QUANTUM MECHANICS, A HALF CENTURY LATER

PAPERS OF A COLLOQUIUM
ON FIFTY YEARS OF QUANTUM MECHANICS,
HELD AT THE UNIVERSITY LOUIS PASTEUR,
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FOREWORD

The articles collected in this volume were written for a Colloquium on Fifty Years of Quantum Mechanics which was held at the University Louis Pasteur of Strasbourg on May 2–4, 1974, in commemoration of the original work by De Broglie in 1924.

It is our hope that this volume will convey to the reader the idea that quantum mechanics, besides being a fundamental tool for scientific workers today, is also a source of a number of questions and thoughts about the interpretation of the foundation of quantum mechanics itself. This gives rise to problems of a philosophical and logical character and has repercussions on other domains such as the theory of gravitation.

Besides the papers presented at the Colloquium, an article has been included by D. Bohm and B. Hiley. This compensates, perhaps, for the article of S. Kochen, whose manuscript unfortunately did not reach us in time for inclusion in this volume. A few months after this Colloquium we learned of the death of Professor Jauch, who had taken a lively and crucial part in its discussions. We have been extremely saddened by the news of his death, and would like to express our long standing indebtedness to him as a physicist.

We are grateful to Professor B. d'Espagnat who kindly helped us in organizing the Colloquium meetings and to Professor G. Ourisson who, as President of the Louis Pasteur University, gave us encouragement and support to our enterprise. We would further like to express our thanks to all those who have contributed to the work involved in the Colloquium and the publication of this book, and especially to Dr J. Simmons who agreed to check the English version of several contributions.

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THE QUANTUM MECHANICAL ONE-SYSTEM FORMALISM, JOINT PROBABILITIES AND LOCALITY

"Il ne faut pas que l'esprit s'arrête avec les yeux, car la vue de l'esprit a bien plus d'étendue que la vue du corps".

Malebranche

R. Magritte

1. INTRODUCTION

Professor Wigner[1] has proved a theorem which is believed to establish the impossibility of associating with any state vector a joint probability of the position and momentum variables. In this work we study this important theorem and we show that in fact it does not rule out the joint probability concept, but that instead it leads to a locality problem inside the one-system formalism of quantum mechanics, similar in certain respects to the problem formulated by Bell[2] inside the two-systems formalism of quantum mechanics.

The analyses which we carry out draw attention to the superposition states with non-connected support, raising doubt concerning the truth of certain quantum mechanical predictions for such states.

J. Leite Lopes and M. Paty (eds.), Quantum Mechanics, a Half Century Later, 107–146. All Rights Reserved

2. STUDY OF WIGNER'S THEOREM ON JOINT PROBABILITIES

2.1. *Wigner's Demonstration*

We start by reproducing Wigner's demonstration. This will be done in detail, in order to facilitate any eventual comparison.

Given a one-system wave function $\psi(q)$ (in one-dimensional notation), Wigner studies a joint function $P(q,p)$ of the positional variable q and the momentum variable p, on which he imposes the following conditions:

(a) that it be a 'hermitian form of $\psi(q)$', i.e.

$$P(q,p) = (\psi, M(q,p)\psi), \tag{1}$$

where M is a self-adjoint operator depending on p and q, and

(b) that $P(q,p)$, if integrated over p, give the proper probabilities for the values of q, as

$$\int P(q,p)\,dp = |\psi(q)|^2, \tag{2a}$$

and, if integrated over q, give the proper probabilities for the momentum, as:

$$\int P(q,p)\,dq = (2\pi\hbar)^{-1}\left|\int \psi(q)\,e^{-ipq/\hbar}\,dq\right|^2. \tag{2b}$$

The condition (b) admits the somewhat milder substitute that $P(q,p)$ should give the proper expectation value for all operators which are sums of a function of p and a function of q, as

$$\iint P(q,p)(f(p)+g(q))\,dq\,dp = \left(\psi, \left(f\left(\frac{\hbar}{i}\frac{\partial}{\partial q}\right)+g(q)\right)\psi\right). \tag{2}$$

A third 'very natural' condition on $P(q,p)$ would be that it is non-negative for all values of q and p:

$$P(q,p) \geqslant 0. \tag{3}$$

But Wigner demonstrates that the conditions (a) and (b) are incompatible with (3). This is realized by showing that the assumption that a $P(q,p)$ satisfying all three conditions (a), (b) and (3) can be defined for *every* ψ, leads to a contradiction.

The contradiction is obtained for wave functions $\psi(q)$ of a par-

ticular form, namely for ψ which are linear combinations $(a\psi_1+b\psi_2)$ of any two fixed functions such that ψ_1 vanishes for all q for which ψ_2 is non-null, and vice versa. Wigner starts with the following lemmas:

LEMMA 1. If $\psi(q)$ vanishes in an interval I, and if $g(q)$ is zero outside this interval and nowhere negative therein, one has for the P corresponding to the $\psi(q)$ above:

$$\int P(q,p)g(q)\,dq = 0, \tag{4}$$

for all p (except for a set of measure zero).

This follows from (2) with $f=0$: the integral of (4) with respect to p vanishes because the right side of (2) vanishes

$$\iint P(q,p)g(q)\,dp\,dq = (\psi, g(q)\psi) = 0. \tag{4a}$$

However, the integrand with respect to p, that is the left side of (4), is non-negative for the g postulated, as long as (3) holds for P. It follows then that the integrand with respect to p must vanish except for a set of p of measure zero, q.e.d.

Furthermore, (4) is valid for every function $g(q)$ which satisfies the conditions of Lemma 1. It can then be concluded in a similar way that:

LEMMA 2. If $\psi(q)$ vanishes in an interval I, the corresponding $P(q,p)$ vanishes for all values of q in that interval (except for a set of measure zero).

Wigner's demonstration then continues as follows:

Let us consider two functions $\psi_1(q)$ and $\psi_2(q)$ which vanish outside of two nonoverlapping intervals I_1 and I_2 respectively. Because of (1), the distribution function $P_{ab}(q,p)$ which corresponds to $\psi = a\psi_1 + b\psi_2$ will have the form:

$$P_{ab}(q,p) = |a|^2P_1 + a^*bP_{12} + ab^*P_{21} + |b|^2P_2. \tag{5}$$

Setting $b=0$, we note that P_1 is the distribution function for ψ_1, and similarly, setting $a=0$, P_2 is the distribution function for ψ_2. Let us consider (5) for the q outside the interval I_1. Since (according to Lemma 2) P_1 vanishes almost everywhere for such q, the distribution function (5) cannot be positive for all a and b unless both P_{12} and P_{21} vanish if q is outside I_1 (except for a set of measure zero in q and p). A similar conclusion can be drawn when q is outside I_2. Hence, we have instead of (5), almost everywhere,

$$(6) \qquad P_{ab}(q,p) = |a|^2 P_1(q,p) + |b|^2 P_2(q,p).$$

This means that the distribution function P_{ab} is almost everywhere independent of the complex phase of a/b. But this is impossible if P_{ab} is to give the proper momentum distribution for $\psi = a\psi_1 + b\psi_2$, i.e. is to satisfy (2b). Indeed, let us denote the Fourier transforms of $\psi_1(q)$ and $\psi_2(q)$ by $\Phi_1(p)$ and $\Phi_2(p)$. Equation (2b) then reads

$$(7) \qquad |a|^2 \int P_1(q,p)\,\mathrm{d}q + |b|^2 \int P_2(q,p)\,\mathrm{d}q = |a|^2|\Phi_1(p)|^2 + |b|^2|\Phi_2(p)|^2 + 2Re\, ab^* \Phi_1(p)\Phi_2^*(p).$$

Since this must be valid for all a and b, it requires identically in p:

$$(7a) \qquad \Phi_1(p) \cdot \Phi_2^*(p) = 0.$$

But this is impossible, since $\Phi_1(p)$ and $\Phi_2(p)$, being Fourier transforms of functions restricted to finite intervals, are analytic functions (in fact, entire functions) of their arguments, and cannot vanish over any finite interval.

Professor Wigner formulates the result of his demonstration in the following terms (p. 28):

"no non-negative distribution function can fulfil both postulates (a) and (b)".

2.2. *Bearing of Wigner's Theorem*

Preliminaries

There seems to be a tendency to interpret Wigner's theorem as the expression of an absolute impossibility of a joint probability of the position and momentum associable to the quantum mechanical state vectors. Such a tendency betrays the real conceptual situation.

Quite generally a *demonstrated* absolute impossibility is impossible: the framework inside which an impossibility is demonstrated ineluctably restricts its bearing. Some of these restrictions cannot be suppressed without disintegrating the studied problem, but some of them might not be essential to the definition of the problem, or even might vitiate it. Obviously only an explicit examination of the logical relativities of a proposition to the framework of its proof can show which restrictions can or must be dropped.

Furthermore the bearing of a theorem is relative also to the inner structure of the proof (via one counter example, or directly for the whole class considered).

We shall now examine the various logical relativities of Wigner's theorem, which define its bearing.

Framework of the proof

The framework consists of the postulates: (a) (hermitian forms defined by (1)), (b) (the two marginal conditions (2) for any ψ), and the non-negativity condition (3). The assumptions of non-negativity and of hermiticity are entailed by the significance of a probability required for the distribution $P(q,p)$, hence they cannot be dropped without disintegrating the very problem chosen for examination, which consists precisely in the possibility of a *probability* distribution $P(q,p)$. Thus eventual unnecessary restrictions can be implied only in Definition (1) and/or in Postulate (b).

DEFINITION (1). Definition (1) is not the most general one conceivable. The distribution operator M is required self-adjoint and dependent exclusively on q and p. The second requirement entails for M independence on ψ, and this entails $P(q,p)$ as a *sesquilinear* form of ψ. Now the functional $P(q,p)$ is researched such as to accept the significance of a probability. Then the concept of a probability requires by its definition the reality of $P(q,p)$ so that $P(q,p)$ must be indeed a hermitian form of ψ: the condition that M be self-adjoint cannot be dropped. But the independence of M on ψ is not imposed via the probabilistic significance desired for $P(q,p)$, so that in the examined context it is an arbitrary a priori restriction. We shall now show that:

PROPOSITION. In absence of the arbitrary restriction to a sesquilinear form for $P(q,p)$, Wigner's demonstration cannot be realized.

Proof. Instead of (1) we start out with the most general definition a priori conceivable for a joint probability distribution of q

and p, namely

$$(1)' \qquad P(q, p) = (\psi, M(q, p, \psi)\psi),$$

where the distribution operator $M(q, p, \psi)$ is self-adjoint and depends on q, p and ψ. All the other assumptions introduced by Wigner are left unchanged. We introduce the notations: ψ_{ab} is a state vector $a\psi_1 + b\psi_2$ where the supports of ψ_1 and ψ_2 are disjoint; P'_{ab}, P'_1, P'_2 are respectively the distributions obtained for ψ_{ab}, ψ_1 and ψ_2 by use of Definition (1)′; P'_{12} and P'_{21} are respectively the analogs of P_{12} and P_{21} from (5) obtained by use of (1)′. With these notations the expression of the joint distribution for ψ_{ab} yielded by Definition (1)′ is

$$(5)' \qquad P'_{ab}(q, p) = |a|^2(\psi_1, M(q, p, \psi_{ab})\psi_1) + a^*bP'_{12} + ab^*P'_{21} + |b|^2(\psi_2, M(q, p, \psi_{ab})\psi_2).$$

In Wigner's expression (5), the factor of $|a|^2$ in the first term and the factor of $|b|^2$ in the last term identify respectively with the distribution P_1 yielded for ψ_1 by Definition (1) and with the distribution P_2 yielded for ψ_2 by Definition (1). The sequel of Wigner's proof is directly founded on this fact and on Lemma 2, as it can be verified by inspection. But this fact is *not* reproduced in Expression (5)′. Now this is so precisely because of the dependence on ψ of the distribution operator M from (1)′, which introduces ψ_{ab} in the argument of M, instead of, respectively, ψ_1 in the factor $|a|^2$ and ψ_2 in the factor of $|b|^2$. For this reason – even though Lemma 2 continues to hold in the assumed context – Wigner's proof can no more be reproduced with the nonsesquilinear definition (1)′, q.e.d.

If not Wigner's proof, then Wigner's conclusion might be generalizable – by some other proof – to any definition of a joint probability subject to both marginal conditions (2). But in fact this cannot be done either, as a well-known example suffices to show: the 'trivial' or 'correlation-free' distribution $|\psi(q)|^2|\Phi(p)|^2$ (where Φ is the Fourier transform of ψ) is a non-negative hermitian and *non*-sesquilinear form of ψ defined for any ψ and which fulfils both marginal conditions (2). Therefore it can be concluded that Wigner's theorem has no bearing on a non-void class of joint probabilities a priori possible. On mathematical grounds (considerations of continuity) it seems probable that this class is not reduced to the trivial distribution alone. It

cannot be decided whether this class contains or not 'interesting' members, as long as the structure of all the conditions to be imposed upon a joint probability (time evolution, mean conditions, correspondence rules between functions and operators, etc. . . .) has not yet been thoroughly defined and studied as an organic whole. The attempts made up to now in this direction are not numerous and – as far as we know – none of them is both complete and guided by an explicit and coherent system of *physical* criteria for the choice of the mathematical conditions.

Postulate (*b*). Let us now examine the two marginal conditions (2). In a first approach we admit the truth of the quantum mechanical predictions expressed by the second members of (2), for any ψ. In a second approach we question this truth for the particular states described by vectors ψ_{ab}.

First stage: The truth of the predictions from the second members of Relation (2) being a priori posed for any ψ, the conditions of consistency with quantum mechanics expressed by use of the first members of (2) are not the most general ones conceivable. They are in fact very restrictive, requiring the *observability* of the integrated distributions $P(q) = \int P(q, p)\,dp$, $P(p) = \int P(q, p)\,dq$ (even though not necessarily of the values q, p also). The joint probabilities $P(q, p)$ subjected to less restrictive conditions of consistency escape Wigner's theorem.

Second stage: An exhaustive examination of the logical relativities of Wigner's theorem obliges us to raise finally also the question of the truth of the second members of both conditions (2) *for the particular state vectors* ψ_{ab} *with non-connected support*. Indeed Wigner's theorem being based on a counter-example proved for the mentioned states, the theorem would remain without foundation if for these particular states the right-hand members from (2) were not both true. This question of truth, even though brought in merely by logical considerations, seems less irrelevant from the physicist's point of view when it is realized that probably the momentum distribution in a state ψ_{ab} with non-connected support has never been measured, so that the 'existence' of an interference term is so far a purely formal fact; not even the assertion of *measurability* of the momentum 'observable' seems to have an obvious operational meaning, neither for such states in particular, nor in general (more detailed remarks can be found on pp. 132, 134, 135).

Inner structure of the proof
Wigner's theorem is demonstrated by producing a counterexample to the initial assumptions, which holds for the state vectors of the particular type $\psi_{ab} = a\psi_1 + b\psi_2$ where the supports of ψ_1 and ψ_2 are disjoint. Even though via this counterexample a *general* impossibility (for any ψ) is established indeed, this impossibility, nevertheless, has no bearing on the sub-class of state vectors of a type different from ψ_{ab}, which contains the major part of the state vectors coming usually into consideration: the theorem leaves open the question whether yes or not for the state vectors $\psi \neq \psi_{ab}$ a non-negative form (1) can fulfil both marginal conditions (2). In certain contexts this question might appear as non-trivial from the physicist's point of view (if, for instance, the quantum mechanical predictions for the momentum in states ψ_{ab} were false).

Conclusion
The preceding analysis shows that Wigner's proof does not exclude the possibility of any non-negative joint distribution function of the position and momentum variables associated with the quantum mechanical state vectors.

Notwithstanding this conclusion we believe that Wigner's proof has an outstanding heuristic interest. Indeed, once an analyzed knowledge has been obtained concerning its structure and its bearing, this proof suggests developments which disclose questions of a fundamental conceptual importance. The remainder of this article is devoted to these developments.

3. SUPERPOSITION STATES WITH NON-CONNECTED SUPPORT AND NON-LOCALITY OF THE ONE-SYSTEM FORMALISM OF QUANTUM MECHANICS

3.1. *The Problem*

The counterexample on which Wigner's theorem is based possesses characteristics which suggest the possibility of a problem of locality implicit in the one-system formalism of quantum mechanics. Indeed, the state vector directly concerned by the proof is a superposition vector $\psi_{ab} = a\psi_1 + b\psi_2$ with non-connected support. The distributions of the position and of the momentum predicted by quantum me-

chanics for such a state are respectively

$$(8) \qquad |\psi_{ab}(q)|^2 = |a|^2|\psi_1(q)|^2 + |b|^2|\psi_2(q)|^2,$$

and

$$(9) \qquad |\Phi_{ab}(p)|^2 = |a|^2|\Phi_1(p)|^2 + |b|^2|\Phi_2(p)|^2 + 2Re\, ab^*\Phi_1(p)\Phi_2^*(p),$$

where Φ_{ab}, Φ_1, Φ_2 are the Fourier transforms of, respectively, ψ_{ab}, ψ_1, ψ_2. Suppose now a joint probability $P(q, p)$ which fulfils the marginal conditions (2) for any ψ, hence in particular also for ψ_{ab} (by the analysis of Wigner's proof we know that such a joint probability, *if* it exists, cannot have a distribution operator independent of ψ). If the factor $P_{p/q}(p, q)$ of conditional probability of p given q is explicitly written, the marginal condition for q applied to ψ_{ab}, ψ_1 and ψ_2 leads (with obvious notations) to

$$(10) \qquad P_{ab}(q, p) = P_{ab}(q)P_{ab,p/q}(q, p) = |a|^2P_1(q)P_{ab,p/q}(q, p) + |b|^2P_2(q)P_{ab,p/q}(q, p).$$

When we now examine (10) we are struck by the following aspect: For each given pair of values q_1, p_k one of the two terms of (10) is null, since either $q_1 \in I_1$ and then $q_1 \neq I_2$, or vice versa. Nevertheless when the conditional factor $P_{ab,p/q}(q, p)$ is tied to a value of the position variable belonging to I_1 we have in general

$$(11) \qquad P_{ab,p/q}(q_1 \in I_1, p_k) \neq P_{1,p/q}(q_1 \in I_1, p_k),$$

and when $P_{ab,p/q}(q, p)$ is tied to a value of q belonging to I_2 we have in general

$$(12) \qquad P_{ab,p/q}(q_1 \in I_2, p_k) \neq P_{2,p/q}(q_1 \in I_2, p_k).$$

This is so because (9) and the marginal condition for p applied to ψ_{ab}, ψ_1 and ψ_2 entail in general for $P_{ab}(p) = \int P_{ab}(q, p)\, dq$ that

$$(13) \qquad P_{ab}(p) \neq P_1(p) + P_2(p).$$

(Wigner's argument: the product $\Phi_1(p)\Phi_2^*(p)$ from (9) is not null identically in p for any ψ_1, ψ_2.) Thus when $P_{ab,p/q}(q, p)$ is tied to I_1 alone, its *value* is not determined only by ψ_1 with support I_1, it depends on the whole superposition $\psi_{ab} = a\psi_1 + b\psi_2$, and this is so notwithstanding the fact that the support I_2 of ψ_2 is separated from I_1 by an *arbitrary distance* (the symmetric proposition holds when

$P_{ab,p/q}(q,p)$ is tied to I_2 alone). This is a *mathematical* non-locality of the functional dependence on ψ_{ab} of the conditional probability $P_{ab,p/q}(q,p)$, emerging in the confrontation between the supposed joint probability $P_{ab}(q,p)$ and the topological characteristics of the support of ψ_{ab}. What Wigner's proof really shows is that *a sesquilinear definition of* $P(q,p)$ *cannot engender this mathematical non-locality, while the marginal conditions* (2) *do demand it for superposition states* ψ_{ab} *with a non-connected support.*

Now the mathematical non-locality specified above expresses exclusively spatial aspects of the confrontation between the concept of a joint probability and the non-connectedness of the support of ψ_{ab}. Therefore – as it stands – it has no established relation with some *physical* problem of 'locality' in the sense of the theory of relativity, where time plays an essential role. Furthermore this mathematical non-locality might vanish like a non-essential aspect when conditions of consistency less restrictive than (2) are required for $P(q,p)$ on the basis of some more analyzed physical criteria of relevance of a joint probability. The aim of this section is to show that in fact the mathematical non-locality perceived in the example from Wigner's proof is an essential aspect of any relevant joint probability $P(q,p)$ (and of any other probability distribution derived from a relevant $P(q,p)$) and that this formal non-locality does entail a problem of physical non-locality inside the one-system formalism of quantum mechanics.

The pursuit of this aim will draw attention on specificities of the superposition states which distinguish these states fundamentally from the mathematical decompositions permitted by the expansion postulate. Along this path we shall be led to the notion that the superposition principle – even though it materializes a mathematical possibility and even though it permitted to describe so accurately the wave-like aspects manifested by certain *position* distributions of microsystems – might nevertheless introduce inadequate predictions, either false or unverifiable, for the dynamical quantities which depend on the momentum and for the spin.

3.2. *Criterion for the Choice of Conditions of Consistency*

Before researching whether the mathematical non-locality discerned in the example from Wigner's proof entails or not a problem of physical non-locality, we shall first specify conditions of consistency with quantum mechanics such as they determine a joint probability concept $P(q,p)$ at the same time *minimally* restricted and 'relevant'. This of course requires criteria of relevance. We believe that the efficient criterion is that of relevance to the 'reduction problem', which is the core of the multiform and now more than fifty years old controversy on the significance of the quantum mechanical formalism. This problem is well-known: the quantum mechanical formalism yields only a statistical prediction concerning the outcome of one individual act of measurement, while this act brings forth a unique well-defined result thereby 'reducing' the predicted spectrum to a certain certitude. The main purpose of those who desire a hidden variables substitute to quantum mechanics is to obtain a 'deterministic' solution for the reduction problem. Such a solution is researched along the following lines. It is postulated that the studied system possesses, independently of observation, certain intrinsic properties statistically describable by a virtual distribution of values of an appropriate group of hidden parameters (hidden to quantum mechanics but not necessarily also to observation). For one given system, at any given time, only one of all the possible groups of values for this group of hidden parameters is conceived to be realized. Each measurable 'quantity w of a system' is conceived as related with a corresponding function h_w of the hidden parameters. An individual act of measurement of w is conceived as a process of interaction between the system and a w-measurement device, which act induces into a deterministic evolution the unique but unknown value $h_{i,w}$ possessed by h_w at the initial moment of this act of measurement. The unique observed value w_j brought forth by the act of measurement can thus be considered to emerge as an observable result of the system-device interaction, deterministically connected with the unique preexisting initial value $h_{i,w}$ via the interaction evolution. It has to be stressed however that the existence of a deterministic connection between each observed w_j with *one* value $h_{i,w}$ does not entail a one-to-one relation between the *values* $h_{i,w}$ and the values w_i; the assumption of such a one-to-one relation is obviously not essential for a deterministic solution of the reduction problem. Therefore it would be unnecessarily restrictive.

Since the main objective of the hidden variables attempts is to develop a deterministic solution to the reduction problem, we shall discard in what follows the conditions of consistency which engender joint probabilities a priori inadequate for the research of a deterministic solution to the reduction problem.

3.3. *Inadequacy of both Marginal Conditions* (2)

The marginal conditions $\int P(q, p)\,\mathrm{d}p = P(q) = |\psi(q)|^2$ and $\int P(q, p)\,\mathrm{d}q = P(p) = (2\pi\hbar)^{-1}|\int \psi(q)\, e^{-ipq/\hbar}\,\mathrm{d}q|^2$ require the observability of both statistical distributions $P(q)$ and $P(p)$. This does not entail that the individual values of the variables q and p have to be also observable, nor does it fix the physical significance to be assigned to the symbols q, p.

If the possible significances of q, p are considered, it is immediately obvious that the significance of 'pure observables' (i.e. values of some observable entities for which the denominations of 'position' and 'momentum' are decreed, but which are defined *exclusively* by the specification of some experimental circumstances involving the system, and where these entities emerge) cannot be relevant to the reduction problem: the criterion of relevance to this problem requires a definition of q, p independent of observation. Discarding then the pure-observable significance and postulating for q, p a significance independent of observation, we shall now show that, whatever hypotheses are chosen concerning the observability of the individual values q, p, the marginal conditions (2) engender a joint probability $P(q, p)$ which is either unnecessarily restricted or self-contradictory.

The beable significance for q, p. Any property possessed by a system independently of observation has been called by Bell a beable property. We like this denomination and we adopt it. We shall now specify in detail the two important particular concepts, of a beable position and of a beable momentum.

Beable position. By definition this concept consists of the assumption of beable properties of the system which possess characteristics describable with the aid of the classical quantity position, i.e. which in any referential are, at any given time, non-negligible only inside a finite and relatively small spatial domain. Such an assumption is equivalent to a minimal *model* of the object named 'system'. However – by its minimality – this model does by no means entail the naïve atomistic, multitudinist hypothesis concerning the structure of the microreality; the finiteness and the smallness of the domain inside which the conceived beable position properties are 'confined', are only *relative* to some specified (and modifiable) degree of approximation chosen for the description of these properties, while their 'existence' is defined only with respect to some specified but arbitrary range of spatial dimensions characterizing the chosen scale of (imagined) observation. The concept of the object called system itself, to which a beable position is assigned, emerges only relatively to some choices of such approximations and of such a scale. Thus the notion that a beable position is possessed by what is named system has nothing absolute in it. In particular it leaves open the problems of separability of the systems and of locality of the phenomena in which they are involved.

Beable momentum. It is not impossible to conceive a beable position which does *not* perform a continuous dynamics, but which merely consists of a discontinuous juxtaposition of an uninterrupted succession of locations possessed by some properties of the system, in the sense specified above. But this sort of a beable position would reproduce the 'essentially probabilistic' features which a deterministic solution for the reduction problem attempts to remove. Such a beable significance for q in the argument of a joint probability $P(q, p)$ would therefore yield a concept irrelevant to the reduction problem, so that we discard it. If then a beable position which does perform a continuous dynamics is assumed, ipso facto some definite continuous time variation of this beable position is assumed. This – by definition – is what we call a beable momentum.

The beable individual kinematic relation: Thus the assumption of a continuously moving beable position of a system is interdependent with the assumption of a beable momentum of this system. These two united assumptions are *equivalent* to the assumption of the descriptive relevance of a position variable q and a corresponding momentum variable p, *tied* to one another by the individual kinematic relation (in one dimensional writing)

$$p = K\frac{\mathrm{d}q}{\mathrm{d}t}, \qquad (14)$$

where K is a factor of proportionality playing the role of an inertial mass. This individual relation is a non-trivial and important implication of the concept of a continuously moving beable position, because it entails statistical correlations and these can be found to be either compatible or incompatible with a given condition of consistency with quantum mechanics envisaged for a joint probability distribution of beable q, p.

Rejection of the requirement of both marginal conditions (2) *for beable* q, p: We consider two complementary hypotheses concerning the observability of beable values q, p, assumed to exist for a system: either not both these values are observable, or they are both observable. Either of these hypotheses leads to the rejection of the requirement of both marginal conditions (2). Indeed, we consider first an inobservable beable q or p. Then it can be rather trivially pointed out that:

PROPOSITION 1. The marginal conditions (2) entail an unnecessarily restricted statistical distribution of the values of an inobservable beable q or p.

Proof. Suppose that the value of the momentum beable is not observable for some given state of the studied system S. Let us then redenote this value p' in order to distinguish it from the observed value produced by an act of momentum measurement performed on S. Even though the individual values p' are not observable, the marginal condition (2b) requires that the statistical distribution $P(p')$ shall coincide with the observable quantum mechanical distribution $(2\pi\hbar)^{-1}|\int \psi(q)\, e^{-2\pi i p q/\hbar}\, dq|^2$ of the values p (i.e. to each unknown value p' corresponds one observed value p which arises statistically the same number of times). This, however, is an unnecessary restriction on the relation permitted between values p' and values p: For ensuring at the same time consistency with quantum mechanics and relevance to the reduction problem it suffices to require that the observed values p alone have the quantum mechanical distribution and that, furthermore, each one observed value p be connected by the measurement interaction evolution, with *one* preexisting value p' (included in a hidden distribution $P(p')$ in general different from the observed one).

An analogous argument holds for q.

We consider now observable beables q, p. We shall show that

THEOREM 1. A joint probability distribution $P(q, p)$ of observable beables q, p, cannot fulfil both marginal conditions (2) for any state vector.

Proof. We produce an example: Consider the state vector

$$\psi(q) = \frac{1}{\sqrt{2}}\,\phi_{\mathbf{p}_1}(q) + \frac{1}{\sqrt{2}}\,\phi_{\mathbf{p}_2}(q),$$

where $\phi_{\mathbf{p}_1}(q)$ and $\phi_{\mathbf{p}_2}(q)$ are eigendifferentials of the quantum mechanical observable momentum (vector), corresponding respectively to the eigenvalues $\mathbf{p}_1$ and $\mathbf{p}_2$ the directions of which make an angle $\alpha \neq 0$, the norms being equal and non-null $(|\mathbf{p}_1| = |\mathbf{p}_2|) \neq 0$. Since this state requires a two-dimensional description we refer it to two orthogonal axes ox, oz, the axis ox being chosen parallel to the bisectrix of α. The quantum mechanical position distribution $|\psi(x, z)|^2 = |\psi(\mathbf{q})|^2$ is then uniform along ox and periodic along oz; furthermore, this quantum mechanical distribution is *stationary*. We consider now a joint probability distribution $P(\mathbf{q}, \mathbf{p})$ associated with the chosen ψ and fulfilling both marginal conditions (2); $\mathbf{q}$ and $\mathbf{p}$ in the argument of $P(\mathbf{q}, \mathbf{p})$ are assumed to be observable beables. Then the beable character of $\mathbf{q}, \mathbf{p}$ entails that at each given time each instantaneous individual value of the momentum variable possesses a kinematic Definition (14) $\mathbf{p} = K(\mathrm{d}\mathbf{q}/\mathrm{d}t)$ according to which it is generated by the time variation of a corresponding joint $\mathbf{q}$. Via this kinematic definition and the hypothesis of observability of the individual $\mathbf{p}$ the marginal condition (2b) for the *momentum* entails consequences for the time variations of the individual values of the *position* variable, and these in their turn entail consequences for the statistical position distribution $P(\mathbf{q}) = \int P(\mathbf{q}, \mathbf{p})\, \mathrm{d}\mathbf{p}$. Now for the chosen state vector the consequences on $P(\mathbf{q})$ of (14) and (2b) are not compatible with the stationarity of $P(\mathbf{q})$ required by the hypothesis of observability of $\mathbf{q}$ and by the marginal condition (2a) for the position. Indeed (14) and (2b) entail non-null z-components for the time variations of the (observable) $\mathbf{q}$

$$\text{(15)} \qquad \frac{\mathrm{d}q_z}{\mathrm{d}t} = \frac{p_z}{K} = \pm|p_z| \neq 0.$$

This entails that, if at some initial time t_0, (1.2a) is realized, throughout the future $t > t_0$ of this time the location with respect to oz of the maxima and minima of $P(\mathbf{q})$ keep reversing by a continuous process, with a time-periodicity

$$\text{(16)} \qquad \mathrm{d}t = \frac{K\,\mathrm{d}q_z}{|p_z|} = \frac{Ki}{2|p_z|}.$$

where i is the distance at t_0 between two successive maxima of $P(\mathbf{q})$.

This example suffices for establishing Theorem 1. It shows that a joint probability $P(q, p)$ of observable beable values q, p which fulfils both marginal conditions (2) for any ψ, is a self-contradictory concept.

Since a joint probability $P(q, p)$ of beable q, p which fulfils both marginal conditions (2) is either unnecessarily restricted or self-contradictory and since, for a priori relevance to the reduction problem, the beable hypothesis for q, p has to be conserved, we conclude that *at least one of the two marginal conditions* (2) *has to be dropped.*

3.5. *Minimally Restricted Relevant Conditions of Consistency*

We admit by hypothesis that the object denominated one microsystem (S) does possess a continuously moving beable position and the corresponding beable momentum. Statistically this leads to the assumption, for any state vector ψ, of a corresponding joint probability of beable position and momentum variables. We shall now characterize this distribution so as to keep constantly faithful to the *minimality* of the model of a microsystem introduced by the mere assumption of a continuously moving beable position, while ensuring nevertheless a priori relevance to the reduction problem. Then, for the sake of minimality, we start out with a joint probability $P_\psi(q', p')$ where neither the position beable q' nor the momentum beable p' is asserted to be observable.

Condition for the momentum

We examine first the momentum distribution $\int P_\psi(q', p')\,\mathrm{d}q' = P_\psi(p')$ because it seems less queer to admit that it is not observable, i.e. that in general it is different from the quantum mechanical momentum distribution: $(\int \mathcal{P}_\psi(q, p')\,\mathrm{d}q = \mathcal{P}_\psi(p')) \neq |\Phi(p)|^2$ (Φ is the Fourier transform of ψ). For relevance to the reduction problem we have to admit that an individual act of momentum measurement relates the one preexisting beable momentum p' of the respective system, to the observed value p. This leaves (in general) an active role to the momentum measurement device $D(p)$, in agreement with Bohr's ideas: if λ is a parameter characterizing the state of $D(p)$, the observed value p is a function $P(p', \psi, \lambda)$ of p', ψ and λ, the form of this function (unknown) being fixed once ψ and a device $D(p)$ are given. Now, for any *physically realizable* ψ *and inasmuch as the quantum mechanical prediction* $|\Phi(p)|^2$ *is true for* ψ, we have to assume that statistically $p(p', \psi, \lambda)$ is obtained the number of times (normalized) $|\Phi(p)|^2$. This number can also be written $P_\psi(p')R_{D(p)}(\lambda)$ where $R_{D(p)}(\lambda)$ designates the statistical distribution of λ over the ensemble of the states of $D(p)$ realized for the individual acts of measurement which yield $|\Phi(p)|^2$, and where p', λ are taken the same as in the argument of $p(p', \psi, \lambda)$; indeed $P_\psi(p')$ and $R_{D(p)}(\lambda)$ are *independent* densities, since in every individual act of measurement interaction p' *preexists* to the interaction, by hypothesis. The necessity to label somehow the products $P_\psi(p')R_{D(p)}(\lambda)$ in relation with the observed values p, leads then to the mean condition

$$(17)_1 \quad \iiint p(p', \psi, \lambda)P_\psi(q', p', t_0)R_{D(p)}(\lambda)\,\mathrm{d}q'\,\mathrm{d}p'\,\mathrm{d}\lambda$$
$$= \iint p(p', \psi, \lambda)P_\psi(p', t_0)R_{D(p)}(\lambda)\,\mathrm{d}p'\,\mathrm{d}\lambda$$
$$= \int p|\Phi(p, t_0)|^2\,\mathrm{d}p = \left\langle \psi(q, t_0) \middle| \frac{\hbar}{i}\frac{\partial}{\partial q}\psi(q, t_0) \right\rangle.$$

We have written explicitly the *constant* time t_0 elapsed since the state ψ has been prepared for each individual S, when the corresponding individual act of measurement interaction between $D(p)$ and S *begins*: thereby we emphasize that the numerical equality $(17)_1$ does not depend on the time evolution of the measurement interactions, neither on their functional form nor on their duration; it depends exclusively on the connection between their *result* (second member) and circumstances which *precede* them (first member, t_0).

But, beyond the numerical aspects, it is important to understand clearly the conceptual content of the integrand from the first member of $(17)_1$: while the values of the functional $p(p', \psi, \lambda)$ are the observed values p from $\int p|\phi(p, t_0)|^2\,\mathrm{d}p$, the functional form of $p(p', \psi, \lambda)$ represents the hypothetical – individual and deterministic – process which leads from one beable value p' possessed by the supposed momentum beable of the system, at the time t_0 when the measurement interaction began, to the observed value p, defined at another time, by a coordinate attached to a macroscopic part or aspect ('pointer') of $D(p)$. The presence of the parameter λ in the argument of $p(p', \psi, \lambda)$ stresses the assumption that this individual process depends – besides p' *and*

ψ – also on the state of $D(p)$, throughout the time interval taken by the measurement interaction. The definition of this state of $D(p)$ introduces a macroscopic potential (constant or null, for p) which is different in general from the macroscopic potentials having commanded the Schrödinger evolution of ψ from the moment from which ψ has been prepared until t_0 when the act of measurement began. Thus the exact meaning hypothetically assigned to $p(p', \psi, \lambda)$ is this: it represents one *individual* member of a virtual statistical ensemble of p-measurement evolutions, *globally* corresponding to the Schrödinger evolution of the state vector of the 'system + $D(p)$', during the p-measurement interaction. We finally note that, for the sake of maximal generality, we conceive that the functional form of $p(p', \psi, \lambda)$ might depend upon the particular $D(p)$ device utilized. Two different devices $D_1(p)$ and $D_2(p)$ can be conceived to introduce in general two different functional forms $p^{(1)}(p', \psi, \lambda)$ and $p^{(2)}(p', \psi, \lambda)$ and two different distributions $R_{D_1(p)}(\lambda_1)$ and $R_{D_2(p)}(\lambda_2)$. But then a certain correspondence has to be also assumed between $p^{(1)}$, $R_{D_1(p)}$ and between $p^{(2)}$, $R_{D_2(p)}$, such that statistically, in a given $\psi(q, t_0)$ prepared for each microsystem S, both $D_1(p)$ and $D_2(p)$ shall create any given observed value p, with the same relative frequency $|\Phi(p, t_0)|^2$.

Condition for the position

We require for the position the same type of consistency condition as for the momentum, in order to conserve the minimality of the demanded restrictions:

$$(17)_2 \quad \iiint q(q', \psi, \lambda) P_\psi(q', p', t_0) R_{D(q)}(\lambda)\, dq'\, dp'\, d\lambda$$
$$= \iint q(q', \psi, \lambda) P_\psi(q', t_0) R_{D(q)}(\lambda)\, dp'\, d\lambda$$
$$= \langle \psi(q, t_0) | q \psi(q, t_0) \rangle = \int |\psi(q, t_0)|^2 q\, dq$$

(obvious notations). All the comments concerning $(17)_1$ are transposable for $(17)_2$. We make now an important remark concerning $(17)_2$:

In the first place, this *mean* condition for the position, in contradistinction to the marginal condition (2a), leaves open the possibility that the beable position q' of one microsystem $S^{(\psi)}$, lies outside the support of ψ. However shocking it might seem, this possibility cannot be excluded since the purely predictional formalism of quantum mechanics introduces no assertion whatever concerning the way in which the only observationally described object $S^{(\psi)}$ 'exists' independently of observation. However (2a) subsists inside $(17)_2$ as a particular possibility. In consequence of Theorem 1, one at least of the two marginal conditions (2a) and (2b) has to be dropped, but not necessarily both. Since we have dropped the marginal condition for the momentum, we remain free for the moment to assume that the marginal condition for the position is always true. But it will appear that this apparently so natural assumption has a heavy price, if all the quantum mechanical predictions are true.

The other mean conditions (macroscopic dynamical quantities, quantum mechanical dynamical operators, beable dynamical quantities)

For dynamical quantities more complex than q and p, most of the mean conditions posed so far in connection with joint probability attempts – and then criticized – have a structure which does not resist a closer analysis. Given a *macroscopic* classical dynamical quantity $f_m(q, p)$, the corresponding *beable* dynamical quantity of a microsystem $S^{(\psi)}$ is usually conceived in a way which violates the *minimality* of the model of a microsystem introduced by the mere hypothesis of a continuously moving beable position: the beable which corresponds to $f_m(q, p)$ is brutally identified with $f_m(q, p)$ and thereby the naïve atomistic model, made obsolete by de Broglie more than fifty years ago, is implicitly reintroduced. Moreover, the fact that a measurement interaction in general *modifies* the beable characteristics of a microsystem, there*from* yielding an observed value, is not taken into account. Such unanalyzed steps lead to mean condition of the type

$$\iint f_m(q, p) P_\psi(q, p)\, dq\, dp = \left\langle \psi \middle| f_{m,QM}\left(q, \frac{\hbar}{i}\frac{\partial}{\partial q}\right) \psi \right\rangle,$$

($f_{m,QM}$ is the quantum mechanical operator for f_m), and then these are found unsatisfactory, which indeed they are. Before going over to locality analyses we shall express these criticisms more detailedly. This will enable us to specify what mean conditions, for any quantity, can be imposed upon a joint probability both minimally restricted and relevant.

We begin by recalling a well-known fact concerning the time evolution conceivable for a joint probability of beable q', p'. Since

$(17)_1$ and $(17)_2$ are required for $P_\psi(q', p')$ at any time, P_ψ has to perform a time evolution compatible with the Schrödinger evolution of the corresponding ψ. This evolution admits a newtonian representation in consequence of the kinematic definition $p' = K(\mathrm{d}q'/\mathrm{d}t)$ assumed for each p'. Indeed – by definition – the time evolution of P_ψ is newtonian if it is describable by an equation of the form

$$(18) \qquad \frac{\partial P_\psi(q', p')}{\partial t} = \frac{p'}{K}\frac{\partial P_\psi(q', p')}{\partial q'} + F\frac{\partial P_\psi(q', p')}{\partial p'},$$

where the symbols q' and p' are pairwise connected precisely by the kinematic relation $p' = K(\mathrm{d}q'/\mathrm{d}t)$, while the time variation of p' is equated, by application of the fundamental newtonian postulate, to a convenient 'total force' F, classical or *not*,

$$(19) \qquad \frac{\mathrm{d}p'}{\mathrm{d}t} = F,$$

(this force can be conservative, or dissipative, or a sum of a conservative term and of a dissipative term; only in the first case it is derivable from a potential function, and then (18) acquires a hamiltonian form). Now, it is well established that, given the Schrödinger evolution of ψ determined by some macroscopic potential $V_m(q)$, it is in general *not* possible to find a newtonian evolution (18) for an attempted joint probability P_ψ, if F in (19) is required a priori identical with the macroscopic force $F_m = -\text{grad } V_m(q)$: proofs of this impossibility are contained implicitly, but rather obviously, in the text-book studies of the WKB approximation as well as in Feynman's path integral approach[3] or in de Broglie's and Bohm's hidden variable attempts. Thus F in (18) has to be conceived as an unknown non-macroscopic force which cannot be posed, but which has to be determined consistently with the Schrödinger evolution of ψ, as a functional of $V_m(q)$ *via* $\psi(V_m(q))$. This functional would probably yield the most specific descriptive element of a non-naïve model of a microsystem.[1] If, on the contrary, F in (18) is decreed to be identical to $F_m = -\text{grad } V_m(q)$, any hope for a joint probability $P_\psi(q', p')$ performing a time evolution consistent with ψ – for any ψ – is thereby banished.

On the basis of this remark it will now be easy to understand that

PROPOSITION 2. Given a *macroscopic* classical dynamical quantity $f_m(q, p)$, a corresponding *beable* classical dynamical quantity does not necessarily exist; if it does exist, then it is in general different from the corresponding $f_m(q, p)$, so that it cannot be found by reversing the correspondence rule which led from $f_m(q, p)$ to the respective quantum mechanical operator $f_{QM}(q, (\hbar/i)(\partial/\partial q))$.

Proof. Again we produce an example. Consider the macroscopic dynamical quantity total energy $f_m(q, p) = H_m(q, p) = p^2/2m + V_m(q)$. Consider also *one* individual microsystem $S^{(\psi)}$. What can be said concerning a beable total energy of $S^{(\psi)}$? With our previous assumptions $S^{(\psi)}$ possesses a beable position and a corresponding beable momentum $p' = K(\mathrm{d}q'/\mathrm{d}t)$. One can then form for $S^{(\psi)}$ a kinetic energy p'^2/K (where K is not identical to the mass m of $S^{(\psi)}$, a priori). But in order to preserve for a joint probability $P_\psi(q', p')$ attempted for $S^{(\psi)}$, the possibility of a time evolution compatible with that of ψ, the force $F = \mathrm{d}p'/\mathrm{d}t$ which – by newtonian postulate – is equated to $\mathrm{d}p'/\mathrm{d}t$, has to be in general different from the macroscopic force $F_m(q) = -\text{grad } V_m(q)$, $F'(q') \neq F(q)$. If moreover $F'(q')$ is not conservative, then $S^{(\psi)}$ simply does not possess a beable hamiltonian, notwithstanding the fact that the time evolution of ψ is expressed by a hamiltonian (operational) formalism[4]. If on the contrary $F'(q')$ also does derive from a potential, this potential $V'(q') \neq V_m(q)$ is in general different from $V_m(q)$; then $S^{(\psi)}$ does possess a beable hamiltonian $H_b = p'^2/2K + V'(q')$ but this is different from the macroscopic hamiltonian $H_m = p^2/2m + V(q)$ to which corresponds the hamiltonian evolution operator for ψ: $H_{QM} = -(\hbar/2m)(\partial^2/\partial q^2) + V(q)$. Replacement in H_{QM} of $(\hbar/i)(\partial/\partial q)$ by p, and of the multiplicative operator $V(q)$ by the function $V(q)$, yields back H_m but not $H_b(\neq H_m)$.

This example suffices for showing that mean conditions of the form

$$\iint f_m(q, p)P_\psi(q, p)\,\mathrm{d}q\,\mathrm{d}p = \langle\psi|f_{m,QM}(q, (\hbar/i)(\partial/\partial q))\psi\rangle.$$

are not significant. (In particular such a mean condition for the potential energy itself

$$\iint V_m(q)P_\psi(q, p)\,\mathrm{d}q\,\mathrm{d}p = \langle\psi|V_m(q)\psi\rangle,$$

is the very *definition* of a naïve, atomistic postulate on the structure of the microreality). P_ψ and ψ cannot be purely algorismically treated as if they were both fit for relevantly calculating means of *any* and *same* functions. P_ψ can yield relevant means for beable values only while ψ is relevant for calculating means of observed values only. Park and Margenau have explicitly contested – on logical grounds – the relevance of mean conditions written with the macroscopic functions $f_m(q, p)$[5]; Proposition 2 gives a more physical reason of this irrelevance. But obviously there exists a much more radical objection: given a quantum mechanical operator $f_{m.QM}$ corresponding to the macroscopic dynamical quantity f_m, even if the respective beable quantity both does exist and is distinguished from f_m, not *its* mean value is relevant to the reduction problem, but the mean engendered by it via the measurement interactions, which depend also on the measurement device. Bohr's views on measurement were very profound, each act of measurement modifies preexisting characteristics of the system, bringing out from it observed values of *other*, only operationally defined 'quantities of the system'.

Then all that *can* be required of a joint probability $P_\psi(q', p')$ of beable q', p' is to have an analytic expression such as to be *compatible* with mean conditions of the type $(17)_1$ and $(17)_2$, for *any* quantum mechanical dynamical observable w, at any time, i.e.

$$(17) \qquad \iiint w(q, p, \psi, \lambda) P_\psi(q', p', t_0) R_{D(w)}(\lambda)\, dq'\, dp'\, d\lambda$$

$$= \left\langle \psi(q, t_0) | f_{QM.w}\left(q, \frac{\hbar}{i}\frac{\partial}{\partial q}\right) \psi(q, t_0) \right\rangle = \int w |C^\psi(w, t_0)|^2\, dw,$$

where all the notations have obvious meanings by analogy with $(17)_1$. All the comments concerning $(17)_1$ can be transposed to (17), which includes now $(17)_1$ and $(17)_2$. We can rewrite (17) in a form more specifically connected with the dynamical observable w: Given one $S^{(\psi)}$ we denote globally by a *unique* parameter w' *all* the beable characteristics of $S^{(\psi)}$ which contribute, with ψ and λ, to the creation of the observed value w when one act of w-measurement is performed on $S^{(\psi)}$. These characteristics can be conceived as defined *at* q' since q' designates the beable element of $S^{(\psi)}$ to which a beable dynamics is assigned. Then statistically the joint distribution $P_\psi(q', p', t_0)$ defines a corresponding joint distribution $\Pi_\psi(w', q', t_0)$. Rewriting of $w(q', p', \psi, \lambda)$ in function of w'

yields a function of a new functional form $w(w', \psi, \lambda)$ but the values of which continue to be the observed values w, and for which all the considerations made for the particular case of $p(p', \psi, \lambda)$ from $(17)_1$ are valid. So (17) becomes

$$(17)' \qquad \iiint w(w', \psi, \lambda) \Pi_\psi(q', w', t_0) R_{D(w)}(\lambda)\, dq'\, dw'\, d\lambda$$

$$= \langle \psi(q, t_0) | f_{QM.w} \psi(q, t_0) \rangle = \int w |C^\psi(w, t_0)|^2\, dw.$$

The critical remarks which led to condition (17)′ show that all the theorems of impossibility (like that of von Neumann concerning simultaneous measurements of quantities with non-commuting quantum mechanical operators ([16], pp. 255–230), or that of Kochen and Specker[7], as well as all the investigations on joint probabilities based on correspondence rules with the quantum mechanical operators (Moyal[8], Bass[9], Cohen[10])) must be carefully reconsidered. Indeed: If the quantum mechanical operators of two quantum mechanical dynamical observables w_1 and w_2 do not commute, this expresses – by definition – the fact that the quantum mechanical measurement processes yielding the quantum mechanical operational definitions for w_1 and w_2, *cannot* be realized simultaneously in one individual act of measurement. Hence, when one examines the question of the "simultaneous measurability of two observables w_1 and w_2 associated with two non-commuting quantum mechanical operators", *ipso facto* a *non*-quantum-mechanical operational definition is now envisaged for at least one of these two quantities, namely a definition such that, now, the two measurement processes conceived *shall* 'commute' (shall *be* simultaneously realizable in one individual act of measurement). In other terms, this problem cannot concern the *same* initial pair of observables w_1, w_2; it can only concern *another* pair, where at least one member is changed. This does not at all mean that the problem is absurd. Nothing hinders the conception that one given beable property w' assigned to a system can be connected with observable facts via several different operational definitions. But there is no reason then to expect for such different operational definitions the same statistical distribution of observed results; different observed statistical distributions have to be expected for them, in general. All these observable distributions are equally acceptable for 'describing' the unique intrinsic distribution of values supposed for the beable quantity w' assigned to the studied system, under the sole condition that each one of the observable distributions be related in some definite – even though specific – way with this unique intrinsic distribution. These considerations entail that when the question of simultaneous measurability is examined, one at least of the two $w(w', \psi, \lambda)$ functionals intervening, describes an individual measurement evolution that is somehow *not compatible* with the quantum mechanical operator for w. There is then no reason whatever to require the equality (17)′ when such a $w(w', \psi, \lambda)$ acts (as Park and Margenau[5] did, as well as von Neumann[6]). Furthermore, there is no reason whatever either for subjecting the functional forms $w(w', \psi, \lambda)$ from (17)′ to structural correspondence rules with the quantum mechanical operators associated to the w-quantities, nor for requiring for these functionals an algebra identical to that of the quantum mechanical operators. The $w(w', \psi, \lambda)$ from (17)′ represent *processes*, and these, moreover, are posed to be *individual*: this is the essential feature of any attempt of a 'causal' solution to the reduction problem. Whereas any quantum mechanical w-operator is defined in direct formal connection with the function $f_{m,w}$ describing the classical macroscopic *w-quantity*; this operator, moreover, is in a one-to-one relation with a whole family of eigenvectors ϕ_{jw}, to each one of which a joint probability attempt assigns already a *statistical* significance, as it can be seen for instance by writing (17)′ for a ϕ_{jw} and by comparing the contents of the two members:

$$\iiint w(w', \psi, \lambda)\Pi\phi_{jw}(q', w', t_0)R_{D(w)}(\lambda)\,\mathrm{d}q'\,\mathrm{d}w'\,\mathrm{d}\lambda$$
$$= \langle\phi_{jw}(w, t_0) \mid f_{QM,w}\phi_{jw}(q, t_0)\rangle = w_j$$

(w_j is the eigenvalue corresponding to ϕ_{jw} of the quantum mechanical operator $f_{QM,w}$). It simply is not physics to impose upon the $w(w', \psi, \lambda)$ a priori formal constraints. The relevant constraints have to be deduced by means of very analyzed physical criteria brought forth by an improved insight in the joint probability problem. We believe that such an insight cannot be obtained as long as only surface probabilistic relations, connecting probability *measures* alone, are stated explicitly, while the corresponding relations between the *events* concerned by these measures are left more or less in the dark. All the various probability *spaces* which intervene – quantum mechanical probability

spaces and joint probability spaces – have to be studied in their entirety and with their interplay at all the levels ('conditions' defining the 'experiment', elementary events brought forth, field on these, measure on the field), in order to acquire a precise and complete perception of the deep structure of the joint probability problem[12].

3.6. *Generalization to Any Relevant Hidden Distribution*

The 'dynamical' observables associated to S correspond – by their operators $f_{QM}(q, \hbar/i(\partial/\partial q))$ – to the classical dynamical quantities, which are all defined as functions $f(q, p)$ of the position and the momentum. Therefore the concept of a joint probability $P_\psi(q', p')$ of a beable position and a beable momentum variable seems a 'natural' concept for expressing the consistency condition (17), to be required for the quantum mechanical 'dynamical' observables associated to S. This joint probability concept, however, cannot yield a direct representation of the 'field-like' beable properties tentatively conceivable for a microsystem; it reflects such properties only indirectly, via the non-classical forces necessary (in general) in the time-evolution law (18), if one wants to preserve the possibility of some compatibility with the Schrödinger evolution of ψ (pp. 125–128). Therefore the joint probability concept $P_\psi(q', p')$ is not appropriate for expressing a consistency condition concerning the quantum mechanical observables of S to which no classical function $f(q, p)$ corresponds (charge, spin component on a given direction). Indeed, for such an observable it would be a priori restrictive to pose that the beable properties w' of S which lead to the observed values w (via the process $w(w', \psi, \lambda)$) are defined *at* q', as it has been assumed for the dynamical quantities considered in (17). Therefore we generalize (17) and (17)′ by making use of a hidden distribution $P_\psi(\mu')$ instead of the joint probability $P_\psi(q', p')$, and of a functional $w(\mu', \psi, \lambda)$ instead of $w(q', p', \psi, \lambda)$, μ' being a generalized hidden variable which designates globally any sort of beable properties assigned to S and conceived to lead to the observed value w via the interaction process described by $w(\mu', \psi, \lambda)$:

(17)″
$$\iint w(\mu', \psi, \lambda)P_\psi(\mu', t_0)R_{D(w)}(\lambda)\,\mathrm{d}\mu'\,\mathrm{d}\lambda =$$
$$= \langle\psi(q, t_0)| w_{QM}\psi(q, t_0)\rangle = \int wc^\psi(w, t_0)\,\mathrm{d}w$$

(w_{QM} in the second member is the quantum mechanical operator of the observable w, connected or not with a classical function $f(q, p)$). Thus (17)″ englobes now (17) and (17)′: we have finally obtained a condition applying to any hidden distribution – a joint probability, or some other distribution – which is both minimally restricted and still relevant to the reduction problem.

3.7. *Methodological Attitude Concerning the Consistency Condition* (17)″

We want to stress a methodological attitude to which we attach a fundamental importance: we assign to the condition (17)″ a symmetric role with respect to quantum mechanics and with respect to a hidden variable attempt, we do not subordinate inconditionally the hidden variable attempts to quantum mechanics.

The conditions of consistency attempted so far have all presupposed the exceptionless validity of the quantum mechanical predictions, at least in the domain of atomic dimensions and newtonian energies. However the fact that a hamiltonian operator can be written does not ensure the physical realizability of its potential term, neither that, a fortiori, of the corresponding Schrödinger time evolution-law. If now a physically realizable potential and the corresponding evolution-law are considered, the mathematically possible ψ-solutions do not all correspond to physically realizable boundary conditions. And if a physically realizable ψ is considered, very paradoxically, the quantum mechanical 'observables' of the system do not all possess a unanimously admitted and physically realizable operational definition, so that the corresponding prediction is not always verifiable (the most striking example of this sort concerns the fundamental 'observable' momentum: in a state ψ which is not an eigenstate of the momentum, according to the orthodox theory of measurement a rigorous measurement of the momentum for $\psi(t)$ yields the observed results at t' such that $(t'-t) \sim \infty$ (time of flight method)). Finally if one considers a physically realizable ψ and an observable for which an admitted operational definition does exist and the results of its application are observable, then the corresponding quantum mechanical prediction might never have been verified.[2] But a priori restrictions corresponding to unrealizable, or to non-verifiable, or to non-verified features of the quantum mechanical description, are likely to introduce

fatal malformations into a joint probability attempt. For these reasons, while requiring the conditions (17)′, we have no rigid preconception. Even these minimally restricted conditions of consistency are demanded only for physically realizable state vectors and we shall keep in mind the two important problems of the verifiability and of the verification of the involved quantum mechanical predictions. In this way, while quantum mechanics imposes restrictions upon the acceptable joint probability, this, in its turn, can play the role of a test concept concerning the quantum mechanical description. This attitude is novel and it is characteristic of our approach.[3,4]

3.8. *One-System Non-Locality*

Joint probability framework

We place ourselves inside the joint probability framework, which afterwards we shall leave. The joint probability defined by the minimal condition (17)′ might seem a very weak concept, unable to lead to any definite conclusion for some problem. But we shall now show that in fact this minimally restricted, while still relevant, concept of a joint probability is strong enough for entailing a problem of physical non-locality inside the one-system formalism of quantum mechanics.

Preliminaries. We make first two remarks:

(1) According to quantum mechanics, if a microsystem S is at some time t in a superposition state $\psi_{ab} = a\psi_1 + b\psi_2$, whose support I in the physical space is a non-connected union $I = I_1 \cup I_2$ of two spatially disjoint intervals $I_1, I_2 (I_1 \cap I_2 = \emptyset)$, then it is possible to prepare the state ψ_1 for S, out of the state ψ_{ab}, namely by suppressing at t on I_2 – with the help of an obturator or filter acting on I_2 – the characteristics of S described by the term $b\psi_2$ of ψ_{ab}. Indeed, if $\Delta t_{pr} = (t_{pr} - t)$ is the time taken by the action of the filter or obturator ('preparation' time), from $t_{pr} = t + \Delta t_{pr}$ on, the state vector to be assigned to S is ψ_1 alone, renormalized to unity. This type of preparation is particularly interesting from our viewpoint because it asserts a relation between a physical – but not *observational* – operation, carried out with the help of a macroscopic device at the location (namely I_2) of a *descriptive* element (namely $b\psi_2$), and a certain physical modification of the 'state' assigned to the object designated by S, possibly entailing changes in the *beable* properties assumed for this object. Even though the quantum theory asserts nothing whatsoever concerning the location in the physical

space, outside the periods of observation, of the objects described by this theory, the possibility of a preparation of the type specified above might contain some implications as to where this object can 'exist' outside the periods of observation, according to a joint probability theory fulfilling conditions of consistency with quantum mechanics.

(2) As we have already pointed out, the quantum mechanical momentum observable has a peculiar operational definition, namely the time-of-flight method. According to this definition the measurement begins at a moment t_0 by the suppression of all external fields, if they existed, while the interaction with a material *registering* device $D(p)$ (which yields, directly, a *position* value) is relevant only if it occurs at another time t, such that $t - t_0 = \Delta t(p) \sim \infty$. The complete measurement interaction consists here of the passage of the infinite period $\Delta t(p)$ + the final registering interaction with $D(p)$. Now, the infinite value thus required for $\Delta t(p)$ introduces ambiguities at the level of a joint probability theory: in the first place, it rules out a *rigorous* verifiability of the quantum mechanical prediction for the momentum spectra. Moreover, not even an approximate verification of this prediction seems ever to have been made effectively for the various types of preparable states ψ (in particular for the superposition states $\psi_{ab} = a\psi_1 + b\psi_2$ with non-connected support, or with connected support (interference)). Therefore, faithful to the agnostic attitude we choose, we reserve our opinion as to the circumstances in which the consistency condition $(17)_1$ concerning the momentum has to be required. In the second place, in the case of a free Schrödinger evolution of ψ, the quantum mechanical operational definition of the momentum observable permits a *degenerate* relation between the observable p-spectrum asserted by quantum mechanics and the instantaneous structure of the hypothetic beable distribution of a hidden momentum $P_\psi(p') = \int P_\psi(q', p')\,dq'$, corresponding to the joint probability measure from $(17)_1$. Indeed the quantum mechanical p-spectrum is an invariant of a free Schrödinger evolution. Then the whole family of different instantaneous structures taken on by $P_\psi(q', p', t_0)$ from the left member of $(17)_1$ when time translations change the t_0 considered, correspond to one same quantum mechanical p-spectrum in the right member of $(17)_1$, if ψ has a free Schrödinger evolution. However, as soon as the beable properties assigned to the object S are different from those of a material point (which seems rather unavoidable, as the remarks on pp. 125–128 show), the beable momentum distribution $P_\psi(p') = \int P_\psi(q', p' = K\,dq'/dt)\,dq'$ can – in general – *change* during a free evolution of ψ, in consequence of the kinematic definition $p' = K(dq'/dt)$ of the beable momentum. Once more an illustration is yielded by the superposition states, namely those which, like $\psi(q, t) = (1/\sqrt{2})\phi_{p_{\text{i}}}(q) + (1/\sqrt{2})\phi_{p_1}(q)$ from the proof of Theorem 1 (pp. 120–122) take on successively, during their Schrödinger evolution, a connected support first, and then a non-connected support (or vice versa)[13]. The preceding remarks apply as well to any function of the momentum alone. But consider now quantities w not depending on the momentum alone (kinetic momentum, projections of the kinetic momentum, total energy). The quantum mechanical operational definitions of such quantities consist of procedures where the time at which the interaction itself between one $S^{(\psi)}$ and a material device $D(w)$ begins, coincides with the time t_0 from $(17)'$ at which what is called 'measurement' as a whole begins. Moreover, the duration $\Delta t(w)$ required in principle for such a measurement *is not infinite*. The preceding remarks concerning the quantities depending on the momentum alone do not apply to these other quantities. We shall now show that:

THEOREM 2. If it is assumed that the beable properties assigned at a time t to the object denominated one system $S^{(\psi)}$ cannot lie outside the support in the physical space of the quantum mechanical state vector $\psi(t)$ associated to $S^{(\psi)}$, then even the minimally restricted joint probability concept from $(17)'$ is unable to ensure a local deterministic solution to the reduction problem, for any state vector and any dynamical observable.

This theorem will be proved by giving an example. Our choices for an example are the following ones:

For the reasons given in the preliminary remarks (b) we consider a quantity w of which the quantum mechanical operational definition involves a *finite* measurement interaction time

(20) $\Delta t(w) < \infty.$

Furthermore, at some initial time t_i, we consider the three state vectors $\psi_1, \psi_2, \psi_{ab} = a\psi_1 + b\psi_2$ such that the supports in the physical space, I_1 and I_2, of – respectively – ψ_1 and ψ_2, are disjoint. The distance d_{12} separating the two nearest points of I_1 and I_2 is subject to a condition, namely: we denote by Δt_{pr} the time-interval necessary for preparing for S the state described by ψ_1 out of the state described by ψ_{ab}, by the method mentioned in remark (a) (i.e., Δt_{pr} is the time-interval, *finite*, taken by an obturator or a filter for suppressing on I_2 the characteristics of S described in ψ_{ab} by the term $b\psi_2$). The moments $t_i < t_0 < t$ are chosen such that $\Delta t_{pr} = t_0 - t_i$, $\Delta t(w) = t - t_0$. We denote $\Delta t_{pr} + \Delta t(w) = \Delta t$ and we require

$$d_{12} > c\Delta t, \tag{21}$$

where c designates the velocity of light.

With these choices we can now develop the proof of Theorem 2. We shall first show that

LEMMA. The product $w\Pi_{ab}$ intervening in the integrand from the first member of the condition (17)′ written for ψ_{ab}, has a *mathematically* non-local dependence on ψ_{ab}.

Proof. The condition (17)′ written for ψ_{ab}, ψ_1, ψ_2, yields (with obvious notations)

$$\iiint w(w', \psi_{ab}, \lambda)\Pi_{ab}(q', w', t_0)R_{D(w)}(\lambda)\,dq'\,dw'\,d\lambda \tag{22}$$

$$= \langle\psi_{ab}(q, t_0)|f_{QM,w}\psi_{ab}(q, t_0)\rangle = |a|^2\int w|C^{(1)}(w)|^2\,dw$$

$$+|b|^2\int w|C^{(2)}(w)|^2\,dw + a^*b\int w(C^{(1)}(w))^*C^{(2)}(w)\,dw$$

$$+ab^*\int w(C^{(2)}(w))^*C^{(1)}(w)\,dw,$$

$$\iiint w(w', \psi_1, \lambda)\Pi_1(q', w', t_0)R_{D(w)}(\lambda)\,dq'\,dw'\,d\lambda \tag{23}$$

$$= \langle\psi_1(q, t_0)|f_{QM,w}\psi_1(q, t_0)\rangle = \int wC^{(1)}(w)|^2\,dw,$$

$$\iiint w(w', \psi_2, \lambda)\Pi_2(q', w', t_0)R_{D(w)}(\lambda)\,dq'\,dw'\,d\lambda \tag{24}$$

$$= \langle\psi_2(q, t_0)|f_{QM,w}\psi_2(q, t_0)\rangle = \int w|C^{(2)}(w)|^2\,dw,$$

Let us admit tentatively the hypothesis conditionally contained in the formulation of Theorem 2, namely

(h) the beable properties assigned at a time t to what is named one system $S^{(\psi)}$ cannot lie outside the support of $\psi(q, t)$ in the physical space.

Consider now the product $w\Pi_{ab}$ from the left member of (22). The hypothesis (h) entails that this product is null outside the support $I = I_1 \cup I_2$ of $\psi_{ab}(q, t_0)$, because the probability measure $\Pi_{ab}(q', w', t_0)$ is null for $q' \notin I$. Then the non-connected structure chosen for $I = I_1 \cup I_2$ (namely $I_1 \cap I_2 = \emptyset$) entails that the product $w\Pi_{ab}$ from (22) is a sum of two terms

$$w(w', \psi_{ab}, \lambda)\Pi_{ab}(q', w', t_0) = w(w', \psi_{ab}, \lambda)\Pi_{ab,I_1}(q', w', t_0) + w(w', \psi_{ab}, \lambda)\Pi_{ab,I_2}(q', w', t_0), \tag{25}$$

(obvious notations) of which one is null for any given q', since *one* $S^{(\psi_{ab})}$ possesses *one* beable 'position' property, so that either $q' \in I_1$ and then $q' \notin I_2$, or vice versa. However, confrontation of (25) with (22), (23), (24) shows that in general

$$\begin{aligned} w(w', \psi_{ab}, \lambda)\Pi_{ab,I_1}(q', w', t_0) &\neq w(w', \psi_1, \lambda)\Pi_1(q', w', t_0),\\ w(w', \psi_{ab}, \lambda)\Pi_{ab,I_2}(q', w', t_0) &\neq w(w', \psi_2, \lambda)\Pi_2(q', w', t_0), \end{aligned} \tag{26}$$

because the sum of the two last 'interference' terms in the second member of (22) is not null for any ψ_1, ψ_2, a, b, and w. It *is* null for the particular case $w = g(q)$ (because of $I_1 \cap I_2 = \emptyset$) so that for these quantities the non-equalities (26) transform into equalities. But for $w \neq g(q)$ the term $w\Pi_{ab,I_1}$ from (25) depends – as the first non-equality (26) shows – on the whole superposition state vector $\psi_{ab} = a\psi_1 + b\psi_2$, even though this term is defined on I_1 alone and even though $I_1 \cap I_2 = \emptyset$, the distance d_{12} which separates I_1 from I_2 being moreover arbitrarily big, as (21) permits. The symmetric argument holds for the term $w\Pi_{ab,I_2}$ from (25). In *this* sense the product $w(\psi_{ab})\Pi_{ab}$ from (22) has a mathematically non-local dependence on ψ_{ab}, q.e.d. The lemma proved above generalizes to any relevant joint probability from (17)′ the mathematical non-locality of Wigner's joint probability (1) (expressed by (11), (12), (13)).

But $w(w', \psi, \lambda)$ designates a *process*, which, in addition, takes a non-null time-interval $\Delta t(w)$. Therefore, in order to investigate whether or not the mathematical non-locality brought into evidence above does involve *physically* non-local phenomena, time has to be taken into account also. This is what one shall do now:

The hypothesis (h) has a rather obvious consequence, namely:

(c) Given one system S to which a quantum mechanical state vector $\psi(t)$ is associated at the time t, throughout any *local* process which involves the system S during a period $\Delta t = t' - t$, the transforms by this process of the beable properties assigned to S at the initial time t, remain confined inside the portion corresponding to Δt of the light-cone of the support I of $\psi(t)$. (This formulation holds with respect to any given space-time referential and, whether or no, from *t on* the quantum theory continues to associate an individualized state vector with S.)

Consider then a statistical ensemble of systems S for each one of which, at the constant time t_i after the preparation of ψ_{ab}, the new state ψ_1 is prepared out of ψ_{ab}, and then a w-measurement is performed on $S(\psi_1)$ (the choices (20), (21) being fulfilled). Under these conditions, if the consistency relation (22) for ψ_{ab} is satisfied, then the condition (23) for ψ_1 is violated, unless some non-local effects take place. Indeed:

The consequence (c) of (h) together with (20) and (21) entail that throughout the time-interval $\Delta t = \Delta t_{pr} + \Delta t(w)$ taken by the global process [preparation for S of the state described by ψ_1 out of the state described by ψ_{ab} + w-measurement on $S^{(\psi_1)}$] the transforms by this process of the beable properties assigned to S at t_i remain confined inside two disjoint and space-like separated space-time domains. But according to (26) the consistency condition (23) for ψ_1 can be fulfilled only if the product $w(\psi_{ab})\Pi_{ab.I_1}$ changes into the different product $w(\psi_1)\Pi_1$. This is a required *statistical* change, but it can come about only if *individually* the beable properties realized for each $S^{(\psi_{ab})}$ on I_1 at t_i, undergo during Δt a transformation different – in general – from the transformation that would have taken place if the state of that S would have continued throughout Δt to be described by ψ_{ab} (i.e. in absence of the action on I_2 of an obturator or filter). In other words, each one action of preparation of a state ψ_1 out of a state ψ_{ab} for one S, even though it takes place on I_2, must – in general – somehow cause a *change*, and *during* Δt, of the individual properties of that S on I_1. Now, if such a change does indeed happen, it can be only non-local, since the portions corresponding to Δt of the light cones of I_1 and of I_2 are two space-like separated space-time domains. While, if the specified change does not happen, (23) for ψ_1 cannot be fulfilled, in consequence of the first non-equality (26). This example suffices for proving Theorem 2.

Quite independently of any experimental investigation which it might suggest, this conclusion is a theoretical fact.

General hidden variables framework. Theorem 2 can be generalized

for any hidden distribution fulfilling the minimal condition (17)″, and for any observable, as we have shown elsewhere [14]. Thus, when grasped synthetically, the conceptual situation is this: No hidden variable distribution can ensure a *local* deterministic solution to the reduction problem, *if* the object denominated 'one microsystem' cannot 'exist' outside the support in the physical space of the quantum mechanical state vector associated to it. Thus we have been led to a direct confrontation between the one-system quantum mechanics, causality, relativity, and the question *where* the object named 'one microsystem' does 'exist'. The concept of hidden variables has played the role of a revelator of this confrontation. This shows the methodological force of the hidden variables concept.

3.9. *Comparison with Two-System Non-Locality*

In the one-system locality theorem proved above, the question of the relation between the beable location of 'S' in the physical space and the support of the quantum mechanical state vector of 'S', plays an essential role; while J. S. Bell, who discovered the locality problem [2], has brought forth, with the help of his well-known two-system example, a pure and striking confrontation between quantum mechanical predictions, causality and relativity, where no explicit use is made of the question mentioned. In this connection we want to make two remarks.

In the first place: when 'S' designates 'one system' only one mark on a measurement device can be registered for each 'S'. This is what necessitates the explicit introduction of the hypothesis (h) in the demonstrations on one-system non-locality. However, if not a demonstration, a one-system alternative for experimental investigation on locality can be formulated without use of (h). Indeed, one can obtain a conclusion by exclusively taking into account the space-time coordinates of macroscopic events, namely the action of an obturator and the registration of a mark on a measurement device: even though quantum mechanics does not *predict* where and when a mark will be registered, a posteriori this mark is always found with *some* definite space-time coordinates. If, for each individual registration, these coordinates are found to be separated space-like from those of the action of the obturator (following the conditions of the proof of Theorem 2) and if, nevertheless, statistically, the quantum mechanical distribution for ψ_1 is found when the obturator is used, while, when not, the distribution predicted for ψ_{ab} is found, then there is non-locality.

In the second place: when it is tried to define the significance to be assigned to the various possible results of the experiments for verifying Bell's inequality, the question of where the object named a 'two system' does beably 'exist' comes into play irrepressibly, raising novel and fundamental problems [15], even though it is absent from Bell's demonstration, at least explicitly. (Implicitly it must somehow intervene, since the location of the two registering devices used is not chosen *independently* of the maxima of the presence probability for the two 'parts' of 'S', calculated with the help of the state vector of 'S'.)

From these remarks we conclude that the question of the relation between the beable location of 'S' in the physical space and the support of the quantum mechanical state vector of 'S' plays in fact an essential role in any locality problem, no matter whether 'S' designates 'one system' or 'two systems' and notwithstanding the formal descriptive differences.

In this perspective, the explicit presence of this question in the one-system demonstration appears as a specific and interesting feature, drawing particular attention to the relations between reality and the descriptive language of quantum mechanics.

3.10. *Experimental Study*

Theorem 2 and its generalization suggest an experimental study which we shall now indicate.

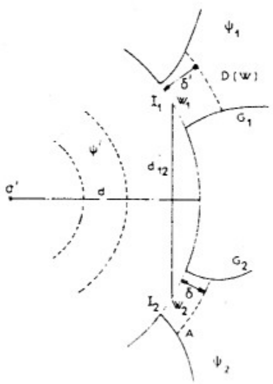

Fig. 1

Preparations (*Figure* 1). A non-monochromatic and low intensity intermittent source σ emits microsystems S. At a distance d from σ is placed a spherical screen S, of radius d, centred on σ. Two circular windows W_1 and W_2 are cut out of S. The distance d_{12} which separates the centres of W_1, W_2 can be chosen arbitrarily big by increasing d. At the right of S the windows W_1, W_2 are continued by widening walls playing the role of guides G_1, G_2 (Figure 1). In these conditions each individual system S emitted by σ is described by quantum mechanics, at the left of S, by a spherical wave packet ψ', the front of which reaches at some given moment t, *simultaneously*, both windows W_1, W_2. From that moment on, at the right of S quantum mechanics describes the considered *one* system S by the superposition $\psi = (I/\sqrt{2})\psi_1 + (I/\sqrt{2})\psi_2$ of the two packets ψ_1 and ψ_2 transmitted respectively by the two windows W_1 and W_2. Because of the guides G_1, G_2 the supports I_1, I_2 of ψ_1, ψ_2, are finite, disjoint, and separated by the arbitrary distance d_{12}. Thus at the right of S one has prepared a superposition state of the type utilized in the proofs of Theorem 2.

The state ψ_1 can be prepared out of $\psi = (I/\sqrt{2})\psi_1 + (I/\sqrt{2})\psi_2$, by introducing an absorbing wall inside the guide G_2, at some distance S at the right of the surface (virtual) of the window W_2 (Figure 1). The state ψ_2 can be prepared similarly.

First stage of experiment: verification of the quantum mechanical predictions for w, ψ_1, ψ_2 *and* ψ. The distribution (and mean value) of w is measured separately in ψ (W_1, W_2 both open), ψ_1 (W_2 constantly shut) and ψ_2 (W_1 constantly shut). The results are compared in order to see whether the quantum mechanical *non*-additivity of the w-spectrum in ψ, with respect to the w-spectra in ψ_1 and ψ_2, is true or not. The problem, in this stage, is to define the theoretical conditions of observability of the sum of the 'interference terms' from the right side of (22) (interference in the *w-space*, even though in the physical space ψ_1, ψ_2 have disjoint supports), and to define a procedure which insures an w-resolution permitting the registration of the w-interference distribution, if it really exists. If in such appropriate conditions the predicted interference w-distribution is not registered, the quantum mechanical prediction from the right side of (22) is not *true* so that the non-locality Theorem 2 is not true either, so that a further stage of locality investigations is irrelevant. If on the contrary the w-interference term is observed, the following stage is relevant:

Second stage: locality investigation. For each system S emitted by σ the superposition state ψ is first prepared (W_1, W_2 both open). Then the preparation of ψ_1 out of ψ is started at a moment t_i by help of an absorbing shutter A dropped inside the guide G_2, at a distance ∂ from the surface of the window W_2. An w-measuring device $D(w)$ is placed inside the guide G_1 at a distance $\partial' = \partial + \varepsilon$ from the surface of the window W_1, ε being very small but sufficient for ensuring that when the front of the wave of the system reaches the level $\partial + \varepsilon$ the preparation of ψ_1 *has* already been accomplished (the term $(I/\sqrt{2})\psi_2$ of ψ has been suppressed) so that it is the wave-packet ψ_1 which reaches the w-measuring device $D(w)$. The condition

$$(21)' \qquad \frac{\varepsilon}{V_{\psi_1}} + \Delta t < \frac{d_{12}}{C}$$

is required, where V_{ψ_1} is the group velocity for ψ_1 (depending on the mean energy chosen for the systems emitted by σ), Δt and c being defined by (21). The device $D(w)$ and the absorbing shutter A are each time set in action simultaneously and $D(w)$ is each time disconnected after a time inferior to d_{12}/C. If by repetition of this procedure the recorded w-distribution is identical with that found for ψ_1 alone in the first stage of experiment (i.e. if the w-interference term, supposed to have been previously found for ψ, is suppressed by the action of the shutter A) then it has to be concluded that either non-local effects have gone from I_1 to I_2, or the object named one microsystem S somehow is not confined on the support of the quantum mechanical state vector associated with its state. The problem to be solved for this stage is to realize the condition (21)' while furthermore ensuring, as in the first stage, conditions of observability of w-interference fringes (in the w-space).

Any observable w for which (20) is fulfilled can be envisaged, spin-components included. Upon a more detailed analysis the spin-component along the direction *perpendicular* on d_{12} might appear to be the most convenient choice. For the moment, however, we reserve our opinion concerning both the choice of w and that of the measurement procedure. If these choices raise questions and seem queer, this is a reflection from the queerness of the quantum mechanical theory of measurement. We believe that this queerness should not be allowed to act as an obstacle to any attempt of *verifying* the concepts and predictions of the orthodox theory of measurement.

4. REMARKS ON THE SUPERPOSITION STATES

Throughout the preceding study, the superposition states have played an essential role which draws attention on them.

There exists a tendency for confounding the superposition states with the mathematical decompositions permitted by the expansion postulate. This tendency has its source in the fact that the quantum mechanical formalism prescribes the same algorism for the calculation of predictions concerning a superposition state or concerning a mathematical expansion. However, quantum mechanics does distinguish – by their definitions – the superposition states from the mathematical decompositions. When this distinction is explicitly taken into account and then confronted with the identity of the algorisms prescribed for calculating the predictions, reasons appear for doubting the truth of certain predictions concerning superposition states. Indeed:

A quantum mechanical state vector ψ is defined at any time by the specification of boundary conditions B which determine an 'initial' form $\psi(q, t_0)$, and of an evolution operator H which determines the transform of $\psi(q, t_0)$ by the passage of time. We shall then write symbolically $\psi = \psi(B, H)$. The physical realization of both B and H is necessary for the physical realization of $\psi(B, H)$.

Let us now adopt the Schrödinger representation:

In a superposition state $\psi = a\psi_1 + b\psi_2$, the boundary conditions are different for ψ, ψ_1, ψ_2, while H can be the same, or not. To take an example, we suppose that H is the same. Then we write $\psi(B_1 + B_2, H) = a\psi_1(B_1, H) + b\psi_2(B_2, H)$, with a, b complex *constants* and ψ, ψ_1, ψ_2 having a time evolution corresponding to H. When ψ_1, ψ_2 'interfere' in the physical space or in some other w-space, this interference concerns two different states both *realized* simultaneously.

Consider now a mathematical decomposition of a state ψ, according to the eigenstates ϕ_{Q_i} of a dynamical quantity Q, $\psi = \Sigma_i c_i^\psi \phi_{Q_i}$, such as is permitted by the expansion postulate. Boundary conditions B and a hamiltonian H are realized *only* for ψ. The ϕ_{Q_i} are constant vectors and the c_i^ψ are complex numbers *depending on B and H* via the definition

$$c_i^\psi(t_0) = \int \psi(t_0)\phi_{Q_i}\, dq, \qquad c_i^\psi(t) = \int \psi(t)\phi_{Q_i}\, dq.$$

Then we have to write $\psi(B, H) = \Sigma_i c_i^\psi(B, H)\phi_{Q_i}$. Here only ψ is physically realized, while on the right side of the equality $\psi(B, H)$ is *represented in terms of the standard state vectors* ϕ_{Q_i} *conceived, but not realized physically.*

When the probability law for some quantity $Q' \neq Q$ is calculated by the *same* algorism

$$\left|\int \psi\phi^*_{Q'_i}\,\mathrm{d}q\right|^2 = \left|\int (a\psi_1 + b\psi_2)\phi^*_{Q'_i}\,\mathrm{d}q\right|^2$$

or

$$\left|\int \psi\phi^*_{Q'_i}\,\mathrm{d}q\right|^2 = \left|\int (\Sigma_i c_i^\psi \phi_{Q_i})\phi^*_{Q'_i}\,\mathrm{d}q\right|^2.$$

applied indistinctly to the superposition $\psi(B_1 + B_2, H) = a\psi_1(B_1, H) + b\psi_2(B_2, H)$ or to the expansion $\psi(B, H) = \Sigma_i c_i^\psi(B, H)\phi_{Q_i}$, this might involve erroneous identifications of statistics of real interactions between physically realized states, with mathematical interferences of standard states, conceived but not physically realized. Therefore we envisage that *the superposition principle might introduce certain false predictions.*

5. CONCLUSION

We have shown that Wigner's proof does not invalidate the concept of a joint probability of the position and the momentum variables, but raises instead a locality problem inside the one-system formalism of quantum mechanics.

In a critical research it might be illuminating to examine in detail a counter-example to a general assertion, instead of using it merely as a sufficient basis for the global rejection of this assertion. In constructive attempts the aim is the perception of some maximally unifying essence, and the choice of the maximal generality in the formulations ensures indeed a progression towards this aim. But in a critical attempt, on the contrary, the progress often lies in the identification of some particular circumstance of which a previous constructive effort has remained unaware, and which has therefore been erroneously forced into a conceptual structure imperfectly fitted for it, where its presence introduces distortions. Thus, in the present case, the connections established between the one-system locality problem and the particular type of state vectors for which this problem arises, suggest that the quantum theory might have erroneously

integrated the description of momentum-dependent distributions, for these particular states. While the study of the position distribution for states $\psi = a\psi_1 + b\psi_2$ with $I_1 \cap I_2 \neq \emptyset$ has contributed to lead towards quantum mechanics, the study of the momentum- or spin-dependent distributions for the states $\psi = a\psi_1 + b\psi_2$ – with $I_1 \cap I_2 = \emptyset$ or $I_1 \cap I_2 \neq \emptyset$ – might contribute to lead beyond the bounds of quantum mechanics.

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NOTES

[1] The de Broglie–Bohm functional of this type is unnecessarily a priori restricted and thereby it introduces distortions [11].

[2] In a preceding work [11] it has been shown that the very *weak* hypothesis according to which the object denominated *one* microsystem cannot progressively extend over a spatial domain indefinitely increasing, suffices to entail a position distribution which in certain states is not rigorously identical with the quantum mechanical one. But the rigorous truth of the quantum mechanical prediction in this case has never been verified.

[3] Discussions on this subject with Dr D. Evrard have strongly contributed to the formation of this attitude.

[4] It seems interesting to compare our considerations on pp. 122–133, with the very pertinent analyses of Belinfante [16] on the hidden variables attempts made up to now.

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