REFLECTION AS AN EXPLANATION OF BELL'S INEQUALITY PARADOX

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This paper develops a quantitative model version of Mugur-Schachter's "reflection" model of Bell's experiment. We show that his nonlocal, but Einstein-separable, model leads to violations of Bell's inequality similar to those predicted by quantum mechanics.

1. Introduction

Experiments with Bell's inequality [1–4] suggest that any hidden variable theory of quantum behavior must be nonlocal. Nonlocalism is usually viewed as synonymous with "action at a distance", i.e. violations of Einstein-separability. But Mugur-Schachter [5,6] presented a nonlocal Einstein-separable model of Bell's experiment in which particles, that do not pass through one detector, may be reflected back toward the other detector. Bordley [7] similarly argued that the *n*-slit interference paradox could be explained by the fact that closing one slit reflects particles – which otherwise would have passed through that slit – toward other slits. (See appendix.) Thus reflection may be an appealing explanation for various quantum mechanical paradoxes.

This paper provides further support for the reflection model of Bell's experiment by showing that a computable variant of the Mugur-Schachter model implies Bell inequality violations similar, though not identical, to those predicted by quantum mechanics.

1.1. The Bell experiment

In the Holt experiment [8], the mercury atom excited by electron bombardment to one of the $9P_1$ states cascades via the $7\,^3S_1$ state to the $6\,^3P_0$ triplet ground state under emission of a first photon of 5677 Å and a second photon of 4048 Å. Two polarization filters with polarization angles a and b with respect to the x direction are placed along the $+z$ and $-z$ direction respectively. Detectors are placed in the $+z$ and $-z$ direction from the emitting atoms so that the detector at $+z$ is sensitive for the first photon only and the detector at $-z$ is sensitive for the second photon only.

Bell's experiment measures the proportion of coincidences, i.e. the proportion of simultaneous observations at both detectors as a function of the polarizer angles a and b. We call two observations simultaneous if, given we observe a particle at one detector, a particle at the second detector is observed within time δt afterwards. All studies of Bell's experiment assume that a coincidence can only occur when a photon of 5677 Å heads toward the 5677 Å detector and passes through the polarizer at angle "a" while a second photon of 4048 Å, generated by the same process as the first, heads toward the other detector and passes through the polarizer at angle "b". We call this an unreflected coincidence.

Let L be the distance from source to detectors. If both photons traveled directly along the z axis, then the 5677 Å photon, assuming it goes through the polarizer, arrives at the detector at some time E_1+L/c, where E_1 is the time the photon is emitted and c is the speed of light. Similarly the 4048 Å photon arrives at the detector at time E_2+L/c. The two photons will be concident if $E_1-E_2<\delta t$.

But there are two other ways in which a coincidence might occur:

(1) A photon of 5677 Å might head toward the 4048 Å detector, fail to pass through the polarizer at angle b, be reflected in the direction of the 5677 Å

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detector and pass through the polarizer at angle a. Simultaneously a photon of 4048 Å might head toward the 5677 Å detector, fail to pass through the polarizer at angle a, be reflected in the direction of the 4048 Å detector and pass through the polarizer at angle b.

More generally, both photons might be reflected N times ($N>0$) before going directly to their detectors. We call this an equally reflected coincidence. In such cases, both photons arrive at the detector at times $E_1+(2N+1)L/c$ and $E_2+(2N+1)L/c$. They will be labelled coincident if $E_1-E_2<\delta t$. There is no way of distinguishing equally reflected coincidences from unreflected coincidences by varying δt and L.

(2) The 5677 Å photon might be reflected M times before going to its detector while the other might be reflected N times ($M\neq N$, $M\geqslant 0$, $N\geqslant 0$). In this case, the photons arrive at the detectors at times $E_1+(2M+1)L/c$ and $E_2+(2N+1)L/c$. They will be coincident if the absolute difference between these two times is less than δt. Hence we need $-\delta t\leqslant E_1-E_2+2(M-N)L/c\leqslant \delta t$. The randomness in emission time and angle of emission makes it possible (through choosing L large and δt small) to reduce, but not eliminate, the number of unequally reflected coincidences [#1].

To relate this model to the Mugur-Schachter model, suppose that all particles travel at the same speed in the z direction. If the two photons created in the nth source emission are called the nth photon pair, then the set of unreflected coincidences is just the set of coincidences from the nth photon pair, for any n, i.e. $\{C(i, i, n, n), n>0\}$ in Mugur-Schachter's notation. The set of equally reflected concidences is just the set of reflected coincidences involving photons from the same photon pair, i.e. $\{C(\rho, \rho, n, n), n>0\}$. Finally the set of unequally reflected coincidences is just the set of reflected coincidences involving photons from different photon pairs, i.e. $\{C(\rho, \rho, k, n), C(\rho, \rho, n, k), C(i, \rho, k, n), k>n, C(\rho, i, n, k), k>n\}$.

We now consider a model involving both equally and unequally reflected photons. This illustrates how a small amount of reflection produces Bell's inequality violations. Since this case can be eliminated in a properly controlled experiment, we then consider a model with only unreflected and equally reflected photons. This model also leads to Bell's inequality violations comparable to those given by quantum mechanics [#2].

2. Equally and unequally reflected photons

Following Belinfante [9], we suppose that a photon of 5677 Å has a "polarization α" such that the probability of the photon passing through a filter polarized at angle a is $\cos^2(a-\alpha)$. If the photon passes through this filter, its polarization angle changes to a; if not, it changes to $a+\frac{1}{2}\pi$. As Belinfante noted, this is consistent with Malus's law and also consistent with a polarizer stopping 50% of a beam of unpolarized light.

Consider a newly generated 5677 Å photon with polarization α. We let Q be the probability that it passes through the filter if it heads in the direction of the 5677 Å detector. Given a photon fails to pass through a detector, let p be the probability it is reflected. Then the photon may reach the 5677 Å detector by any of the following paths:

(1) Go toward the 5677 Å detector and pass through the "a" polarizer. This path has probability $P(5677/\alpha)\cos^2(a-\alpha)=Q\cos^2(a-\alpha)$.

(2) Go toward the 4048 Å detector, fail to pass through the "b" polarizer, be reflected toward the 5677 Å detector and pass through the "a" polarizer. This path has probability $Q\sin^2(b-\alpha)p \times\cos^2[a-(b+\frac{1}{2}\pi)]$.

(3) Go toward the 5677 Å detector, fail to pass through the "a" polarizer, be reflected toward the 4048 Å detector, fail to pass through the "b" polarizer, be reflected toward the 5677 Å detector and pass through the "a" polarizer. This path has probability

$$Q\sin^2(a-\alpha)p\sin^2[b-(a+\tfrac{1}{2}\pi)]p \times\cos^2[a-(b+\tfrac{1}{2}\pi)]\,.$$

Etc.

[#1] If L is 1 m, δt is 5.9 ns and E_2-E_1 is exponential with a mean of 8.3 ns, then almost all unequal reflections with $M-N=+1$ are coincident. But if L equals 10 m, almost no unequal reflections with $M-N=+1$ are coincident.

[#2] Varying the filter polarization rapidly and randomly relative to L/c could prevent any sub-luminal reflected photons from leading to Bell's inequality violations. Current experiments in this direction have not, however, used a fully random variation of filter polarization.

The total probability of the 5677 Å photon reaching the detector is just:

$$Q\cos^2(a-\alpha)+Qp\sin^2(a-b)\{\sin^2(b-\alpha)\times[1+p^2\cos^4(a-b)+p^4\cos^8(a-b)+...]+\sin^2(a-\alpha)[p\cos^2(a-b)+p^3\cos^6(a-b)+p^5\cos^{10}(a-b)+...]\}\,.$$

Note that Q and p both depend on δt. We can rewrite the formula as

$$Q\{\cos^2(a-\alpha)+p\sin^2(a-b)[\sin^2(b-\alpha)+p\cos^2(a-b)\sin^2(a-\alpha)]/[1-p^2\cos^4(a-b)]\,.$$

This gives us PR(5677 detected/α), the probability of detecting a 5677 Å photon given one was emitted with polarization α. Note that it depends upon the polarization at both filters.

Now let $I(\alpha, \beta)$ be the probability that a 5677 Å photon was emitted with polarization α while a 4048 Å photon is emitted with polarization β. Then the probability of simultaneously detecting both photons is

$$\mathrm{PR}(\alpha,\beta)=\int\int \mathrm{PR}(5677\ \mathrm{detected}/\alpha)\times\mathrm{PR}(4048\ \mathrm{detected}/\beta)I(\alpha,\beta)\,. \qquad (1)$$

Following Belinfante, we take $I(\alpha, \beta)=\delta(\alpha-\beta-\frac{1}{2}\pi)/2\pi$. Then (1) becomes

$$Q^2[(1-p^2)^2+(1-p^2)(2+p^2+6p)\sin^2(a-b)+p^2(4+7p)\sin^4(a-b)+p^2(2-p-p^2)\sin^6(a-b)+p^4\sin^8(a-b)]/[8-8p^2\cos^4(a-b)]\,. \qquad (2)$$

2.1. Bell's inequality

Define $\Delta=\mathrm{PR}(a,b)+\mathrm{PR}(a',b)+\mathrm{PR}(a',b')-\mathrm{PR}(a,b')-\mathrm{PR}(a',*)-\mathrm{PR}(*,b)$ where $\mathrm{PR}(a',*)$ is the probability of observing a 5677 Å photon given the second polarizer has been removed and the first polarizer is at angle a' while $\mathrm{PR}(*,b)$ is the corresponding probability with the first polarizer removed and the second polarizer at angle b. Then one version of Bell's inequality specifies that Δ be non-positive.

For $a-b=a'-b=a-b'=0.375\pi$ and $a'-b'=1.125\pi$, quantum mechanics implies $\Delta=+0.21Q^2$, which violates Bell's inequality. At these same angles, formula (2) implies that $\Delta/Q^2>0$ for $p>0.08$ and $\Delta/Q^2=0.21$ for $p=0.18$. Hence our model violates Bell's inequality to the same degree as quantum mechanics. Indeed if $p=1$, we get

$$Q^2[11\sin^2(a-b)+\sin^6(a-b)]/8[2-\sin^2(a-b)],$$

which leads to the same minimum to maximum ratio as given by quantum mechanics. For $a=b$, we get – consistent with quantum mechanics – a zero probability of seeing any coincidences.

This section showed that a fairly small amount of reflection ($p>0.08$) led to violations of Bell's inequality. The next section shows that even if we could eliminate these few unequally reflected photons, a model with only equally reflected photons still gives violations of Bell's inequality comparable to those of quantum mechanics.

3. The model assuming only equal reflection

With no unequally reflected photons, we only get coincidences if:

(1) Both photons go directly to their detectors and pass through their filters with probability $Q^2\cos^2(a-\alpha)\cos^2(b-\beta)$.

(2) Both photons go to each other's detector, are repulsed by the filters, reflect back toward their own detectors and pass through the filters with probability

$$Q^2p^2\sin^2(a-\beta)\sin^2(b-\alpha)\cos^2(b-a-\tfrac{1}{2}\pi)\times\cos^2(a-b-\tfrac{1}{2}\pi)\,.$$

(3) Both photons go to their own detectors, are repulsed by the filters, reflect back toward each other's filters, are repulsed by the filters, reflect back toward their own detectors and pass through the filters with probability

$$Qp^4\sin^2(a-\alpha)\sin^2(b-\alpha)\sin^2(b-a-\tfrac{1}{2}\pi)\times\sin^2(a-b-\tfrac{1}{2}\pi)\cos^2(a-b-\tfrac{1}{2}\pi)\times\cos^2(b-a-\tfrac{1}{2}\pi)\,.$$

Etc.

Thus the total probability of coincidence is

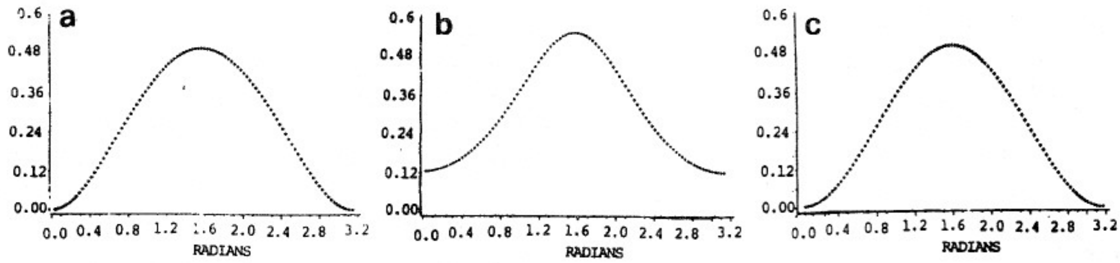

Fig. 1. Probability of simultaneous coincidence. (a) Unequal reflection. (b) Equal reflection. (c) Quantum mechanical.

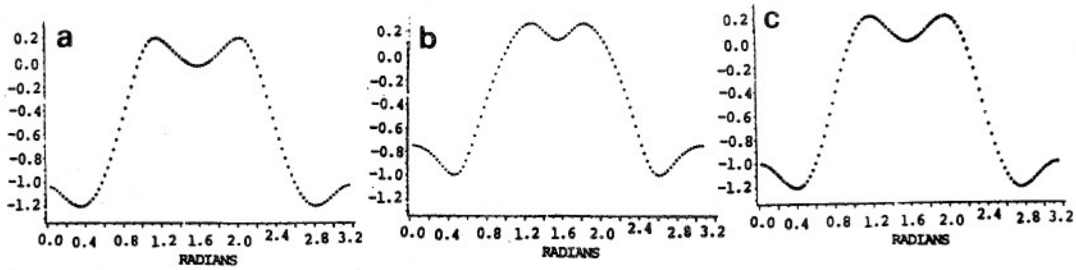

Fig. 2. The Bell inequality factor. (a) Unequal reflection. (b) Equal reflection. (c) Quantum mechanical.

$$Q^2\cos^2(a-\alpha)\cos^2(b-\beta)+Q^2p^2\sin^4(a-b)$$
$$\times\{\sin^2(a-\beta)\sin^2(b-\alpha)[1+p^4\cos^8(a-b)$$
$$+p^8\cos^{16}(a-b)+...]$$
$$+\sin^2(a-\alpha)\sin^2(b-\beta)[p^2\cos^4(a-b)$$
$$+p^6\cos^{12}(a-b)+...]\}\,.$$

Integrating with the Belinfante density for α and β gives

$$Q^2[1+2\sin^2(a-b)]\{1+p^2\sin^4(a-b)$$
$$\times[1+p^2\cos^4(a-b)+p^4\cos^8(a-b)+...)]\}/8$$

or

$$Q^2[1+2\sin^2(a-b)]$$
$$\times\{1+p^2\sin^4(a-b)/8[1-p^2\cos^4(a-b)]\}$$

or

$$Q^2[1-p^2+2\sin^2(a-b)$$
$$+4p^2\sin^4(a-b)]/[8-8p^2\cos^4(a-b)]\,.$$

At $p=1$, we get

$$Q^2[1+2\sin^2(a-b)]/[4+4\cos^2(a-b)]\,.$$

Computing Δ/Q^2 using our formula at the appropriate angles given $\Delta/Q^2>0$ for $p>0.43$ and $\Delta/Q^2=0.21$ for $p=0.70$. Thus we still get violations of Bell's inequality comparable to quantum mechanics.

Fig. 1 graphs the probability of coincidence as a function of $a-b$ for the unequal/equal reflection model with $p=1$, for the equal reflection model with $p=0.7$ and for the quantum mechanical model ($\sin^2(a-b)/2$).

The overall probability of simultaneous concidence for the equal reflection model, $Q^2+Q^2p^2/8$, exceeds that of the quantum mechanical model, $Q^2/4$. Hence polarizers, by encouraging reflection, violate the "polarizers do not enhance detection" assumption of Clauser and Horne [2]. Graphing the Bell factor, Δ/Q^2, for these three models gives fig. 2. The equal reflection model only violates the Bell's inequality lower bounds for p's much higher than 0.43.

3. Conclusions

The original version of Bell's inequality assumes localism, a condition violated by all reflection models. The experimentally testable version [3] further presumes Clauser and Horne's "no enhancement conditions", which is violated by the specific reflection model studied here. This specific reflection model leads to violations of Bell's inequality quite similar to, though not identical with, the violations predicted by quantum mechanics.

Thus Scully's model [10] [#3] and other nonlocal models of quantum behavior (e.g. see refs. [12] and [13]) might be viable.

Appendix. The *n*-slit interference experiment

In the *n*-slit interference experiment, a particle emitted from a source S moves toward a screen with *n* open slits, numbered 1, 2 ..., *n*. If it passes through slit *i*, an event with probability $P_{i/(1...n)}$, it then hits some position E on a second screen with probability $P_{E/i,(1...n)}$. Thus the overall probability of reaching E is

$$P_{E/(1...n)} = \sum_i P_{i/(1...n)} P_{E/i,(1...n)} \, . \tag{A.1}$$

Now suppose that all slits but slit *i* are closed. Let $P_{E/i,(i)}$ be the probability of reaching E given a particle passes through slit *i* and given that only slit *i* is open. The *n*-slit interference paradox is the fact that (A.1) becomes false when we set $P_{E/i,(1...n)}$, in (A.1), equal to $P_{E/i,(i)}$. Hence we can resolve the paradox by showing that $P_{E/i,(1...n)} \neq P_{E/i,(i)}$.

Let p_i be the probability that the first slit an emitted particle reaches is slit *i*. If slit *i* is open, the particle goes to the second screen. If slit *i* is closed, then the probability the next slit it visits is slit *j* is $p_{j/i}$. Let $p_{E/i}$ be the probability that a particle which visits slit *i* first will reach E, given that slit *i* is open. Let $p_{E/-i,j}$ be the probability that a particle which first reaches *i*, fails to go through and then reaches *j*, will reach point E on the screen if slit *j* is open.

Hence $P_{E/i,(i)}$ is given by

$$P_{E/i,(i)} = \frac{p_i p_{E/i} + \sum_j p_j p_{i/j} p_{E/-j,i}}{p_i + \sum_j p_j p_{i/j}} \, . \tag{A.2}$$

When all *n* slits are open, all particles go through the first slit they visit. Thus $P_{E/i,(1...n)} = p_{E/i}$. Then (A.2) implies:

$$P_{E/i,(1...n)} = P_{E/i,(i)} + \sum_{j \neq i} p_{i/j} p_j (p_{E/i,(i)} - P_{E/-j,i}) / p_i \, . \tag{A.3}$$

If $p_{i/j}$ is nonzero, i.e. if a closed slit *j* can reflect a particle toward slit *i*, the $P_{E/i,(1...n)} \neq P_{E/i,(i)}$ which explains the *n*-slit interference paradox.

[#3] Milonni criticized Scully's hidden variables of the third kind theory for implying that some particles have their polarization determined by the polarization of the experimenter's filters [11]. But this implication is always true for our model's reflected particles.

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