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Supplement to a Critique of Piron's System of Questions and Propositions

F. Thieffine,[1] N. Hadjisavvas,[1] and M. Mugur-Schächter[1]

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F. Thieffine,[1] N. Hadjisavvas,[1] and M. Mugur-Schächter[1]

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A previous critique of the relevance of Piron's questions–propositions system as a generator of quantum mechanics by interpretation is reinforced and brought to a close by further investigation.

1. INTRODUCTION

In a recent work[1] we have constructed a critique of a formal system of questions and propositions (qp-s) proposed by C. Piron and claimed to yield by interpretation quantum mechanics, as well as any other known physical theory. We have demonstrated that one of the three axioms upon which this system is built asserts the "existence" of a class of propositions for which neither a syntactic method of construction is explicitely available inside the system nor can a semantic definition be found in consistency with the semantic structure associated with the quantum mechanical formalism inside the quantum theory. We have considered this precise point to be by itself a sufficient reason for rejecting the claim of relevance of the qp-s as a generator of quantum mechanics "by interpretation." Therefore, in a first stage, we have confined our aim to solidly establishing this precise point alone. But now we shall enlarge the view. In the present work we shall close our study of Piron's system by adding briefly new critiques concerning two of the three axioms of the system. These critiques will converge in reinforcing our previously obtained conclusion.

[1] Laboratoire de Mécanique Quantique, Université de Reims, France.

2. SUPPLEMENTARY CRITIQUES

The examined formalism has been systematically reproduced in Ref. 1 (pp. 752–754): this reproduction is treated as an integrating part of the present work also. All the notations are maintained. But, furthermore, we shall make use of Piron's definition of compatibility (and incompatibility) of propositions:

D_{19} (compatible propositions)[2]: Two propositions b and c are said to be compatible (respectively incompatible) if the sublattice generated by $\{b, b', c, c'\}$ is distributive (respectively not distributive). This property will be denoted by $b \leftrightarrow c$ (respectively $b \not\leftrightarrow c$).

2.1. New Bias on the Axiom C

The Axiom C asserts the existence of at least one compatible complement b' for each proposition b. This axiom has been criticized in Ref. 1 via a sequence of three increasingly far-reaching theorems. The second theorem from the sequence (Ref. 1, p. 757, Theorem $\mathscr{T}_2$) states that: For any proposition $a \in \mathscr{L}$, distinct from the trivial proposition, the compatible complement a' is different from the class of the inverses of all the questions of which a is the equivalence class. Graphically, this can be expressed by the diagram

$$\begin{array}{l} \forall a \in \mathscr{L} - \{I\}, \exists \alpha \colon \alpha \;\; \in a \\ \qquad\qquad\qquad\quad\;\; \updownarrow \qquad \updownarrow \\ \qquad\qquad\qquad\quad\;\; \alpha^{\sim} \notin a' \end{array}$$

where $\alpha^{\sim}$ is the inverse of the question α of which the proposition a is the equivalence class. The Theorem $\mathscr{T}_2$ contradicts an assumption implicitely made by several authors, according to which the compatible complement a' of a proposition a from Piron's system ($a \in \mathscr{L}$), simply consists of the class of all the inverses $\{\alpha^{\sim}\}$ of the questions α, $\alpha \in a$. Even though the fact that the mentioned assumption is false is clearly established by the proof of Theorem $\mathscr{T}_2$, the positive content of Theorem $\mathscr{T}_2$ remains obscure. This content is difficult to grasp intuitively in absence of a more analyzed knowledge of the correspondence between Piron's language and the classical predicate calculus. Piron's definitions of a proposition, of a compatible complement for a proposition, of preorder relation, and of truth are, respectively, different from the definitions of proposition, negation, implication, and truth inside the ordinary predicate calculus. In what follows, we sketch out a bridge between Piron's language and the ordinary predicate calculus by help of a set of "connecting definitions" CD. We then offer an insight into the positive content of Theorem $\mathscr{T}_2$ by proving its equivalent—via these particular connecting definitions—in terms of the ordinary predicate calculus.

[2] Cf. C. Piron.[(2)] p. 25.

CD_1: For any question α and any state s in Piron's sense, and for any realization r of α, we denote by $\alpha(s, r)$ the following proposition in the sense of the predicate calculus: the result of the realization r of the yes–no experiment α on the system in the state s is "yes."

Piron's operation of taking the inverse $\alpha^{\sim}$ of a question α is transposed as follows:

CD_2: If in CD_1 the last word "yes" is changed in "no" we denote the obtained proposition from the predicate calculus by $\alpha^{\sim}(s, r)$ and we say that $\alpha^{\sim}(s, r)$ is the negation of $\alpha(s, r)$ in the sense of ordinary logic.

Of course $\alpha(s, r)$ and $\alpha^{\sim}(s, r)$ are not propositions in Piron's sense (Ref. 1, p. 753: D_9), i.e., they are not elements of the "lattice $\mathscr{L}$ of propositions" from Piron's system.

As a function of s and r, $\alpha(s, r)$ is a predicate defined for any α. Now Piron's definition of a certain (or true) question (Ref. 1, p. 752: D_6) will be transposed as follows:

$$CD_3\colon \quad [\alpha \text{ is "certain" in the state } s_0] \underset{\text{def}}{\Longleftrightarrow} [\forall r, \alpha(s_0, r)]^3$$

Furthermore, Piron's definitions of the preorder and of the equivalence relations between questions can be transposed as follows:

$$CD_4\colon \quad [\alpha < \beta] \underset{\text{def}}{\Longleftrightarrow} (\forall s)[(\forall r)\, \alpha(s, r) \subset (\forall r')\, \beta(s, r')]$$

$$CD_5\colon \quad [\alpha \approx \beta] \underset{\text{def}}{\Longleftrightarrow} (\forall s)[(\forall r)\, \alpha(s, r) \equiv (\forall r')\, \beta(s, r')]$$

The structure of connecting definitions $CD_1 \rightarrow CD_5$ builds the research bridge between Piron's language and the ordinary predicate calculus. Other such bridges could probably be imagined. However, the present one enables us already to reach our purpose: state and prove an equivalent $\mathscr{T}'_2$ (relative to the chosen connecting definitions) of Theorem $\mathscr{T}_2$ in terms of ordinary logic.

Theorem $\mathscr{T}'_2$. The transposition CD_2—inside the predicate calculus—of Piron's operation of taking the inverse $\alpha^{\sim}$ of a question α does not conserve the transposition CD_5—inside the predicate calculus—of Piron's equivalence relation between questions.

[3] In such a case, B. O. Hultgren, III and A. Shimony[(3)] write: "Definition (of certainty) is clearly elliptical, and some phrase like 'when the system is in state S' ought to be inserted..." (op. cit. p. 392).

Proof. Immediate, from the rules of predicate calculus and our definitions

$$\alpha^{\sim} \approx \beta^{\sim} \Leftrightarrow (\forall s)[(\forall r)\,\alpha^{\sim}(s,r) \equiv (\forall r')\,\beta^{\sim}(s,r')]$$
$$\Leftrightarrow (\forall s)[(\forall r) \sim \alpha(s,r) \equiv (\forall r') \sim \beta(s,r')]$$
$$\Leftrightarrow (\forall s)[\sim(\exists r)\,\alpha(s,r) \equiv \sim(\exists r')\,\beta(s,r')]$$
$$\Leftrightarrow (\forall s)[(\exists r)\,\alpha(s,r) \equiv (\exists r')\,\beta(s,r')]$$
$$\not\Leftrightarrow \alpha \approx \beta \quad \blacksquare$$

The Theorem $\mathscr{E}'_2$ and its proof yield now an explicit view, in terms of usual logic, on the peculiarity stressed by Theorem $\mathscr{E}_2$: the Axiom C—inside Piron's system—somehow violates the connection between negation and equivalence in the sense of usual logic. We have shown in Ref. 1 that this violation arises when propositions of the form $a \wedge b$ are considered while the trivial case $a \leftrightarrow b$ is excluded.

2.2. Critical Remark on the Axiom A (Covering Law)

The Axiom A asserts that "if p is an atom and if $p \wedge b = 0$, then $p \vee b$ covers b."

To our knowledge, the unique semantic content ever specified for this axiom can be found in a common article by Piron and Jauch.[4] In the particular case in which the proposition b contains a measurement of the first kind (i.e., the answer "yes" for that measurement implies b certain immediately after the measurement) and ideal (i.e., every certain proposition compatible with b is equally certain after that measurement), then the assertion from Axiom A can be obtained deductively for the proposition b.

But consider now the case in which one has in the covering law $b = c \wedge d$. Given a proposition of the form $c \wedge d$, the form of questions belonging to this proposition is either $\gamma \Pi \delta$, $\gamma \in c$, $\delta \in d$, or $\gamma \circ \delta$, where the symbol $\circ$ indicates succession (Ref. 2, p. 72). No example of another form of a question belonging to the proposition $c \wedge d$ has ever been found by some author, as far as we know. But Piron himself has demonstrated the following two theorems:

1. If $\gamma \in c$, $\delta \in d$, and $c \neq d$, then the question $\gamma \Pi \delta$ is never an ideal measurement of the first kind.

2. The question $\gamma \circ \delta$ is an ideal measurement of the first kind if and only if the propositions c and d corresponding to γ and δ are compatible (Ref. 2, p. 72).

Thus, in the case in which in the covering law $b = c \wedge d$, and if the trivial case $c \leftrightarrow d$ is excluded, the Axiom A—according to Piron's own system—remains devoid of any specifiable semantic content.

3. CONCLUSION

Piron's system is built upon three axioms concerning certain defined entities denominated "propositions": Axiom C, Axiom A, and Axiom P. Two of these, namely Axioms C and A, remain devoid of any specifiable semantic content as soon as propositions of the form $a \wedge b$ are considered while the trivial case $a \leftrightarrow b$ is excluded. This conclusion is particularly striking when contrasted against the Moldauer "dialogue" with Piron: "... the program of 'quantum logic' must be conceived thus: construct a non-Boolean lattice of assertions that contains within it all verifiable propositions in such a way that all lattice operations involving propositions can be identified with empirically correct relationships among observables" (P. A. Moldauer,[5] p. 5); "C'est bien le programme que j'ai réalisé dans[2] et qui n'est pas réalisé dans le livre de Jauch" (C. Piron,[6] p. 10).

In fact, the effort of conceptualization in the qp-s has been hypnotically concentrated upon the aim of recovering directly, by the choice of the axioms, the mathematical Hilbert-space structure of which the quantum theory makes technical use for describing microsystems. The semantic organization which characterizes the quantum mechanical description is not reflected by this choice of axioms. This choice leaves implicit the whole semantics of quantum mechanics and it pulverizes it surreptitiously.

An axiomatic formulation of such a type obviously is fundamentally unable to improve the synthetic insight into the physical conception implied by a physical theory. Therefore, it seems useless from the physicist's point of view.

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