#### VI.4.2. Mathematical framework in terms of the theory of categories

We seek now a mathematical representation of the skeleton of MRC. It is crucial to begin by making use of the weakest possible mathematical structure, i.e. which introduces a minimum of formal restrictions not stemming from MRC itself. Only in this way can it be hoped to avoid a too amputating transposition of the content of the verbal presentation. Too often the formalizations, and in particular the mathematical ones, amputate under cover of insuring "generality". Later it will be useful to specify *local* restrictions in order to characterize particular types of MRC-conceptualizations (logical, probabilistic, this or that sort of theory). But the general framework has to be maximally comprehensive. No pre-existing mathematical structure, I think, can yield a fully satisfactory formal expression of MRC. This is so because of the very peculiar character of the basic descriptions (D14.3.1 and D14.3.2) which introduce explicitly into the representation features reflecting fragments of as yet non conceptualized factuality. But the theory of categories seems to be a good candidate for just a start. So we remind briefly of the basic definitions from the theory of categories.

Consider the concept of category (Encyclopedia Universalis Vol. 3, France S.A. 1976, p. 1057) (my translation, where also the notations are correspondingly translated: instead of Fl (flèche) we write Ar (arrow), etc.; these notations, of course, can be optimized later):

«A category C consists of the specification of:

a) a class Ob(C) of *objects*, and a class Ar(C) of *arrows*;

b) two applications s and t from Ar(C) into Ob(C) (for any pair (A,B) of objects one denotes by Hom(A,B) the class of arrows f having the *source* s(f)=A and the *target* t(f)=B; if  $f \in Hom(A,B)$  one writes f: A $\rightarrow$ B, or A $\rightarrow$ B:

c) an application that associates to any pair (g,f) of composable arrows, i.e. such that s(g)=t(f), a composed arrow denoted gof or gf, with source s(f) and target t(g).

The concepts thus defined being subjected to the two following axioms:

(C.1) For any object A there exists a unit arrow  $1_A$ : A $\rightarrow$ A such that  $1_A$  of=f and go1<sub>A</sub>=g, for any arrow f with target A and any arrow g with source A;

(C.2) If f:  $A \rightarrow B$ , g:  $B \rightarrow C$  and h:  $C \rightarrow D$ , then (hg)f=h(gf)

The mathematical structures (sets, groups, topological spaces, etc.) are usually endowed with morphisms (applications, homomorphisms, continuous applications, etc.) and they determine categories (Set, Top., etc.) whose objects are the structured sets and whose arrows are the morphisms; the source and the target of a morphism are here, respectively, the starting set and the arrival set of the morphism. One immediately obtains categories that are not of the preceding type, *via* formal constructions like the following ones: if C<sub>1</sub> and C<sub>2</sub> are two categories, the product category C<sub>1</sub>xC<sub>2</sub> has as objects the pairs formed with an object from C<sub>1</sub> and an object from C<sub>2</sub>, the arrows with source (A<sub>1</sub>,A<sub>2</sub>) and target (B<sub>1</sub>,B<sub>2</sub>) being the pairs (f<sub>1</sub>,f<sub>2</sub>) where f<sub>1</sub>: A<sub>1</sub> $\rightarrow$ B<sub>1</sub> and f<sub>2</sub>: A<sub>2</sub> $\rightarrow$ B<sub>2</sub>. The dual category corresponding to a category C\* is obtained by «reversing» the direction of the arrows from C.

If C and C' are two categories, a functor F from C into C' associates to any object A from C an object F(A) from C', and to any arrow f:  $A \rightarrow B$ , an arrow F(f):  $F(A) \rightarrow F(B)$  such that:

(F.1) for any object A from C,  $F(1_A)=1_{F(A)}$ .

(F.2) if (g,f) are composable in C, F(gf) = F(g)F(f)».

# **IV.4.3.** C<sub>MRC</sub>

Preliminaries. We shall now try to represent the skeleton of MRC, in the terms of the theory of categories. So we shall introduce a category denoted CMRC. This is not attempted under the constraints of the theory of models. Indeed in consequence of the primordial role assigned in it to the consciousness functioning, MRC has a strongly teleological character. Furthermore, because the transferred descriptions root it into pure factuality, beneath language, MRC also has a basically intensive character, namely an actively created and relative intensive character. Whereas nowadays semantics takes its start on the level of languages and of classical logic, so it incorporates the assumption of pre-existing and absolute object-entities and predicates, and its difficulties are well-known: an intensive semantics is not yet accomplished, even the relations to be required between extensive and intensive semantic features are still very obscure. As for pragmatics as a discipline incorporating teleology, it is still very incipient. It would be at the same time hopeless and *pointless* to try to submit a priori an approach like MRC, to requirements induced by other still non-stabilized approaches that start from the current languages and from classical logic. On the contrary, it can be hoped that a free mathematical representation of MRC, as that one attempted below, if it succeeded, would help to build a deep-rooted and sound extensive-intensive pragmatical semantics.

Since  $C_{MRC}$  is attempted as a particular interpretation of the abstract concept of a category, the semantics associated with the involved objects and arrows will be given as much importance as the syntactical constraints imposed by the theory of categories.

### Ob(C<sub>MRC</sub>)

The objects from the class  $Ob(C_{MRC})$  are called *epistemic sites* (in short, sites) and are denoted S. A site is posited to designate a definite sort of conceptual ground – just a semantic receptacle similar to an axis in a graphic representation, or, more generally, to a multidimensional representation space – available for lodging inside it an evolving and unlimited content to which no general structure is pre-imposed (for the representation of particular MRC-problems one can pre-impose a particularly adequate structure, for instance an order). This content, however, is required to have a nature consistent with the general definition of the considered semantic receptacle (to "fit" into it, as, for instance, the red of this flower or the dark of this cat do fit into the semantic dimension labelled by the word "colour", but not into that labelled by the word "form"). The most important feature of the content of a site is that it is not required as given from the start on (though it is permitted such): in general it is conceived of as being created progressively and indefinitely.

The distinction itself between a stable pre-existing conceptual receptacle (a genus, an axis, a multidimensional conceptual space), and a corresponding sort of content of which any constituent or part can always be lodged inside this receptacle, indefinitely, at this or that definite "location" (specific difference, point), is by no means new. Quite on the contrary, more or less explicitly it underlies the whole classical organization of thought (linguistic, logical, mathematical; it was already quite explicit for Aristotle), and it includes also the basic notion of a referential. But neither classical logic nor nowadays mathematics do represent in general and explicit terms the most complete possible process of generation of the content of a pre-posited conceptual receptacle, as specified in the concepts basic transferred descriptions and of subsequent intrinsic metaconceptualizations and modellings. And very often this content is tacitly supposed to somehow be entirely "given" from the start on, to somehow pre-exist all done, "out there", in a Platonic manner. Only if ab initio this hypostatic view is systematically replaced by a genetic one, will it be possible to mimic in the terms of the theory of categories, the fundamental MRC-concepts of basic transferred description and of intrinsic metaconceptualization. This is why here a specific definition of the concept of "site" is needed.

The sites from  $Ob(C_{MRC})$  are:

-  $S_R$  that represents formally the location of the evolving content of the reality R, as defined in D2;

-  $S_{CF}$  that represents formally the *location* of the evolving content of the consciousness-functioning CF, as defined in D1.

-  $S_{ce}$  where have to be located all the *formal representations* of the object-entities  $\alpha_G$  defined in D4, as these emerge;

- S<sub>D</sub> where have to be located all the *formal representations* of the relative descriptions  $D/G, \alpha_G, V/$  (def. D14.1) or metadescriptions  $D^{(n)}/G^{(n)}, \alpha^{(n)}, V^{(n)}/, n=0,1,2,...$  (def. D16), as these emerge.

As already stressed, the explicit distinction between a permanent site determined by a static definition, and the (in general) evolving content located on this site, is quite essential

for Ob(C<sub>MRC</sub>). Furthermore, according to MRC it is necessary to posit explicitly that  $S_R \supset [(Ob(C_{MRC})])$ , which will induce *reflexive* features into the formalization <sup>1</sup>.

In a future elaboration of particular MRC-problems, See and SD will have to be assigned structures. See will have to become a mathematical space lodging in it an evolving content of some sort of specified mathematical beings (real or complex functions, kets, sequences of signs, etc.) generated one by one and in general independently of one another and offering a convenient representation of the considered sort of object-entities (for instance, in the particular case of the Hilbert-Dirac formulation of quantum mechanics S<sub>ce</sub> becomes the Hilbert space of state vectors). SD will have to become another kind of mathematical space, lodging in it an evolving content of some other sort of mathematical beings, again generated one by one and in general independently of one another and representing conveniently the considered type of achieved descriptions (in the case of quantum mechanics SD consists of the space of column-matrixes that represent any state vector in some given basis). These spaces will have to be endowed with general structures such that the formal behaviour of the elements from the space is tied with physical objectentities  $\alpha_{G}$ , when combined with the other elements of the mathematization, shall permit to reflect conveniently the space-time restrictions imposed by the principles P8 and P10, as well as the propositions  $\pi 11$ ,  $\pi 12$ ,  $\pi 13$ . Moreover the two structures posited on S<sub>c</sub> and S<sub>D</sub> will have to be connected with one another consistently from both a mathematical and a semantic point of view. In order to reflect formally this or that particular class of objectentities and/or of descriptions, further more specific structural restrictions can be added.

### Ar(C<sub>MRC</sub>)

Consider now the class of arrows,  $Ar(C_{MRC})$ . The arrows from this class will be called *epistemic arrows*.

Inside the theory of categories, given some category C, an arrow from Ar(C) is currently conceived to represent an already constituted morphism that pre-exists in a Platonian manner. This sort of semantics, however is not coherent with our previous definition of  $Ob(C_{MRC})$  as containing sites with evolving content. For consistency with the definitions from MRC and with our previous definition of  $Ob(C_{MRC})$ , any arrow from  $Ar(C_{MRC})$  will be posited to represent a process of which the action is unlatched inside the

<sup>&</sup>lt;sup>1</sup> Matthieu Amiguet, in a private communication, has made interesting suggestions in this respect.

source-site, at a definite "content-point" which in certain cases is itself created by that process, as its source-point; then the process develops in time (and sometimes in space-time) always ending by the creation at its head of a local contribution to the evolving content of the target-site. In this sense an  $C_{MRC}$ -arrow is posited as a *local genetic arrow*.

The epistemic arrows from  $Ar(C_{MRC})$  themselves are generated inside the consciousness functioning CF *or by its free choices*, in consequence of its interactions with the contents of S<sub>R</sub> and with itself. So:

Though it does not belong to  $Ob(C_{MRC})$ , the generic concept  $Ar(C_{MRC})$  can be best described by making use again of the concept of site, a site bearing an evolving content of arrows.

The set of arrows  $Ar(C_{MRC})$  can be split in two sub-classes of epistemic arrows, a sub-class of *primitive epistemic arrows*  $PAr(C_{MRC})$ , and a sub-class of *composed epistemic arrows*  $CAr(C_{MRC})$ .

 $PAr(C_{MRC})$ . The primitive epistemic arrows from Ar(C<sub>MRC</sub>) are:

- *Data-arrows*  $d \rightarrow$  denoted d, with  $s(d)=S_R$  and  $t(d)=S_{CF}$  (so belonging to Hom(S<sub>R</sub>,S<sub>CF</sub>)), that represent the generation of data inside CF, by the influx of data from the reality R.

- Endomorphic aim-arrows, of two kinds:

\*(*Object-entity-generation-aim*)-arrows GA $\rightarrow$  (in short GA) with s(GA)=S<sub>CF</sub> and t(GA)=S<sub>CF</sub> (so belonging to Hom(S<sub>CF</sub>,S<sub>CF</sub>), that represent the process of constitution inside CF of the aim to know specifically about a somehow pre-figured sort of object-entity  $\alpha_{G}$ .

\*(Qualification-aim)-arrows or, in short, view-aim-arrows, of two kinds,  $V_gA \rightarrow$  or  $VA \rightarrow$ , indistinctly short-noted VA, with s(VA)=S<sub>CF</sub> and t(VA)=S<sub>CF</sub> (so again belonging to Hom(S<sub>CF</sub>,S<sub>CF</sub>), that represent the process of constitution inside CF of the aim to qualify (some object-entity) via an aspect-view  $V_g$  or, respectively, a view V.

- Operational-arrows of two kinds:

\*(*Object-generation*)-operational-arrows or, in short, generation-arrows  $G \rightarrow$  (in short G) that represent the epistemic operations of effective generation of an objectentity. By definition s(G)=S<sub>R</sub> and t(G)=S<sub>@</sub>, so G $\rightarrow$  belongs to Hom(S<sub>R</sub>,S<sub>@</sub>).

\*Qualification-operational-arrows of two kinds, aspect-view arrows  $V_g \rightarrow$  or viewarrows  $V \rightarrow$ , indistinctly called view-arrows (in short V), with  $s(V)=S_{\mathfrak{C}}$  and  $t(V)=S_{\mathfrak{D}}$ (so belonging to Hom $(S_{\mathfrak{C}},S_{\mathfrak{D}})$ ). The view-arrows represent the elaboration of relative descriptions by operations of qualification of an object-entity via, respectively, examination by an aspect-view or a view. Mind that a view-arrow  $V \rightarrow$  represents globally all the processes of examination that establish the one corresponding relative description, so it has to be constructed from aspect-view-arrows  $V_g \rightarrow$  by taking into account the proposition  $\pi 11$ .

- *Aim-activating-arrows* Aa $\rightarrow$ (in short Aa) of three kinds, that represent the passage (decided by the working consciousness functioning) from a given epistemic *aim*, to the corresponding effective epistemic *operation* :

\*(*Generation-aim*)-activating-arrows GAa $\rightarrow$ (in short GAa) with s(GAa)=S<sub>CF</sub> and t(GAa)=S<sub>R</sub>, so GAa $\rightarrow$ belongs to Hom(S<sub>CF</sub>,S<sub>R</sub>);

\*(*View-aim*)-activating-arrows VAa $\rightarrow$ (in short VAa) with s(VAa)=S<sub>CF</sub> and t(VAa)=S<sub>@</sub>, so VAa $\rightarrow$ belongs to Hom(S<sub>CF</sub>,S<sub>@</sub>));

\*(*Descriptional-aim*)-activating-arrows DAa $\rightarrow$  (in short DAa), that just initiate globally the whole descriptional program involved in the choice of an epistemic referential. (An arrow DA $\rightarrow$  itself, a descriptional-aim-arrow, is a *composed* arrow and as such it will be defined below. Nevertheless the corresponding aim-activatingarrow DAa $\rightarrow$  is a monolithic primitive arrow with s(DAa)=S<sub>CF</sub> and t(DAa)=S<sub>R $\cap$ D</sub>, so DAa $\rightarrow$ belongs to Hom(S<sub>CF</sub>,S<sub>R $\cap$ D</sub>) (we have S<sub>R $\supseteq$ </sub> S<sub>D</sub>, so t(DAa), being in S<sub>D</sub>, is also in S<sub>R</sub>).

- *The unit-arrows* required by the theory of categories for each site from  $C_{MRC}$  could be introduced as purely formal arrows. However it is obvious that a fully satisfactory MRC-interpretation of the theory of categories should endow each unit-arrow, with an adequate semantics. This might be possible but it might involve quite non trivial epistemological considerations. It might even lead to certain deep and rigorous explicitations concerning the reflexive features to be assigned to the sites from  $C_{MRC}$ . (For S<sub>CF</sub> the role of unit-arrow

could be assigned to each one of the already defined endomorphic aim-arrows, which arises a problem of choice). So, for the moment, we leave open the question of a meanigful definition of the unit arrows.

Before continuing with the sub-class of composed epistemic arrows, let us note the following. An epistemic referential (G,V) as defined in D6 can be now represented formally by the corresponding pair of operational arrows (G $\rightarrow$ ,V $\rightarrow$ ). In order to represent formally the *a priori* possibility of any MRC-pairing (G,V), inside C<sub>MRC</sub> any pairing (G $\rightarrow$ ,V $\rightarrow$ ) will be permitted *a priori*. An observer-conceptor as defined in D6 can then be represented inside C<sub>MRC</sub> by the association [CF, (G $\rightarrow$ ,V $\rightarrow$ )] between the evolving content CF of a site S<sub>CF</sub> and the representation of an epistemic referential.

 $CAr(C_{MRC})$ . The composed epistemic arrows from  $Ar(C_{MRC})$  are:

- Given two aim-arrows GA $\rightarrow$  and VA $\rightarrow$ , whatever they be, they are always composable in any order, since  $s(GA\rightarrow)=t(QA\rightarrow)=s(GA\rightarrow)=t(VA\rightarrow)=S_{CF}$ . However the MRCsemantics requires to take into consideration only the order GA $\rightarrow$ oVA $\rightarrow$ . So, denoting the result DA $\rightarrow$  (in short DA), we have with  $s(DA)=t(DA)=S_{CF}$ . We call it a *descriptionalaim-arrow* and we write

 $DA = DA \rightarrow = GA \rightarrow oVA \rightarrow$ 

This descriptional-aim-arrow  $DA \rightarrow = GA \rightarrow oVA \rightarrow$ , like a fragment of DNA, holds in it, still non-realized so still a-temporal, the whole descriptional program corresponding to the pairing ( $GA \rightarrow, VA \rightarrow$ ), whether realizable or not <sup>2</sup>.

Given a pair of arrows  $d\rightarrow$ , DA $\rightarrow$ , the composition, in this order, is always possible formally. But it is MRC-significant iff DA $\rightarrow$  corresponds to the content of data supposed to be carried by  $d\rightarrow$  (this, being a fundamentally semantic matter, cannot be established

 $<sup>^2</sup>$  The *selection* - among all the syntactical possibilities offered by a formalism - of exclusively those that translate the *semantic* features to be represented, is unavoidable when an interpretation of a formal system is built. In particular the procedure is quite current throughout mathematical physics. (For instance, in a quantum mechanical problem of square potentials, the general solution of the differential equation of the problem offers exhaustively all the possible formal terms; among these, those which have no physical correspondent in the data of the problem are dismissed, while the conserved expressions are specified as required by these data (limiting or initial conditions, etc.), which cannot follow syntactically. Another example can be found in Schrödinger's solution of the problem of a one dimensional harmonic oscillator where subtle and very constructed physical arguments are introduced in order to identify restrictions that are not imposed mathematically; etc.).

formally). The composition will be taken into account only when it is meaningful. We then call it an *induction arrow*, we denote it ind.DA $\rightarrow$  (in short ind.DA), and we write

ind.DA
$$\rightarrow$$
 = d $\rightarrow$ oDA $\rightarrow$ 

 $s(ind.DA)=S_R$  and  $t(ind.DA)=S_{CF}$ , which represents formally an induction of a descriptional aim from R into CF.

- Consider the representation  $(G\rightarrow, V\rightarrow)$  of an epistemic referential. Formally the two operational arrows are always composable in this order. MRC also requires, for methodological reasons, to systematically admit the composability *a priori*, but to exclude it *a posteriori* if the condition D7 of mutual existence or the condition of individual or probabilistic stability involved by D14, appears not to obtain. So inside CMRC we proceed as follows. First, systematically and tentatively, we do form the composition between  $G\rightarrow$ and  $V\rightarrow$ , in this order, naming it a descriptional arrow  $D\rightarrow$  (in short, D). Thus we write

#### $D \rightarrow = G \rightarrow oV \rightarrow$

with  $s(D)=S_R$  and  $t(D)=S_D$  (so belonging to Hom( $S_R,S_D$ ). But if later it is found that no description arises because D7 or the condition of stability from D14 fails (which, being fundamentally a matter of semantics, cannot follow syntactically), then we cancel *a posteriori* the previously formed arrow  $G\rightarrow oV\rightarrow$  and the corresponding epistemic referential ( $G\rightarrow$ ,  $V\rightarrow$ ). Any epistemic referential considered in what follows is supposed to have been found to satisfy both D7 and D14. The composed arrow  $D\rightarrow=[G\rightarrow oV\rightarrow]$  formed with such a "good" epistemic referential is the operational nucleus of  $C_{MRC}$ . It has to be constructed so as to yield a satisfactory formal expression of all the conditions relevant to the considered description, as required by D14 (so P10 and  $\pi$ 11) as well as by (according to the case) P15, D16, D19:

In consequence of P10 and  $\pi 11$ , D $\rightarrow$  involves an (in general) non-commuting algebraic structure imposed upon the set of arrows V $\rightarrow$ .

- Given an epistemic referential (G $\rightarrow$ , V $\rightarrow$ ), the following corresponding composition, called a *complete-description-arrow* (in short CD) is always possible and significant:

$$CD \rightarrow = CD = d \rightarrow oDA \rightarrow oDAa \rightarrow oG \rightarrow oV \rightarrow = indDA \rightarrow oDAa \rightarrow oG \rightarrow oV \rightarrow a \rightarrow oDAa \rightarrow$$

with  $s(CD)=S_R$  and  $t(CD)=S_D$  (so belonging to  $Hom(S_R,S_D)$ ). Which reeds: data from the reality R induce a descriptional aim into the consciousness functioning, this is activated, and so first an object-entity is generated out of R (which brings on the site of object-entities) and then this object-entity is qualified, whereby a description is obtained (which brings on the site of descriptions). The explicit "sites-trajectory" of a complete descriptional process arrow CDP $\rightarrow$  is

The triplet  $S_{CF}$ - $S_{CF}$ - $S_{CF}$  expresses satisfactorily the dominant role of the consciousness functioning in a descriptional process.

- Other compositions also are permitted by the introduced definitions (for instance  $GAa \rightarrow oG \rightarrow$ ,  $VAa \rightarrow oV \rightarrow$ , etc.). But it seems not necessary to examine them exhaustively.

Notice that the MRC-definition D2 of reality requires to extend now the previous assumption  $S_R \supseteq [(Ob(C_{MRC})])$  by positing explicitly  $S_R \supseteq [(Ob(C_{MRC})+Ar(C_{MRC})])]$ .

#### The axioms C1 and C2

They seem to raise no problems.

#### Representation of the evolving contents of the CMRC-sites

The theory of categories does not specify a general modality for expressing individualizations *inside* an object from Ob(C), as being the source or the target of an arrow tied with that object. While MRC involves such individualizations quite essentially. So we construct the necessary individualizations as follows.

We consider only the operational arrows  $G \rightarrow$  and  $V_g \rightarrow$  that form the hard core of C<sub>MRC</sub>. This will suffice.

Each arrow  $G \rightarrow$  can be labelled by a pair of indexes  $(R_G, \mathfrak{a}_G)$  defining respectively its local start inside  $S_R$  (by the "spot"  $R_G$  where G has to be applied (D4)) and the element  $\mathfrak{a}_G$ from the evolving set { $\mathfrak{a}$ } that constitutes the content of  $S_{\mathfrak{a}}$  by the creation of which the considered  $G \rightarrow$  arrow ends. So for each definite arrow  $G \rightarrow$  we shall write  $(R_G, \mathfrak{a}_G) \rightarrow$ , which distinguishes it from any other arrow  $G \rightarrow$ . Thereby the set { $(R_G, \mathfrak{a}_G) \rightarrow$ } associated to the generation arrows  $G \rightarrow$ , itself also an evolving set, is now *connected with the evolving inner contents of the two sites*  $S_R$  *and*  $S_{\alpha}$  represented, respectively, by the evolving sets {  $R_G$  } and { $\alpha_G$ }. This connection can be then organized more by putting mutually compatible structures on the sets { $R_G$ }, { $\alpha_G$ } and {( $R_G, \alpha_G$ ) $\rightarrow$ } (physical operations of object-entity generation are subject to the frame-principle P8, which requires a convenient extension of the principle P10 of mutual exclusion, to operations of objectentity generation also).

*Mutuatis mutandis* one can connect in a similar way each definite processual arrow  $V_g \rightarrow$ , with a "pair" of indexes ( $\alpha_G$ , {gk}), k=1,2,..., by re-writing ( $\alpha_G$ , {gk}) $\rightarrow$ , k=1,2,... where k takes on a unique value if the attempted descriptional process reveals an individual stability, or a whole set of different values if it reveals a probabilistic stability ((D5.1),  $\pi$ 12,  $\pi$ 13, D14). In ( $\alpha_G$ , {gk}) the index  $\alpha_G$  defines the element from the discrete evolving content of the source-site  $S_{\mathfrak{C}}$  where ( $\alpha_G$ , {gk}) $\rightarrow$  begins, and {gk}, k=1,2,... defines the element from the discrete evolving content of  $S_D$  by the creation of which ( $\alpha_G$ , {gk}) $\rightarrow$  ends. So the (evolving) set {( $\alpha_G$ , {gk}) $\rightarrow$ } of aspect-view arrows is connected with the evolving content of the sites  $S_{\mathfrak{C}}$  and  $S_D$ , expressed respectively by the sets { $\alpha_G$ } and {gk} (where {gk}, k=1,2,..., g fixed, amounts to the description of  $\alpha_G$  *via*  $V_g$ , which is an element from {D}). The connection between the evolving sets { $\alpha_G$ }, {( $\alpha_G$ , {gk}) $\rightarrow$ } and {D} can be then organized more, by putting on these sets mutually compatible structures obeying all the MRC-requirements and furthermore conveniently reflecting the particular considered class of descriptional processes (the nature presupposed for the object-entities and the aspect-view-examinations).

The procedure can be extended to the class of arrows V $\rightarrow$ : in consequence of D5.2 each *definite* V $\rightarrow$  arrow is a *set* of arrows {( $\alpha_G$ , {gk}) $\rightarrow$ , k=1,2,...}, g=1,2,...m, m finite.

Then a relative description D/G, $\alpha_G$ ,V/ from MRC becomes in C<sub>MRC</sub>. a completedescription-arrow [CD $\rightarrow$ =CD=d $\rightarrow$ oDA $\rightarrow$ oDAa $\rightarrow$ oG $\rightarrow$ oV $\rightarrow$ ] where G $\rightarrow$ oV $\rightarrow$  is indexed: (R<sub>G</sub>, $\alpha_G$ ) $\rightarrow$  o ( $\alpha_G$ , {gk}) $\rightarrow$ , k=1,2,...}, g=1,2,...m, m finite

## C<sub>MRC</sub> versus quantum mechanics

We consider the Hilbert-Dirac formalism of quantum mechanics. The Hilbert-space H of the state-ket-vectors  $|\psi\rangle$  of the studied microsystem corresponds to the C<sub>MRC</sub>-site S<sub>ce</sub>

where are lodged mathematical representations of the considered class of object-entities. The set { $|\psi>$ } of state-ket-vectors  $|\psi>$  from H corresponds to the evolving set { $\infty_G$ } from S<sub> $\infty$ </sub>. The vector-space structure assigned in quantum mechanics to { $|\psi>$ } is a particular feature entailed by the principle of superposition posited for quantum states, a principle justified by the wave-like features manifested by what is called quantum states. So in general such a structure has no semantical counterpart, so it will have to be dropped.

The  $C_{MRC}$  generation arrows  $(R_G, \alpha_G) \rightarrow$  have no **general** correspondent in the quantum mechanical formalism: they are represented only in the particular case of microstate-generation by a measurement process.

This is a striking lacuna (which is suppressed in meta[quantum mechanics)].

The quantum mechanical (in general) non-commuting linear differential "dynamical" operators defined on H correspond to the C<sub>MRC</sub>-aspect-view arrows ( $\alpha_G, \{gk\}$ ) $\rightarrow$ , k=1,2,....

The quantum mechanical representation of a state-ket  $|\psi\rangle$  with respect to the basis of eigenvectors introduced by a given quantum mechanical operator A, namely as a columnmatrix of which the elements are calculated with the help of  $|\psi\rangle$  and the considered eigenvectors, corresponds to a *basic transferred* description D(0)/G(0), $\alpha$ (0), $V_g$ (0)/ from S<sub>D</sub> created for a basic object-entity  $\alpha$ (0) by a basic aspect-view-arrow ( $\alpha$ G,{gk})) $\rightarrow$ , k=1,2,....).

The set of all the column-matrix representations of a given state-ket  $|\psi\rangle$  with respect to all the bases of eigenvectors introduced by all the quantum mechanical dynamical operators, corresponds in C<sub>MRC</sub> to a complete-description-arrow

$$CD \rightarrow = CD = d \rightarrow oDA \rightarrow oDAa \rightarrow oG \rightarrow oV \rightarrow$$

(with G $\rightarrow$ oV $\rightarrow$  indexed: (R<sub>G</sub>, $\alpha_G$ ) $\rightarrow$ o( $\alpha_G$ , {gk}) $\rightarrow$ , k=1,2,...}, g=1,2,...m, m finite).

So it will be possible to attempt a systematic transposition of the Hilbert-Dirac formulation of quantum mechanics, in terms of the theory of categories, *via* MRC with its central concept of basic transferred description.

It is of course obvious from the start on that the explicit  $C_{MRC}$ -representations of reality and of the consciousness-functionings have no correspondent in quantum mechanics

where not even the actions of object-entity generation are represented mathematically, nor are they at least conceptually and verbally clearly distinguished from the *qualifying* actions *via* measurements. By comparison with  $C_{MRC}$  quantum mechanics appears as flawed by very flattening lacunae.

Nevertheless, once the main relations  $C_{MRC}$ -(quantum mechanics) have been established, the quantum mechanical formalism becomes a precious guide for a subsequent development of  $C_{MRC}$  (any non-necessary restriction suggested by the – particular – case of quantum mechanics having to be carefully avoided). One first important step in the mentioned direction will be the identification of the individualized *MRC-meaning* of Dirac's dual space of linear functionals defined on the Hilbert space of state-ket-vectors, and of the various sorts of scalar products from the Hilbert-Dirac formulation of quantum mechanics. Then the  $C_{MRC}$ -transposition of these, as well as the *individualized*  $C_{MRC}$ -transposition, will have to be conveniently achieved.

### IV.4.4. Concluding comment on CMRC

The outline indicated above is no more than a sketch that needs development. For instance, the condition  $S_R \supset [(Ob(C_{MRC})+Ar(C_{MRC}))]$  imposed by MRC entails reflexive characters that might raise difficult syntactical problems connected with the definition of the categorial concept of a sub-object.

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