real coefficients).

— the spatial domains $\Delta_j(\chi, t')$, where the presence probabilities corresponding to the state vectors $|\Phi_j(x, t')\rangle$ are not practically zero, become mutually disjoint up to an approximation that can be improved without limitation by increasing t'.

This condition requires H_Λ such that out of the studied state vector $|\psi(x, t)\rangle$ it shall materialize approximately in the *physical* space, at times $t' > t$, the abstract spectral decomposition of $|\psi(x, t)\rangle$ on the basis of eigenvectors of Λ.

4.4. A Measurement Theorem

The condition CH_A, if it is realized, entails the following theorem.

Measurement Theorem MT. The event "registration for $|\psi(x, t)\rangle$ of an eigenvalue a_j of A" can be represented by the event ["registration for $|\chi(x, t')\rangle$ of the presence inside the domain $\Delta_j(\chi, t')$"] $\approx [x \in \Delta_j(\chi, t')]$, namely, in the following sense. The numerical equality

$$\pi(\psi, a_j) = \pi(x \in \Delta_j(\chi, t'))$$

where $\pi(\psi, a_j)$ and $\pi(x \in \Delta_j(\chi, t'))$ are, respectively, the quantum mechanical probabilities of the first and the second event specified above, is realized with an arbitrarily improvable accuracy for any measurement M_A of the first kind.

Proof. Consider the superposition state vector $|\chi(x, t')\rangle = \sum_j |c(\psi, a_j)|\, e^{i\alpha j}|\, \Phi_j(x, t')\rangle$, $t' > t$, as defined in CH_A. At any individual spacepoint x we have for $|\chi(x, t')\rangle$ a presence probability which (at most) is reduced to only one term

$$\pi(x, \chi) = |\chi(x, t')|^2 = |c(\psi, a_q)|\, e^{i\alpha q}\Phi_q(x, t')|^2 = |c(\psi, a_q)|^2\, |\Phi_q(x, t')|^2$$

where the index q designates, among all the disjoint spatial domains $\Delta_j(\chi, t')$, that one to which belongs the considered point x. Then the *total* quantum mechanical presence probability inside the domain $\Delta_q(\chi, t')$ is, from the expression of $\pi(x, \chi)$ and because of the norm 1 of the $|\Phi_j(x, t')\rangle$,

$$\pi(x \in \Delta_q(\chi, t')) = \int_{\Delta q} |\chi(x, t')|^2\, dx = |c(\psi, a_q)|^2 \int |\Phi_q(x, t')|^2\, dx = |c(\psi, a_q)|^2$$

which by the postulate (2) is also the quantum mechanical probability for the realization of the eigenvalue a_q. This is true only approximately but with an accuracy which accordingly to CH_A can be improved arbitrarily by increasing t', i.e., by improving the mutual disjunction of the spatial domains $\Delta_j(\chi, t')$ and thus the mutual orthogonality of any two distinct state vectors $|\Phi_j(x, t')\rangle$ and $|\Phi_k(x, t')\rangle$). So, with an arbitrarily improvable accuracy, we have indeed

$$\pi(x \in \Delta_q(\chi, t')) = |c(\psi(t), a_q)|^2 = \pi(\psi(t), a_q), \qquad t' > t \quad ◆$$

This proof, trivial as it is, establishes a crucial *connection* between the two fundamentally *distinct* concepts of spectral decomposition and of superposition of states. More, in fact. It establishes, for the general quantum mechanical predictional *postulate* (2) $\pi(\psi, a_j) = |\langle u_j | \psi \rangle|^2$

$|\langle u_j | \psi \rangle|^2$], an "explanation" *deduced* from the condition CH_A and the particular acceptance of (2) concerning exclusively the position observable, $\pi(\psi, x) = |\psi(x)|^2$. And notice that the deduction is founded upon the *distinction* between spectral decompositions of a state vector and superpositions of several state vectors.

Via the condition CH_A *and the theorem* MT, *the spectral decomposition of the studied state vector* $|\psi(x, t)\rangle$ *with respect to the eigenvectors of a measured observable* A *appears as an only* abstract conceptual prefiguration *of the superposition state* $|\chi(x, t')\rangle$ *actually prepared in the physical space, at later times* $t' > t$, *by the quantum mechanical operator* H_A *of measurement evolutions* M_A.

By a rotation inside the Hilbert space of the system, the measurement propagator H_A brings asymptotically the conceptual spectral decomposition with respect to the eigenkets of A, of the studied state vector $|\psi(x, t)\rangle$, down onto the physical space. The *abstract* "disjunction" represented by the spectral decomposition $|\psi(x, t)\rangle = \sum_j c(\psi, t, a_j)\, |u_j(x)\rangle$ distinguishes inside $|\psi(x, t)\rangle$ between the elements of a family of mutually exclusive "how's" represented by eigenvectors $|u_j\rangle$, no matter *where* in spacetime, since $\langle u_k | u_j \rangle = 0$ for $j \neq k$ but the $|u_j\rangle$'s are time-independent and in general distinct $|u_j\rangle$'s do not possess disjoint spatial supports. The measurement propagator H_A transposes this abstract disjunction into a "disjunction" in the physical space, represented by the superposition state vector $|\chi(x, t')\rangle = \sum_j |c(\psi, a_j)|\, e^{i\alpha j}|\, \Phi_j(x, t')\rangle$, $t' > t$, that distinguishes between the elements of a family of mutually exclusive "where's," the $\Delta_j(\chi, t')$, while *how* is what populates the disjoint spatial domains $\Delta_j(\chi, t')$ is devoid of pragmatic significance: With respect to the pair of qualifications how-where the initial situation and the final one are *opposed.*

Consider in particular the following degenerate situation. A = H, where H is the operator of total energy of a microscopic *bound* state. The spectrum of eigenvalues $a_j = E_j$ of the considered observable H is discrete and the corresponding eigenvectors identify with a discrete family of normed state vectors, so the spectral decomposition of the studied state $|\psi\rangle$ with respect to H identifies with the superposition state $|\chi\rangle$ involved in the condition CH_A while the time parameter loses its importance, the situation being stationary. In these conditions it seems necessary to assume in a "self-referent" way that $H_{A=H}$ is H itself. This suggests that, in this case, that is particular from a logical standpoint but of outstanding pragmatic importance, it has to be assumed that *a corresponding measurement evolution of the first kind has already been accomplished by the natural process that has brought forth the considered quantized bound state.* This natural process did produce the mutually disjoint spatial domains $\Delta_j(\chi, t')$

also, but because of the spatial confinement of the whole studied superposition state vector inside a microscopic domain, the estimation of *presence* probabilities as a substitute for the experimental estimation of the probabilities $|c(\psi(t), E_j)|^2$ is impossible. Only another indirect method can be used in this case for the experimental estimation of the probabilities of the eigenvalues E_j of H.

Since quantum mechanics was built while attention was focalized mainly upon the atomic bound states, the remarks made above offer an explanation of the tendency toward an identification between superpositions of state vectors and spectral decompositions of a state vector.

5. BEYOND ORTHODOX QUANTUM MECHANICS: (*) THE [PARTICLE + MEDIUM] INDIVIDUAL MODEL

5.1. Probabilistic Insufficiencies of the Orthodox Theory and Questions of Interpretation

The analyses performed in the preceding section produced, we hope, a reiterated and increasingly striking perception of the absence of any definite and formalized description, inside the orthodox quantum mechanics, for the individual processes corresponding to the elementary quantum mechanical chain experiments. These, by their very definition, involve only *one* realization of an operation of state ~~preparation,~~ generation of a measurement evolution M_A, and of a registration of an observed datum V_j. The orthodox formalism involves them quite fundamentally: *They are the operational-processual substratum of the quantum mechanical elementary events in the sense of probabilities, the observed data V_j formally represented by corresponding eigenvalues a_j. They constitute the reproducible procedure (4) from the random phenomena (5), (5′) that introduce the quantum mechanical probability spaces.* Nevertheless they are devoid of mathematical descriptors. They are even devoid of only an explicit convention for symbolization. They are just left nonrepresented inside the merely spoken accompaniments of the formalism. In this sense—quite independently of any philosophical issues concerning essential indeterminism—the orthodox formalism is incomplete. It is incomplete with respect to itself, with respect to the abstract theory of probabilities which it applies and even develops implicitly. This, at least partially, explains why the probabilistic organization of quantum mechanics remained so cryptic. In Sec. 4 we have partially remedied this incompleteness by constructing, inside the orthodox formalism, a mathematical representation of the operations of state ~~preparation~~ generation and by associating a mathematically expressed condition to the individual measurement evolutions previously symbolized *by us* by M_A

(*) Aujourd'hui j'écrirais beaucoup plus simplement le chapitre 5. Vous pourriez, soit ne pas le lire, soit le

(which led to a measurement theorem). But the *object* of one single realization of an operation of state preparation followed by an individual evolution M_A still remains ambiguous. On the one hand, the orthodox formalism associates a state vector $|\psi\rangle$ with an operation of state ~~preparation,~~ generation, but with *each* one reiteration, or only with the *ensemble* of all of them? On the other hand, we have shown that at the very bottom of the orthodox formalism, in the principle of superposition, is encapsulated a *model* concerning that on what one operation of state preparation acts, a model which at one and the same time is camouflaged, incompletely specified, and obscurely perceived but nonunderstood, thereby generating paradoxical problems. In what follows, transgressing now decidedly the domain of the orthodox theory, we shall try to specify this *orthodox* model, but only qualitatively for the moment. It will be remarkable to find that this suffices for triggering a whole chain of "explanations" concerning the physical significances of: the principle of superposition, superselection rules, spectral decompositions of a state vector versus superpositions of state vectors, the projection postulate, and the reduction problem. Thereby the loops of interrogations opened up in the preceding sections will be closed by stippled lines.

5.2. A [Particle + Medium] Specification of the Minimal Model Involved in the Principle of Superposition

Concerning an individual "system" described by the quantum theory, let us admit tentatively that: An individual entity concerned by the quantum mechanical description *cannot* adequately be called a "microsystem," because each such entity consists of:

— (a) What is currently called the (universal) "medium" or the "void," simply *all* of it.

— (b) A *highly localized* part of the universal medium—*this* to be called a "microsystem" or a "particle"—which is separated inside this medium *conceptually*, for *methodological* reasons, and is regarded as a "source of perturbation" of the whole "rest," with respect to itself, of the universal "medium," the perturbations being posited to spread out with some *finite* phase velocity[20].

This sort of system will be called "[particle + medium] individual system," in short a [p + m] individual system.

Concerning a "state" of a $[p+m]$ individual system, let us admit that it is characterized by:

(a) A medium state.

(b) A "particle state" consisting of the association between

* An *invariant* "intrinsic particle state" of a given "type" determined by several quite definite qualifications (constant numerical values defining a "rest mass," a "spin," an "electric charge," etc.). So, using a perhaps more pertinent language, we can now say that a particle *is* a definite sort of highly localized state of movement of the universal medium.

* A *variable* "dynamical particle state" described by *other*, "dynamical" qualifications ("position," "velocity," "velocity-dependent mass," "momentum," "angular momentum," "energy," etc.) tat are relative to the state of observation.

The particle states are posited to interact with the medium state in the following sense. The particle state is supposed to *act* on the medium state via perturbations that are somehow dependent on the type of the intrinsic particle state, while the medium state is supposed to (in general) act *back* on the particle state, on the dynamical particle state, or even on the intrinsic particle state (creations or annihilations). This restores the unity of the universal medium, conceptually broken for methodological reasons.

Concerning the ~~*preparations*~~ generations of states, let us admit that:

— Each one operation of preparation of a state of a [p + m] individual system *does* introduce *one* particle and *only* one (if it introduces none, we shall say that it is only a preparation of a state of the medium, but not of the whole metaentity called a [particle + medium] individual system, while if it introduces two or more particles, we shall say that it is a preparation of a many-particle state, so that it exceeds the one-system quantum mechanics).

— Each one preparation of a state of a given sort of a [p + m] individual system introduces always one and the same typical invariant intrinsic particle state of that [p + m] individual system.

— Each one preparation of a state of a [p + m] individual system introduces a dynamical particle state which, in contradistinction to what happens for the intrinsic particle-state, belongs to *a whole ensemble* of possibilities. The dynamical particle states are *nonspecified and nondescribed individually inside quantum mechanics as it now stands.*

— Each one preparation of a state of a [p + m] individual system *does* introduce one and *only* one medium state which belongs to *a whole ensemble* of possibilities. The medium states are *nonspecified and nondescribed individually inside quantum mechanics.*

Concerning observation, let us assume that

— The registration of an eigenvalue of a quantum mechanical observable can be produced *only* by the interaction of a convenient macroscopic apparatus, with the sort of highly localized state of movement of the medium that has been called here a particle.

Concerning the relation between the states of a [p + m] individual system and the "corresponding" quantum mechanical state vector $|\psi\rangle$, let us admit that:

— The state vector $|\psi\rangle$ connected with the results of the actions of the operator of state preparation represented by the normed projector $(1/\langle\psi \mid \psi_i\rangle)\, P_\psi$ is *a statistical metadescriptor* representing the whole statistical ensemble of individual dynamical particle states and of correlative individual medium states of the involved individual [p + m] system. Via the algorithms of the quantum theory, this state vector $|\psi\rangle$ yields only a knot of mutually nonseparated characterizations of the one invariant intrinsic particle state of the studied [p + m] individual system, of its variable individual dynamical particle states and medium states introduced by the reiterations of the operation of state preparation represented by $(1/\langle\psi \mid \psi_i\rangle)\, P_\psi$, and of the relations between these.

Together, the ensemble of assumptions listed above constitute "the [particle + medium] model." The sort of individual system involved in this model is *both* "microscopic" by its "particle" part *and* arbitrarily extended, "cosmic," by its "medium" part: It obeys the requirements of the minimal orthodox model involved in the principle of superposition, and it completes this model explicitly.

The explicitly declared unlimited extension of the medium part is a new and fundamental feature with respect to the well-known de Broglie–Bohm model. More or less correlatively there are also other differences.

— In the first place, the [p + m] model does not *include* the extended "field" (called here the "medium") into the concept of "particle" or "microsystem." It juxtaposes it (methodologically only) to this concept. This difference is not devoid of significant consequences. Indeed, it follows from it that the [p + m] individual model does *not* introduce notions like "wave **of** the particle," or "wave-particle duality involving wave aspects **of** a particle," or "a particle which passes through both Young holes," or "a particle *of which* the corpuscular part passes through one Young hole while *its* wave passes through both Young holes," etc., *All this sort of paradoxical language is entirely effaced* by the [p + m] individual model. Such is the power of the choices of semantical assignations, to words, conventional as they are. We are left with just a state of movement of the universal medium involving one highly localized particle movement, only conceptually—but radically—separated inside the universal medium for methodological reasons favoring the descriptibility.

— In the second place the mutual distinction and characterization of the "particle" and the "medium" is, qualitatively, more specific.

— In the third place, the relation between the quantum mechanical state vectors and the defined individual system is *different*. The distinction between the two descriptional levels where these two descriptors are placed is more radical: It is posited to concern *also* the states of the medium part of an individual [p + m] system, not exclusively those of its particle part ("the position of the particle inside *its* wave"). This entails, for instance, in contradistinction to what is done in the de Broglie–Bohm interpretation, that the action of the *individual* medium state, upon the corresponding dynamical-particle-state, cannot *in general* be represented with the help of functions (like the quantum potential and the forces derived from it) of the amplitude and the phase *of the quantum mechanical state vector* which has a **statistical** meaning. In a future work this will be found possible only if the quantum mechanical state vector has in particular a one-to-one relation with the corresponding individual medium state, as probably happens in the case of a microscopic bound state and of certain macroscopic interference states.

— Finally, with respect to Bohm's concept of "holomovement" also there are differences. The [p + m] model of an individual system does not refer to the "*sub*quantum medium," nor to fluctuations of this medium; it refers directly to the object of the quantum theory itself, for which no other object is supposed.

5.4. The [Particle + Medium] Model and the Principle of Superposition

Consider the principle of superposition, formally expressed in our terms: for any pair of preparable state vectors $|\psi_a\rangle$ and $|\psi_b\rangle$ there exists, for any complex numbers λ_a and λ_b, an "initial" state vector and an operation of state preparation $(1/\langle\psi_{ab} \mid \psi_i\rangle)\, \mathrm{P}_{\psi ab} = (\lambda_a/\langle\psi_a \mid \psi_i\rangle)\, \mathrm{P}_{\psi a} + (\lambda_b/\langle\psi_b \mid \psi_i\rangle)\, \mathrm{P}_{\psi b}$ such that

$$(1/\langle\psi_{ab} \mid \psi_i\rangle)\, \mathrm{P}_{\psi ab} \mid \psi_i\rangle = [(\lambda_a/\langle\psi_a \mid \psi_i\rangle)\, \mathrm{P}_{\psi a} + (\lambda_b/\langle\psi_b \mid \psi_i\rangle)\, \mathrm{P}_{\psi b}] \mid \psi_i\rangle$$
$$= \lambda_a \mid \psi_a\rangle + \lambda_b \mid \psi_b\rangle = |\psi_{ab}\rangle$$

The [p + m] individual model permits us to understand now the principle of superposition as the assertion that, out of the initial ensemble of states of perturbation of the universal medium represented by $|\psi_i\rangle$, it is **in principle**

possible to produce for the studied [p + m] individual system *involving an indefinitely extended medium part*, a new ensemble of states of perturbation corresponding to any chosen quantum mechanical superposition state vector $|\psi_{ab}, t_0\rangle = \lambda_a |\psi_a, t_0\rangle + \lambda_b |\psi_b, t_0\rangle$. No matter how big is the spatial distance that separates from one another the two finite spatial supports $\Delta(a, t_0)$ and $\Delta(b, t_0)$ where the quantum mechanical presence probabilities defined respectively for the two state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$ are not practically null (according to some definition of practical nullity, any one but fixed), this can indeed be conceived to be possible via reiterations of some global action that takes place *on and in the indefinitely extended medium parts* of the involved replicas of the studied system. This global action, represented by the normed projector $(1/\langle\psi_{ab}|\psi_i\rangle)\, \mathrm{P}_{\psi ab}$, can be conceived to stem from the two separate actions represented by the two normed projectors $(\lambda_a/\langle\psi_a | \psi_i\rangle)\, \mathrm{P}_{\psi a}$ and $(\lambda_b/\langle\psi_b | \psi_i\rangle)\, \mathrm{P}_{\psi b})$ that combine in the expression of $(1/\langle\psi_{ab} | \psi_i\rangle)\, \mathrm{P}_{\psi ab}$. Each one of these two separate actions consisting of some corresponding process triggered by a human observer via some interactions with (or stemming from) macroobjects, thus necessarily *local* in spacetime, but that can be followed by a process extending inside the medium on arbitrarily big spacetime supports and *ending*, in each reiteration of the global process of preparation represented by $(1/\langle\psi_{ab} | \psi_i\rangle)\, \mathrm{P}_{\psi ab}$, at the time t_0 (statistical, measured with respect to the zerotime of that reiteration) when has been achieved an individual factual situation corresponding to the quantum mechanical descriptor $|\psi_{ab}, t_0\rangle = \lambda_a |\psi_a, t_0\rangle + \lambda_b |\psi_b, t_0\rangle$: The paradoxical notion that it is possible to act "simultaneously" *on one microscopic "particle"* at two arbitrarily distant place, disappears (Figs. 3A and 3B).

5.5. The [Particle + Medium] Model and Interference with Respect to the Summed State Vectors

The interferences relative to the summed state vectors appear as intimately related with the above interpretation of the principle of superposition. The "multiple" character of the operation of state preparation of a superposition state vector plays the main role. From the different localized sources of perturbation of the medium involved by an operation of state preparation of this sort spread out, inside the medium, perturbations arriving from different directions at that or that space-time point. This creates an "interference field" that acts on the dynamical particle state (and possibly also on the intrinsic particle state, namely on its intrinsic mass[15]) according to laws of interactions which probably will have to be assumed to be essentially of the type of those posited by the de Broglie–Bohm model: It is well known that according to the de Broglie–Bohm

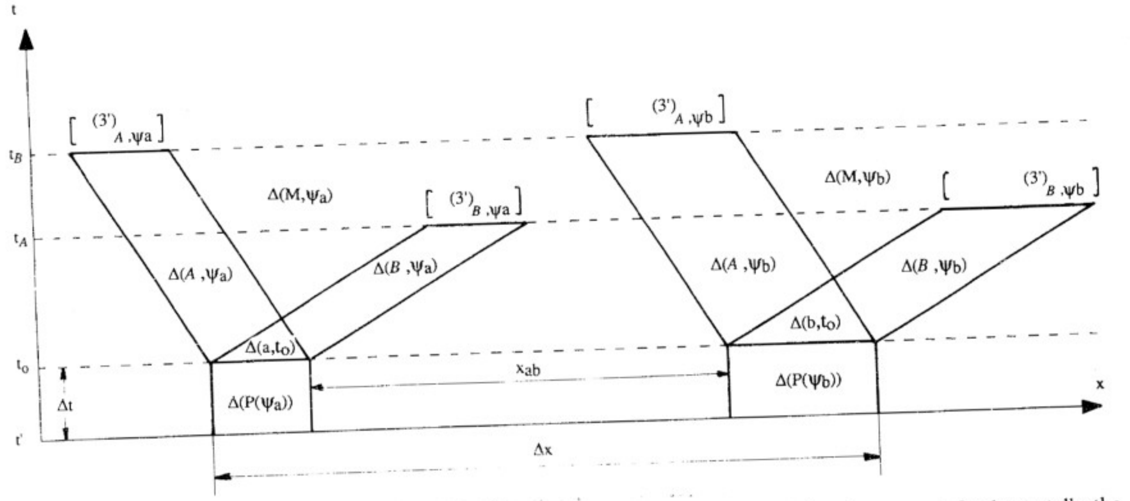

Fig. 3A. The principle of superposition asserted for the state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$ concerns fundamentally the corresponding operations of state preparation $P(\psi_a)$ and $P(\psi_b)$. These involve processes contained in the spacetime domains $\Delta(P(\psi_a))$ and $\Delta(P(\psi_b))$ imbedded in a more extended spacetime domain $\Delta x \cdot \Delta t$ where the temporal extension $\Delta t = t_0 - t'$ is limited by the time t_0 when the studied state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$ emerge. The principle of spectral decomposition concerns measurement operations involving spacetime domains $\Delta(A, \psi_a)$, $\Delta(B, \psi_a)$, etc., imbedded in a more extended spacetime domain $\Delta(M, \psi_a)$, or spacetime domains $\Delta(A, \psi_b)$, $\Delta(B, \psi_b)$, etc., imbedded in a more extended spacetime domain $\Delta(M, \psi_b)$, the temporal extensions of $\Delta(M, \psi_a)$ and $\Delta(M, \psi_b)$ being posterior to the time t_0. The spatial distance x_{ab} that separates the domains $\Delta(a, t_0)$ and $\Delta(b, t_0)$ where the state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$, respectively, assert a nonnegligible presence probability, is arbitrarily big. As in Figs. 1 and 2, the notations $[3'_A, \psi_a]$, $[3'_B, \psi_a]$, $[3'_A, \psi_b]$, $[3'_B, \psi_b]$ symbolize probability spaces brought forth by the considered measurement processes.

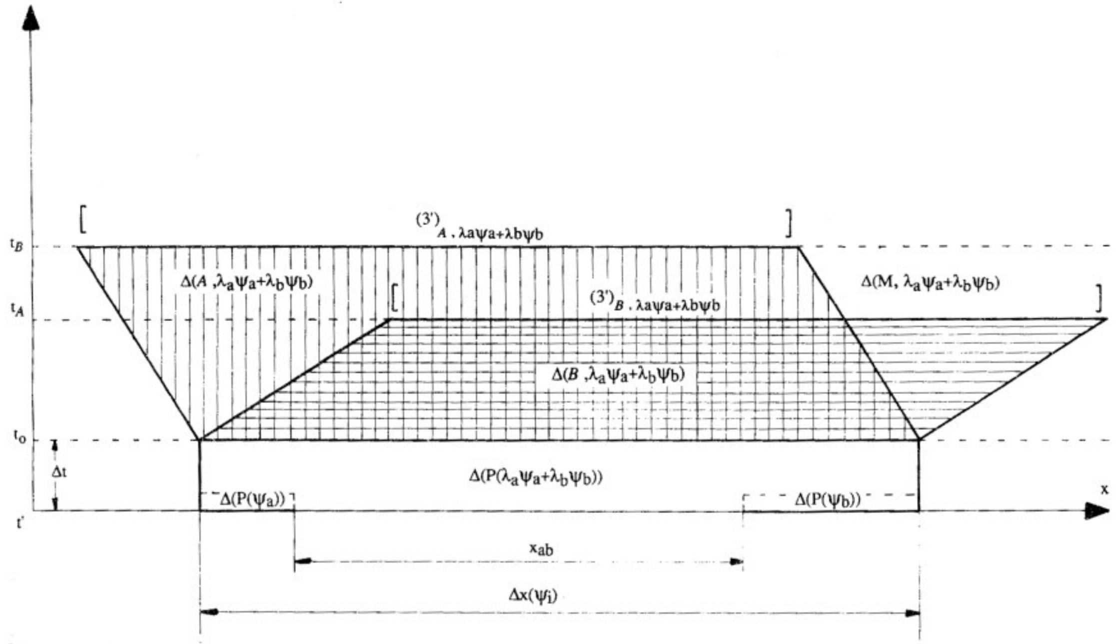

Fig. 3B. According to the principle of superposition asserted concerning the state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$, no matter how big is the distance x_{ab} defined in Fig. 3A, the separately realizable operations of state preparation $P(\psi_a)$ and $P(\psi_b)$ can be combined in a unique operation $P(\psi_{ab})$ acting on an initial state $|\psi_i\rangle$ of the studied system that possesses a corresponding extension $\Delta x(\psi_i)$. The unique operation $P(\psi_{ab})$ which out of $\psi_i\rangle$ brings forth the superposition $|\psi_{ab}, t_0\rangle = \lambda_a |\psi_a, t_0\rangle + \lambda_b |\psi_b, t_0\rangle$ develops inside a spacetime domain $\Delta(P(\lambda_a \psi_a + \lambda_b \psi_b))$ imbedded in the more extended domain $\Delta x(\psi_i) \cdot \Delta t$, where $\Delta t = t_0 - t'$. Measurement operations of two noncommuting observables A and B performed on the superposition state vector $|\psi_{ab}, t_0\rangle = \lambda_a |\psi_a, t_0\rangle + \lambda_b |\psi_b, t_0\rangle$ develop, respectively, inside spacetime domains $\Delta(A, \lambda_a | \psi_a + \lambda_b \psi_b)$ and $\Delta(B, \lambda_a | \psi_a + \lambda_b \psi_b)$ of which the temporal extensions $t_A - t_0$ and $t_B - t_0$ are posterior to the time t_0. These measurement operations bring forth probability spaces $[3'_A, \psi_a + \lambda_b \psi_b]$, $[3'_B, \psi_a + \lambda_b \psi_b]$.

model particles of which the movement is commanded by laws of that type can (in certain geometrical conditions) produce, on the statistical level, observable "interference patterns" of impacts, while such patterns never arise in the absence of a "multiple" character of the operation of state preparation.

This consequence of the [p + m] model associates an interpretation to the calculus with whole probability trees discerned (and completed) inside the orthodox formalism.

5.6. The [Particle + Medium] Model and Superselection Rules

Our definition of operators of state preparation, associated with the interpretation of the principle of superposition offered by the [p + m] individual model, permit us to specify in quite general terms a possible source of superselection rules. Consider two operators of state preparation $(1/\langle\psi_a|\psi_i\rangle\, P_{\psi a}$ and $(1/\langle\psi_b|\psi_i\rangle)\, P_{\psi b})$ corresponding to two state vectors $|\psi_a, t_0\rangle$ and $|\psi_b, t_0\rangle$. The two finite spatial supports $\Delta(a, t_0)$ and $\Delta(b, t_0)$, where at the time t_0 the quantum mechanical presence probabilities are not practically null (according to some definition of practical nullity, any one but fixed), are data that can be caculated inside the orthodox formalism. But the dynamics of progression of the perturbations of the medium part from the studied [p + m] *individual* system, represented by the normed projectors $(1/\langle\psi_a|\psi_i\rangle)\, P_{\psi a}$ and $(1/\langle\psi_b|\psi_i\rangle)$, are *not* described inside orthodox quantum mechanics. Imagine now that these dynamics were such that it is not possible to find two places in the medium starting from which at some initial times t_a and t_b, $t_a < t_0$, $t_b < t_0$, the perturbations reach at t_0, with the required form, the domains $\Delta(a, t_0)$ and $\Delta(b, t_0)$. In this case the superposition state vector $|\psi_{ab}, t_0\rangle = \lambda_a|\psi_a, t_0\rangle + \lambda_b|\psi_b, t_0\rangle$ cannot be prepared. On the other hand, the unlimited character of the medium part—which entails the possibility of arbitrarily big time intervals $t_a - t_0$ and $t_b - t_0$—suggests that any impossibility of the type just specified should be tempered perhaps in terms of high improbability of a natural realization or practical impossibility of an intentional one. This would amount to suppress *in principle* the superselection rules, thus conserving rigorously the structure of vector space for the ensemble of state vectors. This permits one then to associate this ensemble with linear algebras of operators in a way that in principle remains *free* of limitations.

5.7. The [Particle + Medium] Model Versus Projection, Preparation by Measurement, and Reduction

Consider first only *one* measurement evolution M_A concerning a quantum mechanical observable A. This measurement evolution belongs to

some elementary quantum mechanical chain experiment [preparation $P(\psi)$—measurement evolution M_A—registration of a needle position V_j of D_A] (for the sake of simplicity we admit $|\psi_0\rangle \equiv |\psi\rangle$). According to the [p + m] individual model the operation of state preparation $P(\psi)$ realized at the beginning of this elementary quantum mechanical chain experiment *does* introduce a particle, and only *one*, and the eigenvalue registration from this elementary quantum mechanical chain experiment can be produced only by the interaction of that one particle with a convenient macroscopic apparatus. This entails already—exactly as in the case of the de Broglie–Bohm model—that *one* elementary quantum mechanical chain experiment can produce only *one* observable result V_j tied with only *one* eigenvalue $a_j = f_A(V_j)$: *There is no need of a "reduction" in order to explain why each one elementary quantum mechanical chain experiment produces only one observable result* V_j.

What are we now to coherently assume concerning the medium-state during the one measurement evolution M_A involved by one chain experiment? With respect to this question also the [p + m] model works in a way similar to the de Broglie–Bohm model. But it permits one to go further. Let us come back to the condition CH_A and the measurement theorem MT. These complete the orthodox formalism by statistical statements. But the [p + m] model permits one translate them as follows into *individual* terms:

Inasmuch as it is "good" for yielding a registration of an eigenvalue $a_j = f_A(V_j)$ of the observable A, an individual measurement evolution M_A from an elementary quantum mechanical chain experiment [preparation $P(\psi)$—measurement evolution M_A—registration of a needle position V_j of D_A] is such that it transforms the initial individual [p + m] state (unknown and nondescribed inside the quantum theory) produced by the *one* realization of the operation of state preparation $P(\psi)$ from *that* experiment into a new individual [p + m] state where:

— On the **finite** spatial support $\Delta_j(\chi, t')$ where the eigendifferential $|\phi_j(u, t)\rangle$ corresponding to the eigenvector $|u_j\rangle$ asserts a nonzero presence probability (see p. 1441), the type of medium state that *surrounds* the *one* highly localized particle involved is *well* characterized mathematically by the *one*, $|u_j\rangle$, among all the quantum mechanical *eigen*vectors of A (while *nothing* is specified concerning the state of the medium outside $\Delta_j(\chi, t')$).

— By interaction with the medium state from the vicinity $\Delta_j(\chi, t')$ of the one particle involved, well characterized by the functional form of a mathematical descriptor $|u_j\rangle$, the *dynamical* particle state acquires characters that "mirror" that medium state. These—not defined and thus nondescribed inside quantum mechanics—are *such* that by the final interaction of the particle with the apparatus for registration of eigenvalues of A there emerges the observable datum V_j that leads to precisely the particular quantum mechanical eigenvalue $a_j = f_A(V_j)$.

According to this view, *the generally nonnormed eigenvectors* $|u_j\rangle$ *and the corresponding eigenvalues* a_j *from the orthodox quantum mechanical formalism describe conjugated aspects of the [p + m] individual system*:

— The quantum mechanical eigenvectors $|u_j\rangle$ point toward *definite individual* though arbitrarily extended factual designata, namely *types of* **patterns** *of medium state*. Each eigenvector $|u_j\rangle$ can be regarded as an abstract, idealized specification, in mathematical terms, of a particular "value" of another more general qualification (semantic dimension), namely that of "medium state created by a measurement evolution M_A," and which *a priori* admits of an infinite ensemble $\{|u_k\rangle\}$ of distinct "values." An eigenvector $|u_j\rangle$ yields a Platonic specification of the more general qualification of an M_A-generated medium state, a specification comparable, say, with some given nuance of blue that specifies the more general qualification of color in a way *freed* from any physical spacetime confinement: a pure definition of a certain "how."

— The eigenvalue a_j corresponding to the eigenvector $|u_j\rangle$ is a character of the dynamical-particle state that corresponds to the pattern of medium state characterized by $|u_j\rangle$.

(Notice that in orthodox quantum mechanics the qualifications of space and of time—Kant's *a priori* frame-qualifications, the "pure" where's and when's involved by any how but *prior* to it—possess, each one, a particular status of its own. The eigenvectors $|x_j\rangle = \delta(x - x_j)$ (representations of atomic here's) tied with the position operator X (representation of somewhere) are highly singular functions. As to the time qualifications, they are devoid of a corresponding operator. Furthermore the individual and the statistical level of temporal description are currently mixed up with one another or confounded. The time parameter from the argument of a state vector $|\psi(x, t)\rangle$ possesses exclusively a statistical meaning, while inside certain nonformalized definitions of measurement evolutions (consider, for instance, the method of the time-of-flight for momentum measurements) individual times come in, concerning the individual, elementary quantum mechanical chain experiments and their individual outcomes $a_j = f_A(V_j)$: Obviously the quantum mechanical incorporation of the time qualifications is still very primitive. In such conditions, what hope is there to achieve a unification with relativity?).

In short, imagine that by *one* measurement evolution M_A the *one* individual unknown [p + m] state initially produced by the operation $P(\psi)$ from one given elementary quantum mechanical chain experiment,

acquires a new state well characterized—inside an arbitrarily extended vicinity of the one particle involved—by *one* pair of quantum mechanical descriptors $(|u_j\rangle, a_j)$. Then, *iff* the registration of the datum V_j tied with the eigenvalue $a_j = f_A(V_j)$ is *nondestructive* (as in a Stern–Gerlach spin registration), *after* this registration the metaentity [particle + medium] remains indeed in an individual state which, around the "particle," *continues* to be well characterized by the same pair $(|u_j\rangle, a_j)$ of quantum mechanical descriptors of which the relevance has been ensured before the registration by the individual measurement evolution M_A. This pair of descriptors is then indeed *adequate* for the estimation of future probabilities concerning possible *observable* manifestations of *that* individual state of the studied [p + m] individual system. While the characters of the medium state far away from the "particle" do not produce observable effects. (This last assumption might come out to be false if an interference (of the medium-state with itself) were deliberately produced after the eigenvalue registration, as Wigner has suggested.[21] Which would only confirm the [p + m] individual model and would permit one to study in more detail the individual medium state produced by a measurement evolution M_A.) Anyhow:

We can understand now both the orthodox definition of operations of state preparation, as measurement evolutions, and the projection postulate,

without entirely accepting them, of course. The significance of the normed probabilistic quantum mechanical state vector $|\Phi_j\rangle$ that "corresponds" to the eigenvector $|u_j\rangle$ in the statement of the condition CH_A, becomes also clear. According to the [p + m] model the descriptor $|\Phi_j\rangle$ has to be regarded as a statistical metarepresentation of the individual system involved. According to this model the superposition state vector $|\chi(x, t')\rangle = \sum_j c(\psi, a_j)|\, e^{i\alpha(j)}\, |\Phi_j(x, t')\rangle$, $t' > t$, can emerge out of the studied state vector $|\psi(x, t)\rangle$ only by a large ensemble of *reiterations* of the action of the measurement propagator H_A. So $|\Phi_j\rangle$ has to be regarded as *a statistical metarepresentation* of the specification of the more general qualification of an M_A-generated medium state that is individually represented by the pair $(|u_j\rangle, a_j)$. A statistical mathematical representation ("packet") where the statisticity involves adulterating *fluctuations* around the precise individual mathematical qualification given by the eigenfunction $|u_j\rangle$. (However, the "imperfect" descriptor $|\Phi_j\rangle$ alone ensures the possibility—crucial from a pragmatic point of view—of the measurement theorem MT.)

Finally, consider the reduction problem. Since according to the [p + m] model the process of passage from $|\psi(x, t)\rangle$ to $|\chi(x, t')\rangle = \sum_j c(\psi, a_j)|\, e^{i\alpha(j)}\, |\Phi_j(x, t')\rangle$, $t' > t$, concerns a *statistical ensemble* of individual processes, the generation by the measurement propagator H_A as well as the conservation by the unitarity of H_A, of *all* the terms of the superposition state vector $|\chi(x, t')\rangle$, concern exclusively this statistical ensemble. They do *not* concern also the *individual* elementary chain experiments.

The reduction problem simply disappears when the level of individual description is clearly distinguished from the metalevel of statistical description: On the statistical level **there is no suppression of superposition terms** *while on the individual level* **there is no superposition of terms.**

As to the *informational* reduction which appears in the observer's mind when he becomes aware of an individual eigenvalue registration, this is not a specific feature of the quantum theory. It is introduced by any probabilistic representation.

These last interpretations achieve our distinction between superpositions of state vectors and spectral decompositions of a state vector and our connection between these two notions.

6. CONCLUSION OF PART I

We have constructed an integrated view concerning the probabilistic organization of the quantum mechanical formalism. This view brings in four hierarchically connected descriptional levels:

— The elementary quantum mechanical chain experiments (eqmce).

— The basic probability chains (1′), (5) which are metastructures with respect to the elementary quantum mechanical chain experiments.

— The probability trees of a state preparation $\mathscr{T}(\mathrm{P}(\psi_0), |\psi\rangle)$ which are metastructures with respect to the basic probability chains (1′), (5).

— Linear superpositions of probability trees which are metastructures with respect to the probability trees, namely compositions of several entire probability trees entailed by the principle of superposition (we do not mention the quantum mechanical algorithms representing successive measurements which, by use of the projection postulate, identify confusingly the preparable states of a microsystem and the eigenfunctions of an observable).

The integrated view concerning the probabilistic organization of quantum mechanics has acted as an instrument for critical analyses and for constructive developments. It permitted to complete the orthodox theory

by the explicit definition of operators of state preparation and the calculus with these, and by complements to the quantum mechanical theory of measurements. In a second phase, it led outside the orthodox theory, guiding the definition of a model founded on a concept of [particle + medium] *individual* system that is microscopic by certain aspects while by other aspects it is arbitrarily extended, cosmic. We have shown that this model permits to better "understand" the principle of superposition, the emergence of observable interference patterns of impacts, the orthodox definition of the operations of state preparation by measurements and the projection postulate, and the reduction problem.

The quantum mechanical calculi as well as the verbal accompaniments of these convey only very mutilated indications concerning the underlying probabilistic organization of the formalism. Vectors, operators, equations, probability measures, operational definitions of measurements, are manipulated accordingly to algorithms. But the more globalized concepts of an elementary quantum mechanical chain experiment, of a random phenomenon (5), (5′) of a basic probability chain (1′), (1″), of a probability tree $\mathscr{T}(\mathrm{P}(\psi_0), \psi\rangle)$, with their formal features and their specific semantic contents, seem to have remained so far nonperceived. Not even the algorithmic shadow (1) of only an isolated basic probability chain (1″) has been clearly recognized as a probabilistic whole. *A fortiori*, the distinction between formal entities and factual entities remained so dispersed and so vague that the central connecting role of the identities (7) has not been realized fully. This, no doubt, is due to the particular complexity of the random phenomena (5) studied in quantum mechanics and to the unusual potential-actualization nature of the roots [eqmce] of the elementary events V_j produced by these. The conjunction of these two characters acted as a barrier.

We have overcome this barrier by a systematic reference to the basic concepts of the abstract theory of probabilities and by *an explicit specification of the cognitive operations by which the "observer," the "conceptor," produces the entities to be qualified (quantum mechanical states) and the processes of qualification of these (measurement evolutions).* In the second part of this work, in the second issue of this journal dedicated to Sir Karl Popper, we shall generalize this method. Thereby we shall obtain a general representation of the "relativized descriptions" of any kind where remarkable relations with Sir Karl Popper's concept of propensity will appear. But the most efficient feature of the approach practised above is the fact that we have taken into account systematically *the spacetime aspects of all the phenomena involved.* This is what has induced an organization, a unifying form, into the pile of probabilistic algorithms. Imagine a balloon of some sophisticated form. Take away the space from it by letting out the air. What remains? A heap of randomly folded surface. Extend then to a spacetime situation, to a sort of form-dance of the balloon, and the example becomes still more sad.

Der Lattenzaun.

Es war einmal ein Lattenzaun, mit Zwischenraum, hidurchzushaun. Ein Architekt der dieses sah stand eines Abends plötzlich da und nahm den Zwischenraum heraus und baute draus ein grosses Haus. Der Zaun indessen blieb ganz dumm, mit Latten ohne was herum. Ein Anblick hesslich und gemein. Drum zog ihn der Senat auch ein. Der Architekt jedoch entfloh nach Afri-od-Ameriko.

Christian Morgenstern, *Galgenlieder Der Gingganz.*

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