

**PRINCIPLES OF
A SECOND QUANTUM MECHANICS**
rooted in factuality via computational assistance¹

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Abstract

This work is not a ‘reinterpretation’ of the nowadays Hilbert-Dirac quantum mechanics, QM_{HD} . It offers the principles of a new representation of microstates, called ‘a second quantum mechanics’ and denoted $QM2$, freed of ‘interpretation problems’ and fully reconstructed conceptually and formally in its structural principles.

First a *qualitative but formalized* representation of microstates is developed quite independently of the quantum mechanical formalism and outside it, under *exclusively [epistemological-operational-methodological] constraints*. This is called ‘infra-quantum mechanics’ and is denoted IQM . The specific aim of this representation is to constitute a *reference-and-embedding-structure* directly rooted into a-conceptual physical reality, able to insure comparability with QM_{HD} and thus to endow with criteria for estimating its adequacy from defined points of view and in defined terms. IQM is the very first realization of a new basic kind of scientific discipline.

By systematic reference to IQM are then first worked out preliminary critical examinations of several aspects of the Hilbert-Dirac formulation of QM that play a key role in the nowadays quantum mechanical representation of measurements. These reveal that *nowadays QM_{HD} is devoid of any general formal representation of individual, physical, actual ‘microstates’ and of individual operations on these that bring them from still a-conceptual physical factuality, into scientific knowledge*. For free microstates only predictive statistics of numbers are represented, posited to be results of quantum measurements but determined – practically – directly and exclusively by calculi. But the quantum measurements themselves - that somehow contain individual features - are devoid of an acceptable representation: *QM_{HD} is simply devoid of a theory of quantum measurements*.

This lacuna is entirely compensated for free microstates that do not involve possibility of quantum fields. But for free microstates that do involve quantum fields it is found that, not only the representation of the acts of measurement is defective but furthermore the QM_{HD} predictions themselves of results of measurements of the momentum observable *are not verifiable*. A way is defined to confront this situation by recourse to the de Broglie-Bohm approach. *The possibility of the mentioned way is conditioned by the – very likely – success of a well-defined experiment*.

In order to close the conceptual exploration, it is then admitted by hypothesis that the mentioned experiment has been realized and has succeeded.  this basis is delineated a theory of quantum measurements that takes into account *all* the classes of microst mutually distinguished in this work, free without quantum field, or free with quantum fields, or bound. In **this** **predictional** phase this theory can make use of Schrödinger equation of evolution whenever this equation can effectively be defined and solved. But the purely mathematical formalism *is quite generally duplicated by a factual-formal, computationally assisted procedure of establishing the predictions*.

Around this core are then finally sketched out the general principles (source-domains, postulates, and inner structure) of a second quantum mechanics, $QM2$, where all the major problems raised by QM_{HD} vanish by construction.

$QM2$ is an intimate synthesis between IQM , QM_{HD} , and basic elements from the Broglie-Bohm representation that are explicitly drawn into the realm of operational consensual observation.

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Dedication

This work is dedicated to Louis de Broglie
whose deep unconventional work has founded Quantum Mechanics
and 90 years later permits to re-found it.

Some useful notational conventions.

1. G is an operation of generation.
2. Different operations of generation will have different subindexes (e.g. G_1 and G_2).
3. When several operations of generation G are realized (for statistical purposes for example), the n -th realization will be referred by $G(n)$; A composed operation of generation, where G_1 and G_2 are virtual, will be denoted by $\mathbf{G}(G_1, G_2)$.
4. Generating by a unique operation of generation G one microstate that involves several microsystems S_1, \dots, S_k will be denoted by $G[S_1, \dots, S_n]$.
5. The realization, at a time t_0 , of an operation of generation G that is posited to produce a microstate ms_G , followed by a time interval $(t - t_0)$ during which ms_G evolves in definite external conditions denoted by EC , is equivalent to a new operation of generation denoted by $G^{(t)}(G, EC, t - t_0)$ that generates a microstate denoted by $ms_{G^{(t)}}$.

Then, for example: $G_2^{(t)}[S_1, \dots, S_7](5)$ refers the 5th realization of the operation of gen G_2 , for creating one microstate of 7 microsystems (called S_1, \dots, S_7), followed by any measurement after t units of time (from its generation at t_0).

6. Quantities are indicated by A, B, \dots, X, Y, \dots , operators that measure these quantities are represented by bold letters, $\mathbf{A}, \mathbf{B}, \dots, \mathbf{X}, \mathbf{Y}, \dots$, and apparatuses that measure these quantities are represented by caligraphic letter $\mathcal{A}, \mathcal{B}, \dots, \mathcal{X}, \mathcal{Y}, \dots$.
7. For a given qualification X , we denote by $X_j, j = 1, \dots, J$ is possible qualification values.

GENERAL INTRODUCTION

The first attempts at a representation of microscopic physical entities started in terms of usual 'objects' endowed with delimited spatial volumes. Therefrom classical models and ways of reasoning were more and more deeply lowered into the domain of small space-time dimensions. This process however has come to a clear crisis around 1900: the connections with classical physics ceased being compatible with the experimentally established facts. Therefore Bohr and Planck introduced *non*-classical but ad hoc principles.

And then, de Broglie's model fractured the evolution: It changed the *origin* on the vertical that connects knowledge of macroscopic physical entities, to knowledge concerning microscopic entities. Indeed de Broglie's model is placed just upon the extreme frontier between the microscopic still *a-conceptual* factual physical reality, and the realm of the already conceptualized. This instilled a necessity to also *reverse* the direction and the nature of the actions of construction of knowledge along the mentioned vertical. Instead of continuing to try to guess top-down starting from the classical level and advancing downwards into the realm of microscopic space-time dimensions via mental procedures trussed up unconsciously into inertial strings, there appeared a new tendency to construct down-top by a sort of conceptual climb in the dark along strictly operational-observational-formal requirements. This inversion involved fundamental changes in the process of conceptualization. And this induced obscure mental confrontations between ancestral habits of thought and new procedures still devoid of definite contours and unnamed, but of which the imperative presence was strikingly sensed. The thought about physical reality was undergoing mutation.

The mathematical representations of Schrödinger and Heisenberg led to impressing successes and these neutralized the conceptual disquietudes. Bohr, struck by the radically new characters of the emerging theory of the presence of which he was strongly aware but of which the source and nature withstood identification inside his mind, tried to protect these characters maximally by a preventive interdiction of any model of a microsystem. He founded this interdiction upon a general philosophical requirement of a strictly positivistic attitude in science. This was an over-dimensioned interdiction that indeed protected the development of the emerging Schrödinger-Heisenberg representation, and later its mutation into the nowadays Hilbert-Dirac reformulation. But on the other hand this interdiction inhibited heavily a genuine understanding of the formalism. Indeed de Broglie's model, though rejected by Bohr's positivistic diktat, remained quite essentially involved in the quantum mechanical formalism. But it remained there in an only hidden way, immobilized in atrophy by absence of a declared and definite conceptual status. So up to this very day it keeps acting inside the formalism without being exposed to overt control and optimization. And much more generally, the formalism occults many conceptual, factual, and operational features that are quite basically involved. Therefore, since 90 years the representation of microstates nourishes endless questioning and groping that systematically pulverizes against a paradoxical 'negative' dike of absence of definite criteria for estimating the adequacy of the mathematical representation. The formalism itself proliferated densely, and it still does so, but at its core there subsists a deleterious semantic magma that entails an urgent need of overtly organized *meaning*.

What lacks – dramatically – is a structure of *insertion-and-reference constructed independently of the quantum mechanical formalism and outside it*, that offer an explicit, clear and thorough understanding of the non-classical specificities required by a human representation of non-perceptible microscopic entities.

Only this could permit an exhaustive and coherent critical examination of the way in which this formalism manages to signify.

In the first part of this work I construct such a structure of insertion-and-reference, the first one of this kind.

In the second part, by reference to the constructed structure, I identify the main lacunae from nowadays Quantum Mechanics, explicate the model of microstates that does work inside it, and introduce some essential semantical elucidations.

In the third part I outline the principles of a second quantum mechanics freed of interpretation problems via an explicit control of semantic-syntactic consistency.

Part I

Infra-Quantum Mechanics: A qualitative but formalized structure of reference built outside the Quantum Mechanical formalism

Introduction to part I

" In order to reach the truth, for once in the life one has to free oneself from all the received opinions and to reconstruct the whole system of knowledge, starting from the foundation".
René Descartes

A human being who wants to construct knowledge concerning ‘microstates’ makes use of physical entities to which he associates this denomination, of instruments and operations, and he introduces representational *aims* and corresponding *methods of acting and thinking*. Thereby the human observer introduces severe constraints that structure the process of construction of knowledge. It is not possible to preserve this process from such constraints, they are precisely what ‘forms’ it. Nor is it possible to eliminate a posteriori the effects of these constrains from the constructed knowledge, these are essentially incorporated to the achieved form to which they have led. The constructed knowledge remains irrepressibly relative to its whole genesis. So, if the observer-developer wants to stay in control of the knowledge that he has generated, to be able to understand and to freely optimize it – he has to be thoroughly aware of the epistemological-operational-methodological weft of this knowledge.

In what follows – quite independently of the mathematical formalism of quantum mechanics – the necessary and sufficient features of *a procedure that is appropriate for creating ‘scientific’ knowledge on microstates* will be structured in qualitative but explicit terms, formalized² and finite (effective),. The result is called in advance *infra-(quantum mechanics)* and is denoted *IQM*.

In order to insure for the exposition self-sufficiency and controllable inner coherence the trivialities will not only be implied, they will be spelled out, insisently. This will permit a direct and contrasted perception of the specificities involved by the cognitive situation in which a human being places himself if his aim is to create knowledge on microstates, not on perceivable entities and changes of these. Historically this aim is very new. And only when all the involved cognitive specificities and the ways to *deal* with them will be known explicitly, will the mathematical structure of nowadays quantum mechanics stay openly face to face with the *meanings* that it should express.

I would like to convey to the reader from the start what follows.

Nothing – throughout the construction elaborated below – is conceived as an assertion of ‘objective intrinsic factual *truth*’. I just figure out a succession of *methodological* steps, each one of which, in order to instil intelligibility, is *tied* deliberately to the structure of our classical thought-and-languages that have emerged and settled in our minds by interactions with entities that *are* perceived. But on the other hand, each one of the mentioned steps transgresses our classical forms of thought by definite features commanded by the novelty of the aim to establish how to proceed in order to create ‘scientific’ knowledge concerning a limiting sort of entities that not only cannot be perceived, but furthermore are drawn out – directly – from a still a-conceptual physical factuality. *IQM* is the global

²We employ the word ‘formalized’ in the sense that all the specific basic terms are endowed with explicit and finite definitions, the posits are explicitly stated, and the elements thus introduced are constructed as general and syntactically related void loci for receiving in them unspecified particular semantic data.

procedural whole that is obtained when these methodological steps are put together. It is a *procedural reference-and-insertion-structure*.

The main aim of this work is to construct a mathematically represented intelligible knowledge – *QM2*– on how to *predict-and-verify concerning microstates*. But I begin by constructing a reference structure because, *IQM* because I think that this is an unavoidable pre-condition for reaching the mentioned main aim.

I am convinced that ‘factual truth on how intrinsically is’ what we posit to exist outside ourselves, transcends *scientific* knowledge quite essentially, radically and definitively. The sequence of words ‘factual truth on how intrinsically is a physical entity’ is meaningless. It designates a vicious circle that is so vast and drawn on a ground so irregular that we do not make out its contour and therefore we ignore the imprisonment inside it. Any search for entirely ‘neutral’, objective descriptions of ‘how this or that fragment of physical reality truly is in itself’, manifests a naive illusory sort of realism that modern microphysics irrepressibly dissolves. Any scientific knowledge is communicable and consensual description, and any *description* is marked in a non-separable and non-removable way, as much by *what* is described as by *how* the description has been worked out, via what *aims*, constraints, choices. Only non-analysable dense lumps of such what-s and how-s can emerge inside what we call knowledge. These lumps however can be subjected to an explicit *genetic* organization, optimized with respect to definite methodological requirements of coherence and intelligibility. When this is explicitly achieved for some domain of reality, a corresponding independent "infra"-discipline should be brought forth that works like an insertion-and-reference structure for later constructing also a mathematical scientific representation for the considered domain of reality. *IQM* is the very first such "infra"-discipline ever constructed and *QM2* is the corresponding mathematical representation ³.

³ While the general Method of Relativized Conceptualization (MRC) ([26, 27, 28]) yields the previously elaborated general framework for constructing various such infra-disciplines in a unified way.

1 THE FIRST GERM OF A DESCRIPTION OF A MICROSTATE

1.1 ‘Definition’ of a microstate

In agreement with Dirac we distinguish between stable characteristics assigned to a micro-*system* (mass, spin, etc.), and unstable dynamical characteristics assigned to a micro-*state* (position, momentum, etc.). So in this work we consider *microstates*: so far just a verbal sign to be used like a sort of coordinate of where the attention is to be focalized: Each considered microstate is presupposed to be a physical thing that is entirely unknown as to all its specificities.

A basic question. In current languages and in classical grammars and logic, an object-to-be-qualified is usually supposed to pre-exist. Its definition is realized by use of grammatical predicates (“bring me the brown thing from that drawer”) or even by just pointing toward it. But how can a non-perceivable and unknown *micro-state* be introduced as what-is-to-be-studied? How can it be defined in some stable way so as to be kept available for further cognitive action upon it, when it is not even known whether it pre-exists, nor where and when?

Obviously as soon as it is *named* an unknown microstate is already a priori conceived to possess some minimal class-characters. But in order to become a possible subject of *factual* study, it has to be factually *generated as such* via some definite, macroscopically controllable physical *operation of generation* that – accordingly to some previously established knowledge – should produce it on some specifiable space-time support: If not we cannot even think of it; so a fortiori we cannot study it. Let us then denote such an operation of generation by G , and by ms_G the physical and individual microstate produced by G .

The aim of constructing scientific knowledge concerning ms_G requires possibility of verifications. So *repeatability* of G and of its result ms_G are unavoidable pre-conditions. But how can we know that when G is repeated the result denoted ms_G is systematically the ‘same’? How can we know that G itself emerges ‘the same’? Well, *we cannot know this a priori nor can we insure it*, because ‘ G ’ can be inter-subjectively specified only by some finite definition that, quite essentially, is unable to constrain the whole factual singularity of any realized replica of the operation ‘ G ’. (Umberto Eco has said that as soon as we speak or write we conceptualize and thereby we quit and lose irreversibly the infinite singularity possessed by any realized factual entity). However giving up because of this, the project of establishing how one can create knowledge on microstates, would be an unacceptable weakness from the part of a human mind. This difficulty has to be dominated. Therefore we introduce a first **methodological decision**, *MD1*. Namely:

MD1. We posit that each time that an operation of generation of a microstate denoted G , is realized in agreement with a definition expressed in terms of macroscopically controlled parameters, this operation comes out the ‘same’, and that *what* emerges in consequence of this realization of G is a *singular specimen* of something that is also each time the ‘same’. This same something we label by ‘ ms_G ’ and we call it ‘the microstate corresponding to G ’, *whatever be the a priori unknown factual singularities of its specimens*. This amounts to denote $ms_G = \{\sigma(ms_G)\}$ and to posit by a choice of language a one-to-one and repeatable relation between a conceptual-factual operation and a class :

$$G \Leftrightarrow ms_G \tag{1.1}$$

This statement introduces ‘ ms_G ’ in a way that is purely *factual-operational*. That what is denoted ms_G is still *void* of any specified semantic content asserted for ms_G *itself* and thereby circumvents the full absence of preexisting knowledge on the involved specimen of ms_G . Nevertheless this *suffices* as a ground for just *starting* a subsequent experimental research on ‘a microstate’. And here our local aim is precisely and exclusively this. It is an essentially provisional aim. Indeed *MD1* acts as a *methodological provisional definition*. It endows for the moment with the crucial possibility to speak, to think and to act for trying to create later some definite genuine semantic content – some knowledge – tied with that toward which points the symbol ‘ ms_G ’ introduced in (1.1).

Mutation of the classical concept of ‘definition’. By this very first step the construction attempted here has already imposed upon us a quite notable egress from the domain of classical thinking. The microstate ms_G to be studied has been brought in as a void locus for an as yet *entirely* unknown physical factuality. It is true that this void locus is conceived in advance to belong to a certain class called ‘microstates’ that, on the basis of previously constructed knowledge is admitted to point toward something that is posited to be conceivable and possible to be brought into existence, and has been named. These however are no more than minimal instrumental pre-requisites for just connecting as yet non-specified *subsequent* cognitive actions and the knowledge entailed by these, with previously organized thought, language, and knowledge. But the connective strings involved by (1.1) assert nothing on – specifically – the particular given outcome $\sigma(ms_G)$ of the microstate ms_G produced by each one realization of the operation of generation G . So *MD1 places us systematically, repeatedly, on a sort of local platform of strictly zero-level of specific knowledge on the considered singular outcome*⁴ of what is denoted ms_G . This is entirely new with respect to the classical concept of definition where usually already known qualifications of the defined entity yield support for new qualifications of this same entity. The direct perceptibility permits this comfortable ellipsis where the operation of generation ‘ G ’ is absorbed. But for microstates this is not possible *originally*. So in order to deal with this limiting case, the action of ‘defining’, in the classical sense, has been explicitly split into a succession of steps. A preliminary step that introduces a void conceptual receptacle for semantic content, and subsequent steps that will have to construct a way to pour into this receptacle specific singular semantic contents that will constitute a posteriori a factual definition of the class denoted in advance ‘ ms_G ’. Thereby the cognitive procedures can be *started* on the basis of a formal, methodological and consensual nature, instead of a factually perceived nature. And in this way we have avoided stagnation in the circumstance that a microstate is a conceived physical entity that cannot be qualified via direct perception⁵.

Composed operations of generation: a principle of composition. From its start, the study of microstates has brought into evidence a class of microstates that have been called ‘(auto)-interference-states’ and that played a founding role in the emergence of quantum mechanics (the paradigmatic case is Young’s two slits experiment). The process of generation of an interference-state permits to distinguish at least two operations of generation G_1 and G_2 that are ‘involved’, but in the following very peculiar sense: Each one of these two operations *can* be produced separately, in which case two different corresponding microstates ms_{G_1} and ms_{G_2} emerge. But when G_1 and G_2 are ‘composed’ into only *one* operation – let us denote it $\mathbf{G}(G_1, G_2)$ ⁶ – accordingly to (1.1) there

⁴ The expression ‘one outcome of ms_G ’ is to be understood only as ‘the microstate tied with one given realization of the operation G ’ (our time-and-space where we are imprisoned forces us to distinguish between the realizations). So, if we introduce a numerical indexation in a sequence of N successive realizations of G by writing $G(1), G(2), \dots, G(n), \dots, G(n)$, the result of $G(2)$ is the particular outcome of ms_G that is denoted $ms_{G(2)}$.

⁵ By the posit (1.1) the construction we enter upon quits from the start the realm of classical representations and acquires a formal deductive character. This character is comparable to that imparted to the natural syllogistic of Aristotle via the fact that each syllogism is founded – by construction – upon a universal ‘major hypothesis’ that cannot be verified but closes the explored domain of facts by rendering it – hypothetically – absolute, whereby it permits to reach consequences that can be expressed as certainties; which too is just a procedure for generating a dynamic of progressive investigation. (One can however keep into full evidence the hypothetical character by choosing to express a syllogism in the form of an implication).

⁶ This notation stresses that only one operation of generation has been effectively achieved by composing other operations of generation that could have been achieved separately but have not been separately achieved.

emerges only *one* corresponding microstate $ms_{\mathbf{G}(G_1, G_2)}$ that manifests ‘auto-interference effects’. On this factual basis tied with the just indicated way of speaking, we introduce here an only *qualitative* – but nevertheless general – ‘*principle of composition of operations of generation*’ according to which *certain operations of generation of a microstate, two or more such operations – deliberately produced by human researchers or brought forth by natural processes – can ‘compose’ while acting upon one preliminary unspecified microstate, so as to generate together ‘one’ microstate-to-be-studied, in the sense of MD1*. When this happens we shall speak of one microstate $ms_{\mathbf{G}(G_1, G_2, \dots, G_k)}$ with composed operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$ ⁷. When this does not happen, for contrast or precision we can sometimes speak of a ‘simple’ operation of generation.

Though its global domain of applicability is still only very feebly defined, the principle of composition of operations of generation of a microstate posited above will entail most essential consequences in the second part of this work.

1.2 Qualification - inside IQM- of one outcome of a microstate

Classical qualification. Inside the classical thinking an act of qualification involves more or less explicitly a *genus-differentia* structure. The *genus* can be conceived as a *semantic dimension* (or space) and the *differentia* can be regarded as *values* from a *spectrum of values* carried by this semantic dimension. The spectrum can be numerical or not, ordered or not, and it can be specified by material samples or otherwise. Let us denote the semantic dimension by X and by X_j , $j = 1, 2, \dots, J$, the values from the spectrum posited to be carried by X (X can be ‘colour’, for instance, and then the spectrum of values X_j can be posited to consist of a number of definite colours $\{red, green, blue, etc.\}$; for effectiveness we consider only *finite spectra*). The semantic dimension and the spectrum of values carried by it are currently imagined inside classical logic and grammars to somehow pre-exist. But here, even for classical acts of qualification, we conceive them as being in general freely constructed by the human observer who conceptualizes accordingly to his local qualification-aims and under the general cognitive aims that are acting like global a priori constraints. According to this classical conception there usually exists some possibility to estimate what value X_j of X is found for a given entity-to-be-qualified when it is examined via X : this is a way of imagining more or less explicitly a sort of act of ‘measurement-interaction’ between some ‘measurement apparatus’ $\mathcal{A}(X)$ – biological or not – and the entity to be qualified. Let us denote it $MesX$. The result X_j of an act $MesX$, when perceived by the observer, becomes a knowledge concerning the examined entity: by the definition of the concept, ‘knowledge’ of some thing is qualification of this thing, since what is not qualified in any way is not known.

The operation $MesX$ cannot be defined otherwise than by some finite specified set of controllable parameters. Unavoidably these are transcended by circumstances that cannot be conceived a priori. So again, like in the case of (1.1), there is no other way than just *admit* that *all* the realizations of $MesX$ are the ‘same’ (which means the *same with respect to the defining parameters*⁸).

When the estimation of a value X_j of a quantity X assigned to an ‘object’ in the classical sense is performed directly via a human biological sensorial apparatus, the ‘measurement-interaction’ generates in the observer’s mind a *quale*, a strictly subjective perception of a definite particular ‘quality’ that cannot be *described* but of which the subjective existence can usually be communicated by words, gestures, or other signs that label it consensually in connection with its exterior source that is publicly perceivable (the classical ‘object’)⁹. Let us denote globally this classical coding procedure by

⁷ We do not try to specify the conditions that restrict the possibility of such a composition (in particular, the space-time conditions) though such conditions do certainly exist. Nor do we try to specify some limit to the possible number of composed operations of generation. These are features that are still unexplored from both a factual and a conceptual point of view because inside nowadays quantum mechanics – together with the concept of operation G of generation of a microstate itself – they remain hidden beneath what is mathematically expressed.

⁸ Suppositions of this kind are made everywhere inside science.

⁹ For instance – as it is well known – one experiences the feeling of a quality that he calls ‘red’ and says the word ‘red’ while showing the source to which he connects the quality (say a flower). Thereby that quale acquires by learning a common inter-subjective verbal labelling that points inside each given mind toward a strictly subjective non-communicable quale. So in classical circumstances each very usually arising quale acquires an inter-subjective labelling.

cod.proc(X_j). So, in short, a classical *grid of qualification* (gq) is a structure that can be symbolized as

$$gq[X, X_j, MesX, cod.proc(X_j)] \quad (1.2)$$

Qualification of one specimen of a microstate. But how can a specimen of a microstate ms_G be qualified? Obviously, the operation of generation G must be followed *immediately* by a measurement interaction $MesX$ realized inside the space-time neighbourhood of the space-time support supposed for the operation G . Indeed each outcome of ms_G is by definition a *dynamical state*, a changing physical entity, an entirely unknown changing entity but a *physical* entity. So the human observer, though any specific knowledge of this changing entity is still lacking in his mind, assigns it irrepressibly some space-time support, and to this ¹⁰ he *can* assign an only vague location, on the basis of previously constructed knowledge and of assumptions of continuity ¹¹. But because of the dynamical character assigned by definition to a ‘microstate’ this location cannot be supposed to last. Furthermore, usually $MesX$ destroys the involved outcome of ms_G . In short:

A whole succession [G.MesX] has to be realized for achieving each one act of measurement.

This is well known, but usually it is not explicitly mentioned. Also, an act of measurement on a microstate necessarily requires a *non-biological* apparatus, and its result must consist of publicly observable marks. And so on. All these questions have been already discussed very much indeed and they have suffered heavy trivialization. But curiously, a huge gap seems to have been unanimously left open:

What procedures – *exactly* – permit to endow the publicly observable marks produced by a given sort of ‘measurement-interaction’ $MesX$ performed upon a given outcome of ms_G , with *meaning* in terms of a given value X_j

I call this the *coding problem*. We shall specify it more below.

The general content of a grid for mechanical qualification of a microstate, accepts the same general *form* (1.2) of a classical grid. But when a microstate is the object of qualification, the signs $X, X_j, MesX, cod.proc(X_j)$ point toward entities and circumstances that, with respect to the human observer, involve cognitive constraints radically *different* from those that act in the case of classical ‘mobiles’.

- That what is to be qualified – one outcome of [‘the micro-*state* ms_G ’ for which the one-to-one relation $G \Leftrightarrow ms_G$ is posited] – has been extracted by the operation G from, *directly*, the as yet a-conceptual physical reality. It is still radically unknown in its specificities. It is only posited to exist and is labelled.

- The involved individual outcome $\sigma(ms_G)$ of the studied microstate remains constantly and entirely non-perceptible *itself* by the observer. *Exclusively* groups $\{\mu_{k_X}\}$ of some marks $\{\mu_{k_X}\}$ – with $k_X = 1, 2, \dots, K_X$ and K_X an integer tied with X – can be observed on registering devices of some measurement apparatus $\mathcal{A}(X)$ when one act of measurement $MesX$ is performed on that outcome of ms_G .

- So no qualia directly tied with the entity-to-be-studied are ever triggered in the observer’s mind: *the observer gets no inner subjective feeling tied with the nature of X and with a specimen of the studied microstate ms_G .*

- The meaning of the registered group of marks $\{\mu_{k_X}\}$ – whatever it be – cannot be conceived in terms of some ‘property’ assignable to the involved specimen of the studied microstate *alone*. These marks, by construction, characterize exclusively the achieved measurement *interaction* as a whole, where both the involved specimen of the studied microstate *and* the utilized apparatus have been active, and in the considered cognitive situation no criteria are conceivable for separating inside $\{\mu_{k_X}\}$ the contributions from these two sources.

We will indicate globally the circumstances mentioned above by speaking of results of measurement-interactions that are radically *transferred* on the registering devices of apparatuses, in the form of marks that do not entail qualia tied with the studied microstate; marks that involve a still *meaning-less*, brute, and very first – primordial – qualification of the considered specimen of the studied microstate

¹⁰ We mention this in order to stress how human thinking comes in, irrepressibly.

¹¹Cf. [10]

that previously was strictly unknown in its specificities. In short, we shall speak of measurements with *transferred primordial* results. This concept will be thoroughly established in Section 2.2.

- And most important but much less claimed:

How are we to *conceive* an act of measurement-interaction $MesX$ in order to found the assertion that the registered marks do qualify the involved specimen of ms_G in terms of a given value X_j of a given measured quantity X ? *In what a way can the observable marks $\{\mu_{k_X}\}$ be brought to signify in terms of one definite value of X_j ?* How has an interaction $MesX$ to be conceived in order to mean something at all?

This specifies now more the coding-problem. It is a highly non-trivial problem. This problem is not addressed. And *it cannot be treated inside a mere reference-and-insertion-structure for any theory of microstates*, because no particular model of a microstate can be asserted inside a general pre-structure required by construction to define the features of any acceptable theory of microstates. But we just want to draw attention immediately and strongly upon the existence of this problem, because in the second part of this work it will play a central role. As for now, let us clearly note that whatever be the still unknown solution to the problems raised by the nowadays mathematical representation of measurements on microstates, in order to be able to specify what *sort* of measurement-interaction is convenient for measuring a given dynamical quantity X for a given sort of microstate, and to assign meaning in terms of a value X_j of the measured quantity X to the observable marks produced by each measurement-interaction, it is imperative to dispose of a general model of a microstate.

In the absence of *any* model no criteria can be formulated for specifying pertinent measurement-interactions and for assigning meaning to the observable results of these.

This refutes the very possibility to obey Bohr's positivistic interdiction of any model. Which in its turn proves that in fact this interdiction has never been taken into account. It has only enormously intimidated the minds of the physicists and pushed them into passivity.

1.3 Graphic representation of the definition and qualification of one outcome of a microstate

The global content of the two basic points exposed so far are summarized graphically in Fig. 1.1 where: $(\mathcal{A}(G))$: apparatus for producing the operation of generation G ; $\mathcal{A}(MesX)$: apparatus for producing the measurement interaction for the dynamical quantity X).

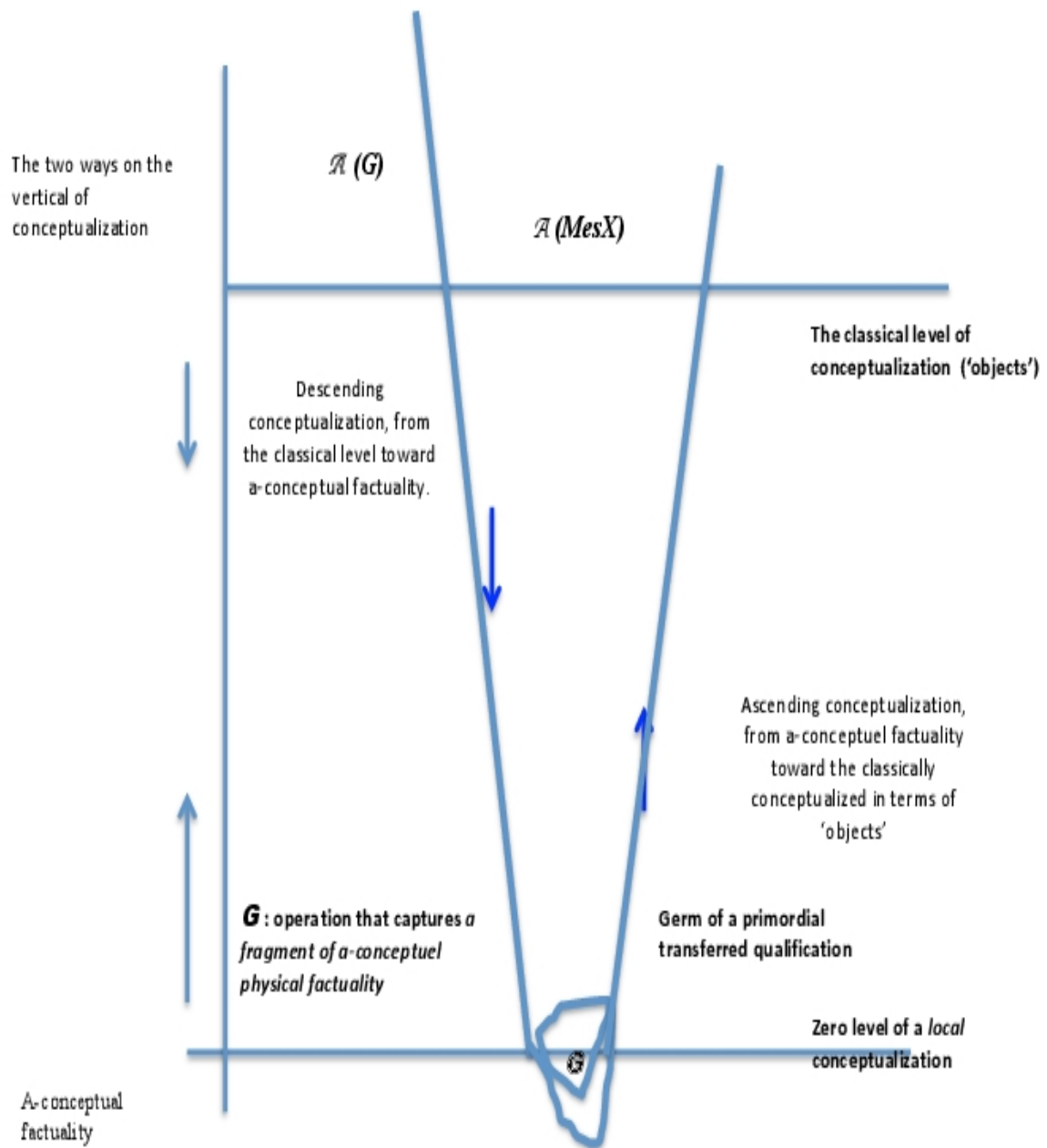


Figure 1.1: Summary of the definition and qualification of **one** outcome of a microstate

This figure introduces one chain

$$\begin{aligned}
 & [(G \Leftrightarrow ms_G) - [G.MesX] - \{\mu_{k_X}\} \text{ coded in terms of one } X_j] \\
 & k_X = 1, 2, \dots, K_X, j = 1, 2, \dots, J
 \end{aligned}
 \tag{1.3}$$

This chain concerns only one act of measurement-interaction performed upon one outcome of a specimen of the microstate ms_G defined in (1.1). It will be called a coding-measurement-succession. It is the very first germ of the representation of generation of knowledge on a microstate.

In what follows this germ is developed into the general representation of an exhaustive and stable deliberate procedure for creating scientific knowledge on microstates.

2 DESCRIPTION OF A FREE MICROSTATE AND OF THE HUMAN GENESIS OF IT

2.1 Preliminary construction of language: fundamental definitions

Consider a measurement-interaction involving a specimen of a microstate generated by an operation G . This produces observable marks that have to be translatable in terms of *one* value X_j of of what, exactly? Of only *one* measured dynamical quantity X for any sort of microstate, or possibly of several such quantities for *some* sorts of microstates? Shall we organize our concepts-and-language so as to imply that *one* act of measurement on only *one* outcome of the studied microstate me_G brings forth *necessarily* only *one* value X_j of each measured dynamical quantity X ? Or that it might imply necessarily (at most) only one set of ‘compatible’ quantities (which is not the same thing as in the preceding question)? What restrictions are we prepared to accept? The answers are not obvious *because the words ‘micro-state’ and ‘micro-system’ designate different concepts*. And one *micro-state* tied in the sense of (1.1) with one operation of generation G can involve one or *more* *micro-systems*. But how are we to count such ‘one’ and ‘two’? What presuppositions have to be incorporated in order to stay in agreement with the current ways of speaking that accompany the formal quantum mechanical writings as well as those from the theory of elementary particles? The expressions ‘*micro-system*’ and ‘*micro-state*’ are made use of inside a hazy conceptual zone. Introducing clear distinctions might come out to be a major advantage. Indeed it seems clear that to introduce a perfectly clear conceptual ground we must pre-organize explicitly our language ¹².

So consider a microstate.

- (a) It *necessarily* involves some *micro-system* (or several) of which it is the *micro-state*. Indeed the whole human conceptualization associates the concept of ‘state’ to some stable support that can be called ‘system(s)’. Violating such a fundamental slope of natural human conceptualization would uselessly waste energy.
- (b) *MD1* imposes $G \Leftrightarrow ms_G$ that is a basic posit of this approach. So by definition [one operation of generation G] produces [one microstate] while the number of the involved ‘systems’ is *not restricted* by (1.1).
- (c) Beneath the *current* ways of speaking and writing inside microphysics, we have discerned the following conceptual organization – more or less obscure and moving but *general*:

Definition [(micro-state) and (micro-system)]. One *micro-state* - according to 1.1 - the effect of one realization of an operation of generation G ; furthermore one definite *micro-system* is - accordingly to a basic implication structure of the language of physics - the concept delimited by the set of stable characters assigned to a definite set of mutually distinct microstates, namely the set of all microstates of that sort of system.

Definition [(one micro-system) and (one micro-state of one micro-system)]. Consider a *micro-state* that is such that *one* act of measurement accomplished upon *one* outcome of this micro-

¹² We embed structures of thought in structures of language and the structures of language are often beds of Procust. The aim of natural languages is to be contextual in order to maximally permit allusive, suggestive transmissions of meaning, or poetic connotations, etc. While the aim of a scientific language is to maximally avoid confusion. Nevertheless quantum mechanics, like all the mathematical theories of physics that are not strictly axiomatic, is imbedded in natural language. This induces much confusion. All the more so as the Hilbert-Dirac mathematical formalism does not represent the individual concepts that are involved, as it will appear in the second part of this work.

state can bring forth only one group $\{\mu_{k_X}\}$, $k_X = 1, 2, \dots, m$ of observable marks. We shall say that this micro-state brings in one micro-system and so we shall call it a *micro-state of one micro-system*.

Definition [one micro-state of n micro-systems]¹³. Consider now $n > 1$ micro-systems of a type of which we know that, for each one of them separately, it is possible to generate a micro-state in the sense of the preceding definition, which, if done, would lead to '**n micro-states of one micro-system**' in the sense of the preceding definition. But let $G[S_1, \dots, S_n]$ be only *one* operation of generation that, acting upon some physical initial support regarded as 'prime matter', has generated *one common* micro-state for the micro-systems S_1, \dots, S_n ¹⁴; or even, out of some initial substratum, has simultaneously generated altogether the n micro-systems themselves with their common *one-micro-state*. In both these cases we shall say that the microstate generated by $G[S_1, \dots, S_n]$ is one micro-state of n micro-systems and we shall denote it by $ms_{G[S_1, \dots, S_n]}$ ¹⁵.

Definition [complete measurement on one micro-state of n micro-systems]. One act of measurement performed on one outcome of a microstate $ms_{G[S_1, \dots, S_n]}$ of n micro-systems, can produce **at most** n distinct groups of observable marks signifying n observable values of dynamical quantities. An act of measurement that effectively realizes this maximal possibility will be called a *complete* act of measurement on one outcome of a micro-state $ms_{G[S_1, \dots, S_n]}$ of n micro-systems. We permit by definition the quantities X and the values X_j to which these n distinct groups of marks are tied, to be *either identical or different*.

Definition [incomplete measurement on one micro-state of n micro-systems]. One act of measurement accomplished upon one outcome of a microstate $ms_{G[S_1, \dots, S_n]}$ of n micro-systems, that produces less than n distinct groups of observable marks, will be called an *incomplete measurement on* $ms_{G[S_1, \dots, S_n]}$.

Finally, for self-sufficiency of this set of definition, we restate here telegraphically the definition from section 1.1 of a micro-state $ms_{\mathbf{G}(G_1, G_2, \dots, G_k)}$ generated by a composed operation of generation:

Definition [one micro-state generated by a composed operation of generation]. Consider *one* micro-state of either *one* micro-system or of $n > 1$ micro-systems S_1, \dots, S_n . If this micro-state has been generated by a composed operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$ in the sense defined in section 1.1 then we call it '*a microstate with composed operation of generation*'. When $n = 1$ we simply write $\mathbf{G}(G_1, G_2, \dots, G_k)$, and when $n > 1$ we write $\mathbf{G}(G_1, G_2, \dots, G_k)[S_1, \dots, S_n]$.

Definition [one 'bounded' micro-state of several micro-systems]. This is the usual verbal designation of the result of a '*natural* operation of generation' – accomplished in consequence of the 'laws of nature', before any human aim of investigation (like in the case of the natural realization of an atomic structure). But in principle it can be also thought of in terms of the result of a 'composed' operation of generation, so much more so as a bounded micro-state of several micro-systems manifests systematically interference effects.

With respect to free microstates, the features of bounded micro-states are *exceptional*, for at least two reasons. The first one is that a bounded state can pre-exist any desired investigation *just as it is supposed for classical 'objects'*; the second reason is that furthermore a bounded state can be assigned a *definite spatial delimitation*, again as in the case of a classical mobile. This might explain why the mathematical representation of bounded microstates has constituted the passage from classical physics to quantum mechanics. But in this work we want to explicate and stress the radical novelties imposed by the representation of microstates. So the bounded microstates with their quasi-classical characters will occupy a marginal position.

We shall mainly consider free microstates. These will permit to bring into evidence:

¹³ This definition is crucially fertile: it will permit to open a constructed door toward unifying fundamental quantum mechanics and the fields-theories.

¹⁴ This is the case, for instance for $n = 2$, where $G[S_1, S_2]$ consists of an interaction between two pre-existing elementary particles that brings forth 'a pair'.

¹⁵ The posit (1.1) entails that the uniqueness of the operation $G[S_1, \dots, S_n]$ is to be a priori conceived as a source of global observational specificities of each specimen of ' $ms_{G[S_1, \dots, S_n]}$ ' and so of ' $ms_{G[S_1, \dots, S_n]}$ ' itself (for instance of what is called 'auto-interference' aspects).

To what a degree the scientific representations can become deliberate constructions of which *the necessary and sufficient conditions of possibility depend strongly on the involved cognitive situation.*

This will permit a critical attitude with respect to the choice of a descriptive aim. More generally, this will modify our conception on scientific representation.

Throughout what follows the definitions from this section are adopted firmly, because we hold that they insure global coherence relatively to the implications carried by the language practised inside nowadays microphysics, as well as continuity with the basic principles of the classical language. If one contests the adequacy of some feature from these definitions then he must specify the reason why he does so and propose a better usage of words.

2.2 Primordial transferred description of a microstate

What follows in this section is formulated in terms that are valid for any microstate.

Preliminary requirements. Inside current thinking and speaking the qualifications are in general just asserted freely (this tree is big, today the air is cold, etc.). But a ‘scientific description’ is required to be communicable with precision, to be endowed with a consensual definition, and to be verifiable. These requirements entail constraints. In particular verifiability entails *repeatability* as well as the existence of some definite descriptive *invariant* with respect to repetitions that shall permit corresponding predictions. In the case of microstates these implications entail specific and basic consequences among which the following three are the most important:

1. Repeatability. In general a microstate-to-be-studied does not pre-exist in some known and attainable way, but has to be first generated in order to be able to create some knowledge on it; while furthermore in general the studied microstate is destroyed by a measurement interaction. So in general one cannot consider a measurement operation $MesX$ separately from an operation of generation G as it is currently done in the case of a ‘mobile’ in the classical sense. For each observation of a result, one has to realize a whole coding-measurement-succession $[G.MesX]$ (1.3). So when repetitions are necessary, *sequences of such successions $[G.MesX]$ have to be realized.*

2. Descriptive invariant: “factual (ϵ, δ, N_0) -probabilities”. Consider now the constraint of existence of some descriptive invariant with respect to repetitions of successions $[G.MesX]$. In general when a given succession $[G.MesX]$ is repeated one obtains *different* results X_j . This is an *experimental fact*, notwithstanding that in each succession $[G.MesX]$ the operations ‘ G ’ and ‘ $MesX$ ’ are both ‘the same’ with respect to the parameters that define them.

Thereby we come to an arm-wrestling between *IQM* and the classical requirements for scientific knowledge. Indeed: The current assertion that the micro-phenomena possess a “*primordial statistical character*” points toward precisely this fact – a physical-*cognitive* fact – that nowadays arises irrepressibly by repetition of the strictly **first** acts of qualification represented in the figure 1.1¹⁶. While in classical mechanics the basic laws are conceived and formulated as individual in-variants with respect to repetition.

Now, in order to succeed to formulate some sort of ‘law’ that permit predictions and verification of these, *some* invariant with respect to repetition *has* to be identified for also the case of a scientific study of microstates. And since one starts on an observational ground that has a primordially statistical character –with respect to knowledge –, the only possible observational invariant that could be asserted is the existence of a *primordially ‘probabilistic’* invariant for the global result of a big number N of repetitions of the succession $[G.MesX]$. But the classical concept of probability is founded upon the weak law of large numbers that is a *non-effective* mathematical concept, while here we have chosen to develop from the start a strictly effective approach (cf. the introduction to Part I). So we have to

¹⁶ In consequence of the development of nanotechnology this circumstance – as well as that one posited by Heisenberg’s principle of uncertainty – might change in the future. It is not a conceptual necessity, but a cognitive technical line, that evolves.

specify an effective concept of probability. For this we proceed as follows. From the weak theorem of large numbers

$$\forall j, \forall (\epsilon, \delta) (\exists N_0 : \forall N \geq N_0) \Rightarrow \mathbf{\Pi} [|n(e_j)/N - \pi(e_j)| \leq \epsilon] \geq (1 - \delta) \quad (2.1)$$

we extract explicitly the following well-known finite implication. Consider a universe of events $U = [e_1, e_2, \dots, e_J], j = 1, 2, \dots, J$, with J a finite integer. If the probability $\pi(e_j)$ of an event e_j is postulated to exist for any e_j , then 2.1 insures by construction that for any pair of two arbitrarily small real numbers (ϵ, δ) there exists an integer N_0 such that – for any $N \geq N_0$ and with an uncertainty not bigger than δ – the meta-probability $\mathbf{\Pi}$ of the event $|n(e_j)/N - \pi(e_j)| \leq \epsilon$, expressing that the relative frequency $n(e_j)/N$, observed for the event e_j inside a sequence of N events from U , does not differ from $\pi(e_j)$ by more than ϵ . This statement, with N_0 chosen freely, and the corresponding pair (ϵ, δ) will be considered in what follows to define the general **factual** and *finite* concept of an (ϵ, δ, N_0) -probability $\pi(e_j)$ of the event e_j ¹⁷. In our case U consists of the finite spectrum of values X_j assigned to X . And we make the strong assumption that the one-to-one relation $1.1 \text{ } ms_G \Leftrightarrow G$ together with the systematic repetition, for any X , of the corresponding succession $[G.MesX]$, are sufficient constraints for entailing ‘convergence toward an (ϵ, δ, N_0) -probability $\pi(X_j)$ ’, for any association between a chosen pair (ϵ, δ) and the relative frequency $n(X_j)/N$, with $N \geq N_0$, found for a value $X_j, j = 1, 2, \dots, J$ that is present inside the chosen qualification grid (1.2) $\text{gq}[X, X_j, MesX, cod.proc(X_j)]$ ¹⁸. In short, given a definite microstate $ms_G \Leftrightarrow G$, the stated assumption introduces for any couple of pairs $((G, X), (\epsilon, \delta))$ a corresponding (ϵ, δ, N_0) -probability law

$$\{(\epsilon, \delta, N_0)\text{-}\pi(X_j), j = 1, 2, \dots, J\} \quad (2.2)$$

3. ‘Compatibility of quantities’ versus ‘specific’ knowledge on a given microstate. The aim to construct a ‘description of a microstate ms_G ’ amounts in fact to the aim to substitute to the initial only formal and general definition (1.1) of this microstate via a posited one-to-one correspondence $G \Leftrightarrow ms_G$, a factual and verifiable definition of any particular microstate in terms of semantic contents that establish specific *knowledge* on this microstate *itself*. We want to convert the very first germ of knowledge on a given microstate ms_G defined in part I, into a stable, consensual and verifiable piece of specific knowledge on this particular entity. This should express a *factual specificity* of the considered microstate. But nothing entails that a probability law (2.2) established for this microstate ms_G relatively to only *one* dynamical quantity X , cannot be observed also for *another* microstate different from ms_G . It seems likely however that two probability laws (2.2) corresponding to two mutually different dynamical quantities X and $X' \neq X$ – considered conjointly – do already much more likely constitute an observational factual specificity associable to the particular considered microstate.

This draws now attention upon the *way* in which measurement operations of distinct dynamical quantities can be associated in order to reach an observable knowledge that is factually specific of the studied microstate:

Is it possible to subject *one* specimen of the studied microstate ms_G – simultaneously – to operations of measurement of two or several distinct dynamical quantities X defined for a microstate?

¹⁷ In [31] I have examined Kolmogorov’s non-effective, purely mathematical concept of probability and I have constructed in finite terms a corresponding concept of ‘factual and numerically specified probability law’: The abstract probability measure from a Kolmogorov probability space is not numerically specified; it is just posited as an existing void receptacle for numerical specifications of a certain ‘corresponding’ set of statistics, namely those that are factually obtained and manifest relative stabilities: I have shown that a ‘factual statistical-probabilistic law’ consists of a statistic that, with respect to repetition, is endowed with stability relatively to the triad $(\epsilon, \delta, N_0(\epsilon, \delta))$ denoted (ϵ, δ, N_0) , namely with improvement of the stability with respect to increases of N_0 and/or diminutions of ϵ and δ – which can be understood only if it is conceived to stem from a permanent whole of which we cannot acquire an integral perception, but only a fragmented one. In this sense **the abstract concept of probability is just a conceptual explanation of a set of factual consensual statistical stabilities**, or even a deliberate strategy for generating such a consensual stabilities on the basis of some presupposed but unknown factual invariant that is introduced with this aim. Concerning the conceptual status of a ‘statistic’, with respect to that of a ‘probability law’, there are huge confusions that last since centuries (and the researchers begin to grow conscious of this (Wasserstein & Lazar [2016], Leek & Penn, [2015],)

¹⁸ X_j being identified starting from a group of observable physical marks, via the utilized coding-procedure that inside *IQM* cannot be defined but is supposed to have been defined inside the employed theory of microstates.

Consider two distinct dynamical quantities X and $X' \neq X$ and *one* outcome of a microstate ms_G of one or *several* microsystems. Suppose that it is *possible* to specify for X and X' one *common* measurement-interaction with a *unique outcome of* ms_G . This involves that it is possible to achieve for X and X' one *common* factual measurement-interaction with *one* specimen of ms_G , so that only *one common* space-time support is covered by the operation and finishes by the registration of a *unique* group $\{\mu_{k_{XX'}}\}$, $k_{XX'} = 1, 2, \dots, m$ of brute observable marks, but out of which, *afterwards* are worked out two *conceptually distinct values* X_j and X'_j , that have to be assigned, respectively, to X and to $X' \neq X$. In other words, this means that X and to X' are mutually compatible in this sense that - in the considered circumstances - factually they are the same quantity, but conceptually they are distinguished from one another exclusively on an abstract level ¹⁹). In such a case we shall say that 'the dynamical quantities X and X' are compatible *with respect to the considered sort of microstate*'. ²⁰.

If on the contrary the considered procedure is *not* possible with respect to the considered pair of dynamical quantities X and X' (or more) we shall say that these are '*mutually incompatible quantities*', in the considered circumstances.

The concepts of compatibility or incompatibility of dynamical quantities that have been defined above are essentially *relative* to: the concept of one *individual outcome* (specimen) of the considered microstate; the sort of considered microstate (in the sense of the definitions from section 2.1); the considered set of quantities; *the available techniques for measuring*; the **model** of a microstate that is presupposed, that constantly plays the central role ²¹.

When X and X' are compatible with respect to the considered sort of microstate, all the corresponding (ϵ, δ, N_0) -probability laws (2.2) involve for that sort of microstate only one same physical substratum. And obviously, for the considered microstate this can happen more frequently with only one group of mutually compatible quantities, than for two or more such groups. So a maximal dynamical specificity of a given microstate is obtained by establishing the statistical behaviour of this microstate with respect to *all* the groups of mutually incompatible dynamical quantities that are defined with respect to it.

Primordial transferred description. The considerations from the preceding point lead us to posit by definition that - notwithstanding that the laws (2.2) do *not* concern the studied microstate ms_G *isolately* from the measurement interactions from the successions $[G.MesX]$, $\forall X$ that led to them. The set:

$$\{ \{ (\epsilon, \delta, N_0) - \pi(X_j) \}_G, j = 1, 2, \dots, J, \forall X \} \quad (2.3)$$

of *all* the factual (ϵ, δ, N_0) -statistical-probabilistic laws (2.2) established with respect to **one** given operation of generation G and **all** the dynamical quantities X defined for a microstate, will be regarded as a *mechanical description* 'of ms_G '.

This seems appropriate. Indeed, to the initial definition (1.1) of the microstate ms_G that only labels this microstate by the operation G that generates it, and to the chain (1.3) that endows us with a very first and feeble dot of meaning tied with this microstate itself, (2.3) substitutes now a specific characterization of ms_G in terms of a whole dense and stable structure of communicable, consensual, predictive and verifiable pieces of statistical data that involve factually the microstate ms_G *itself*. While via the coding-procedures $cod.proc(X_j)$, $\forall X$, involved by the definitions of the

¹⁹If the initially considered microstate is one micro-state of two micro-systems - in the sense of the definitions from section 2.1 - the sort of compatibility between X and X' that has been defined above can cease when one considers one micro-state of one micro-system (cf. the future section 3.2)

²⁰ This happens, for instance, for the classical quantities p and $p^2/2m = T$ for which it is possible to first determine in a physical-operational way the numerical value of the common basic quantity $|p| = m(v_x + v_y + v_z)$, and out of this basic operational determination, to work out afterward, conceptually, the two results ' p ' and ' $p^2/2m$ ' that are mutually distinct from a conceptual point of view as well as by their numerical values).

²¹These relativities draw attention upon the fact that in nowadays quantum mechanics the concepts of mutual compatibility or incompatibility of dynamical quantities are uncritically assigned a rather mysterious absolute nature, which is the source of unending astonishment and confusion.

measurement interactions $MesX, \forall X$, this structure is intelligible because it is connected to the knowledge established in classical mechanics.

This finally installs the concept of a microstate ms_G as a scientific concept endowed with own and definite stable semantic content. In this sense we are now finally in presence of *knowledge* tied with the microstates themselves. ‘Tied with’ but not ‘on’ the microstates themselves, exclusively. For the sort of knowledge represented in (5’) violates strongly the classical ways of thinking in terms of ‘objects’ that – as delimited wholes – are endowed with a definable and stable global space-time location, with a definable inner organization, and can be qualified in terms of ‘properties’ that these ‘objects’ would ‘possess’. It also violates the conventional views on ‘objective’ facts. It violates the classical concept of knowledge of some ‘thing’.

Let us now immediately organize and qualify in detail this new sort of knowledge.

Notations, denominations, comments. In order to deal efficiently with the unusual features of the result established so far we shall begin by introducing a very analytic way of naming these features.

- The grid of qualification introduced by a dynamical quantity X defined for microstates will be called the *aspect-view* X .

- The whole set of all the dynamical quantities defined for a microstate will be called *the mechanical view defined for a microstate* and will be denoted V_{Mec} . So $\{X\} \approx V_{Mec}$

- The set of basic genetic elements

$$[G, ms_G, V_{Mec}] \quad (2.4)$$

will be called the *genetic triad* of (2.3) (it acts like a sort of inorganic physical-conceptual DNA).

- The whole vast set

$$\{[G.MesX]\}, \forall X \in V_{Mec} \quad (2.5)$$

of repeated successions of operations of the general form $[G.MesX]$ achieved by the use of all the genetic triads (2.4) will be called the *genesis* of (2.3).

Let us note that the genetic triad (2.4) of (2.3) *itself* has a physical-operational-methodological character. Correlatively:

*The genesis $\{[G.MesX]\}, \forall X \in V_{Mec}$ of (2.3) is quite essentially, strongly and deliberately endowed with a space-time organization that expresses basic features of the current **human** thought and actions. These have imposed relative **individualizations** and relative **unity** from **outside** the involved elements of physical reality.*

- *The brute result of the genesis $\{[G.MesX]\}, \forall X \in V_{Mec}$ of (2.3) consists of the set-of-sets of observable marks $\{\{\mu_{k_X}\}, k_X = 1, 2, \dots, K_X, \forall X\}$. This will be called the factual data on ms_G and will be denoted by $(fd)(ms_G)$. So we write*

$$\{\{\mu_{k_X}\}, k_X = 1, 2, \dots, K_X, \forall X \in V_{Mec}\} \equiv (fd)(ms_G) \quad (2.6)$$

The totality (2.6) of all the factual data emerges at very dispersed moments, and also very dispersed spatially on various registering devices of various apparatuses for measuring various quantities X . Observationally, it is just a powder of heaps of traces of vanished interactions, transmuted into meaning by a man-made operational-conceptual-methodological-theoretical machine²². Nevertheless this powder hides inside it a very elaborate *unity of human curiosity, project and method*. The emergence of (2.6) can be made possible only on the organizing basis of the *model* of a microstate posited inside the utilized theory of microstates, and of the correlative coding procedures that have immediately converted each observed group $\{\mu_{k_X}\}$ of physical marks, into a *significant* datum. So, in a still non-expressed way, the factual data from $(fd)(ms_G)$ are already marked in their inner content by the organizing relativities that, inside (2.3), have been endowed with an explicit, intelligible and consensual final expression. But, and this is very important to be noticed:

²² Let us stop a moment to realize how simplistic it would be to assert that this knowledge pre-existed and has been ‘discovered’, when so obviously it has been invented and constructed.

Both the factual data from (2.6) and their explicit and utilizable final organization from (2.3) are devoid of any defined own space-time organization and of any qualia tied with the studied microstate ms_G **alone**.

This is a striking feature of any probabilistic description. But here it acquires a limiting degree of purity.

- The definition (2.3) of the probabilistic predictive laws concerning ms_G – separated from its genesis (2.5) – will be *re-noted* now as

$$D_{Mec}(ms_G) \equiv \{ \{ (\epsilon, \delta, N_0) - \pi(X_j) \}_G, j = 1, 2, \dots, J, \forall X \in V_{Mec} \} \quad (2.7)$$

and it will be called the *primordial transferred mechanical description of the microstate ms_G* (‘transferred’: on registering devices of apparatuses; ‘ Mec ’: ‘mechanical’). When only one quantity X is considered we shall write $D_X(ms_G) \equiv \{ \{ (\epsilon, \delta, N_0) - \pi(X_j) \}_G, j = 1, 2, \dots, J \}$, and we shall speak of the primordial transferred description of a microstate ms_G with respect to the dynamical quantity X . As we have already stressed, this description, itself is devoid of space-time organization.

The *whole* that is constituted by both the geneses (2.5) of repeated successions $[G.MesX]$, $\forall X \in V_{Mec}$, and their result (2.7) $D_M(ms_G)$, will be called the representation of the microstate ms_G and it will be denoted

$$D_{Mec}/G, ms_G, V_{Mec} / \quad (2.8)$$

(or $D_X/G, ms_G, X/$ if only one aspect-view X is involved); this symbol stresses the inseparable unity, in the case of microstates, between the studied entity, the gained knowledge, and the conceptual-physical-operational creation of this knowledge by the human observer-conceptor, *wherefrom the intelligibility stems*.

One feels already the challenge involved with respect to realism and ‘objectivity’ in the ancient classical sense.

A remarkable scission. So, even though the human cognitive actions that lead to the primordial transferred description of a microstate are naturally and irrepressibly endowed with space-time features, nevertheless the final result (2.7)-(2.8) of these cognitive actions has *spontaneously* emerged in a quite non-classical state of rigorous absence of an inner space-time structure. *This is a very remarkable spontaneous scission* ²³. A scission of the same kind appears already in any classical statistical or probabilistic description, but never with this radical character, never *entirely* devoid inside the human mind, throughout the whole investigation, of *any* perceptible material substrate of what *is conceived to exist* – in space-time – and is studied. Inside the present construction of a reference-structure for estimating a theory of microstates:

The primordial transferred description of a microstate reveals a *radically non-classical* character of a type that up to now has never as yet been identified explicitly and listed, neither in the current grammars, nor in logic and in the sciences.

Inside any mathematical *theory* of microstates accomplished up to now, and in particular inside fundamental quantum mechanics, the psychological impact of this character – though factually and observationally it fully subsists – is strongly diminished by the fact that a model of a microstate is constantly working inside the minds of the observer-conceptors, in order to conceive ‘appropriate’ measurement operations, coding-procedures, etc. Whereas here, inside the formalized general structure of reference that we are building, the new concept (2.7)-(2.8) of a primordial transferred description emerges *pure, naked* ²⁴, and also free of any mathematical receptacle that withstands the full perception

²³ I became aware of this scission in consequence of a private exchange with Michel Bitbol.

²⁴ Possibly that is what Bohr desired to preserve when he has interdicted any model of a microstate. He might have been trapped in an implicit feeling of contradiction between the extreme peculiarities that he perceived in what is here called a ‘primordial transferred description’ – especially the radical absence of any own globally delimited spatial support and any defined inner space-time organization – and on the other hand, a total unawareness of the fact that the process of conception and of factual realization of a description marked by such a radical degree of lack of own global space-time definition, does unavoidably *require* a model that cannot be imagined outside space and time. For in his time and by himself the crucial role of coding procedures was entirely ignored.

of its semantic peculiarity. So the limiting character of such a description appears strikingly. And it becomes clear that this character – by itself – constitutes a basic conceptual novelty.

This illustrates the peculiar conceptual powers that a qualitative preliminary formalization independent of any mathematical formalization can manifest concerning *mathematical physics*.

3 THE PROBABILITY TREE OF THE PRIMORDIAL TRANSFERRED DESCRIPTION OF A FREE MICROSTATE

3.1 The probability tree of *one* progressive micro-*state* of *one* micro-*system* with non-composed operation G of generation

We consider first the basic case of one free microstate ms_G of one microsystem. We shall elaborate for its genesis and the result of it, a synthetic tree-like graphic variant of the contents indicated by the representations (1.1) to (2.8). Throughout what follows we distinguish radically between the individual level of conceptualization, and the probabilistic one.

Individual level of conceptualization. The very numerous successions of operations $[G.MesX]$, $\forall X \in V_{Mec}$ involved in a genesis (2.5) start all by definition with one same operation of generation G . But afterward – in consequence of individual and relative compatibilities and incompatibilities between dynamical quantities (cf. section 2.2) the set of all the individual space-time supports of these successions of operations $[G.MesX]$ fall apart, in general, in two distinct space-time ‘branches’. So in general there emerges a tree-like structure²⁵. For simplicity we introduce only two non-compatible quantities X and Y (in the sense of section 2.2); the generalization is obvious.

The two considered mutually incompatible dynamical quantities X and Y introduce respectively the two qualification-grids of form (1.2)

$$gq[X, X_j, MesX, cod.proc(X_j)], j = 1, 2, \dots, J; gq[Y, Y_r, MesY, cod.proc(Y_r)], r = 1, 2, \dots, R \quad (3.1)$$

(for simplicity we endow them with the same number M of possible values, X_j and Y_r respectively).

Let $[d_G.(t_G - t_0)]$ denote the invariant space-time support of each one realization of the operation G of generation of the studied microstate ms_G , that plays the role of a common ‘rooting’ into the microphysical factuality; and let $[d_X.(t_{MesX} - t_G)]$ and $[d_Y.(t_{MesY} - t_G)]$ denote the – mutually distinct – space-time supports of a measurement-operation $MesX$ and a measurement-operation $MesY$, the time origin being re-set on zero after each time-registration (obvious significance of the notations). So each realization of one whole succession $[G.MesX]$ covers a global space-time support

$$[d_G.(t_G - t_0) + d_X.(t_{MesX} - t_G)]$$

and produces a group of observable marks $\{\mu_{k_X}\}$, $k_X = 1, 2, \dots, K_X$, that is coded in terms of a value X_j accordingly to (3.1); while each realization of a succession $[G.MesY]$ covers in its turn a global space-time support

$$[d_G.(t_G - t_0) + d_Y.(t_{MesY} - t_G)]$$

²⁵In section 3.2 we have much stressed the various relativities that restrict the concept of mutual compatibility between dynamical quantities as it is defined in this approach. In certain cases these relativities can entail a total absence of mutual incompatibilities *with respect to the studied microstate*. In such a case, for the sake of generality of the defined language, one can speak of a ‘one branch-tree’. This case – that constitutes also one of the ways that lead from the conceptualization of ‘microstates’ to the classical conceptualization in terms of ‘material objects’ – conceptualization will be detailed elsewhere.

and produces a group of observable marks $\{\mu_{k_Y}\}$, $k_Y = 1, 2, \dots, m_Y$, that is coded in terms of a value Y_r of the quantity Y . Thereby for the considered case the genesis (2.5) from the level of individual conceptualisation of the representation (2.8) is achieved.

This individual phase of elaboration of the representation (2.8) has a dominant physical-operational character, so a space-time organization.

- *Probabilistic level of conceptualization.* Let us start now from the fact that one succession $[G.MesX]$ produces one group of observable marks, $\{\mu_{k_X}\}$, with $k_X = 1, 2, \dots, K_X$. This group of marks $\{\mu_{k_X}\}$ is then coded into a value X_j of X via an adequate choice of the definition of a measurement-interaction $MesX$, accordingly to the coding procedure indicated by the utilized theory for the considered pair (G, X) . The coding value X_j is stored. Mutatis mutandis, the same holds for a succession $[G.MesY]$. Suppose now that: a sequence of a big number N of realizations of a succession $[G.MesX]_n$, $n = 1, 2, \dots, N$, has been realized; the relative frequencies $n(X_j)/N$, $j = 1, 2, \dots, J$ have been established ($n(X_j)$ is to be read ‘the number n of values X_j ’); and an (ϵ, δ, N_0) -convergence in the sense of (2.2) has been found to emerge indeed for these relative frequencies. In these conditions the primordial transferred description (2.7) has been specified *fully*, both factually and numerically. Furthermore on the top of the branch we have constructed a corresponding *effective* and ‘**factual**’ Kolmogorov-like probability-space for the pair (G, X) : The universe of elementary events from the probability space is $U = \{X_j\}$, $j = 1, 2, \dots, J$, and the probability law from the space, namely the primordial transferred description (2.7) $D_X(ms_G) \equiv \{(\epsilon, \delta, N_0)\text{-}\pi(X_j)\}_G$, $j = 1, 2, \dots, J$, is *numerically* defined for all the values $\{X_j\}$ considered for the measured quantity X ²⁶. (For the moment the algebra on the universe of elementary events is not considered explicitly). Mutatis mutandis, the same holds for the quantity Y and its values Y_r . Thereby the probabilistic level (2.7) of the representation (2.8) is also constructed. On this level – out of the observable factual data $(fd)(ms_G)$ generated for the quantities X and Y by the individual and physical-operational genetic phase (2.5) – has been worked out a purely *numerical* probabilistic content. So this level of conceptualization has an abstract *mathematical* character. It induces a promontory into the realm of the mathematized: As soon as we count, irrepressibly, we have already ‘spontaneously’ mathematized.

- *A meta-probabilistic level of conceptualization.* But we cannot stop here. The explicit awareness of the role of the *unique* operation G of generation of all the outcomes of the studied microstate ms_G – from both branches – hinders that. Since the two different effective probability laws $\{\pi(X_j)\}_G$, $j = 1, 2, \dots, J$ and $\{\pi(Y_r)\}_G$, $r = 1, 2, \dots, R$ ²⁷ that crown the operational space-time branches from the zone of individual conceptualization, stem both from one same trunk-operation of generation G , the graphic representation must stress that the two branch-probability laws concern one same microstate ms_G , in the sense of (1.1). Indeed in these conditions it seems unavoidable to *posit that there exists some sort of meta-probabilistic correlation between the two factual probability laws $\{\pi(X_j)\}_G$ and $\{\pi(Y_r)\}_G$* . Such a correlation accepts an expression of the general form

$$\begin{aligned}\pi(X_j) &= \mathbf{F}_{X_j, Y} \{ \pi(Y_r) \}_G, \quad r = 1, 2, \dots, R \\ \pi(Y_r) &= \mathbf{F}_{X, Y_r} \{ \pi(X_j) \}_G, \quad j = 1, 2, \dots, J\end{aligned}\tag{3.2}$$

$$\mathbf{F}_G(X, Y) = \mathbf{F} \{ \pi(X, Y) \}_G\tag{3.3}$$

where $\mathbf{F}_{X_j, Y}$ (respectively \mathbf{F}_{X, Y_r}) is a functional that represents, the *individual* probability $\pi(X_j)$ (respectively $\pi(Y_r)$) in terms of the whole probability laws $\{\pi(Y_r)\}_G$ (respectively $\{\pi(X_j)\}_G$), and $\mathbf{F}_G(X, Y)$ is a functional – left unspecified here – that, establishes the global correlation between the

²⁶ As well-known, a complete Kolmogorov probability space has the structure $[U, \tau, \pi(\tau)]$ where τ is an algebra on the universe U of elementary events. As for the probability $\pi(\tau)$ – defined on τ – it designates exclusively the general concept of a probability measure, without specifying it numerically; while nowhere in the mathematical theory of probabilities is it indicated how to construct the numerically specified probability law that works in a given, factual, particular probabilistic situation (MMS [2014A], [2014B]). While it has been explicitly stated how the factual – i.e. the finite and numerical – probability law is constructed in (2.3) and so in (2.7) and (3.4).

²⁷ For the sake of brevity, from now on we cease to always write explicitly the specification ‘ (ϵ, δ, N_0) ’, but it will be constantly presupposed.

two whole laws $\{\pi(X_j)\}_G$, $j = 1, 2, \dots, J$, and $\{\pi(Y_r)\}_G$, $r = 1, 2, \dots, R$. Together, the relations (3.2) and (3.3) will be called the *meta-probabilistic correlations involved by $G \Leftrightarrow ms_G$* with respect to (X, Y) and will be symbolized by $(Mpc(G))_{(X,Y)}$ (*Mpc*: ‘meta-probabilistic correlation’). So the description (2.7) of the studied microstate has to be explicitly completed²⁸ :

$$D_{Mec}(ms_G) \equiv [\{\{\pi(X)\}_G, \forall X \in V_{Mec}\} - \{(Mpc(G))_{(X,Y)}, \forall (X, Y) \in V_{Mec}^2\}]. \quad (3.4)$$

And in order to distinguish clearly between the probability-laws $\{\pi(X)\}_G \forall X \in V_{Mec}$ from (2.7) and the meta-probabilistic correlations $(Mpc(G))_{(X,Y)}, \forall (X, Y) \in V_{Mec}^2$ defined by (3.2), (3.3), we shall say by definition that (2.7) contains *probabilistic qualification of the first order* whereas $(Mpc(G))_{(X,Y)}, \forall (X, Y) \in V_{Mec}^2$ expresses *probabilistic qualifications of the second order*.

The description (3.4) has been developed inside an a priori given cell for conceptualization, namely the pair (G, V_{Mec}) that from now on we call an *epistemic referential*.

The global geometrized result: the ‘probability tree’ $T(G, (X, Y))$. The figure 8.1 represents the totalized result of the preceding genesis. We have remarked that, in contradistinction to its result this genesis itself possesses a definite space-time. If therefrom the succession is abstracted away there remains a geometrized tree-like structure that conserves the marks entailed by the mutual compatibilities or incompatibilities with respect to the considered type of microstate, between the measured dynamical quantities. So let us denote this geometrized structure of the genetic process of a description (3.4), by $T(G, (X, Y))$ (*T*: ‘tree’) ²⁹.

The green zone, of genetic conceptualization – individual, physical-operational – is clearly separated from the yellow zone of abstract, purely numerical conceptualization where only counts according to various representational criteria have been performed upon the observable results drawn from the individual physical-operational zone (supposed to have been *coded* in terms of values X_j or Y_r in order to work out of these predictive probabilities and meta-probabilities, eventhough inside *IQM* the coding procedure is not defined).

²⁸ Mackey [1963], Suppes [1966], Gudder [1976], Beltrametti [1991], and probably quite a number of other authors also, have tried – directly by purely mathematical means – to establish a satisfactory formulation of a meta-probability law associable with a quantum mechanical state-vector. The tree-like structure constructed here explicates the qualitative and semantic foundations of such a law. This, in the future, should much facilitate the specification of a consensual mathematical expression for what is here denoted $(Mpc(ms_G))$.

²⁹ The expression “probability tree” is already made use of, with various significances. All these should be very carefully distinguished from the particular significance represented by the figure 8.1.

More detailed examination of $T(G, (X, Y))$. The concept of probability-tree of a microstate involves significances that are a far from being trivial. They develop Kolmogorov's concept of probability into a new and much more complex, factual concept of probability.

Probabilistic point of view

- *Random phenomenon.* The classical theory of probabilities offers no formalization of the concept of random phenomenon. It just makes use of the current verbal expression. Whereas on the figure 8.1 one literally sees how – from nothingness – a Kolmogorov probability-space emerges for a microstate, factually and conceptually and up to several numerically specified probability laws and meta-probabilistic correlations between these. Thereby the basic concept of random phenomenon acquires for this case a detailed inner structure, expressed in definite terms [G , $MesX$ or $MesY$, marks $\{\mu_{k_X}\}$ or marks $\{\mu_{k_Y}\}$, code X_j or code Y_r], wherefrom Kolmogorov probability-spaces are then constructed. But these are **factually defined probability-spaces**, that contain numerically specified (ϵ, δ, N_0) -probability laws that are effective and relativized in the sense defined in (2.2). And this result can then be generalized and induced in an enlarged theory of probabilities (MMS [2002A], [2002B], [2006], [2013], [2014]).

- *Probabilistic dependence.* The complete Kolmogorov probability spaces that crown the two branches from the figure 8.1 admit, respectively, the denotations

$$[U(X_j), \tau_X, \{\pi(X_j)\}_G], \quad j = 1, 2, \dots, J, \quad [U(Y_r), \tau_Y, \{\pi(Y_r)\}_G], \quad r = 1, 2, \dots, R$$

where τ_X and τ_Y are the respective algebras of events (cf. the note 3 on this chapter). Let us consider now explicitly these algebras also. Inside the classical theory of probabilities the concept of probabilistic dependence is defined *only* for events from the algebra of one given space. Kolmogorov has written (Kolmogorov 1950, p.9) :

«.....one of the most important problems in the philosophy of the natural sciences is – in addition to the well known one regarding the essence of the concept of probability itself – to make precise the premises which would make it possible to regard any given real events as independent.»

And he has *posited* by definition that two events X_j and X_k from the algebra τ of a probability space, are mutually ‘independent’ from a probabilistic point of view if the numerical product $\pi(X_j)\pi(X_k)$ of the probabilities $\pi(X_j)$ and $\pi(X_k)$ of their separate occurrences is equal to the probability $\pi(X_j \cap X_k)$ of their (set)-‘product-event’ $X_j \cap X_k$ from τ ; whereas if this is not the case, then X_j and X_k are tied by a probabilistic ‘dependence’. But *inside the classical theory of probabilities the concepts of probabilistic dependence or independence are not defined for elementary events from one same universe U* . (Such a dependence can be apprehended only indirectly, by *comparison* with the probability law that acts upon a universe of elementary events defined as a Cartesian product of two universes, one of which is U . But this involves another random phenomenon, distinct from the random phenomenon that generates the space where U is the universe of elementary events). Now, these classical definitions are sufficient indeed if each one of the two probability spaces that crown the two branches from the figure 8.1 is considered *separately* from the other one. But consider now an elementary event X_j from the space that crowns the branch $MesX$, and an elementary event Y_r from the space that crowns the branch $MesY$. **Observationally**, these two events are ‘independent’ in the sense of Kolmogorov. Since the quantities X and Y are mutually incompatible, the measurement-operations $MesX$, and $MesY$ cannot be realized together for one outcome of the studied microstate ms_G , so *the elementary events X_j and Y_r cannot even coexist*. Nevertheless the events X_j and Y_r concern the same microstate ms_G , in the sense of (1.1). And even though ‘one’ microstate in the sense of (1.1) cannot be identified conceptually with one *outcome* of this microstate, the considerations that led to (3.2) and (3.3) entail with a sort of necessity the assertion of the meta-probabilistic correlation ($Mpc(G)$) and the explicit extension (3.4) of (2.7). *Which amounts to the assertion of a sort of ‘probabilistic dependence’ of the second order*. The classical theory of probabilities also defines the general concept of probabilistic correlations, quite explicitly. But *it does not singularize inside it a special class of meta-probabilistic correlations that manifests specifically the fact that one same basic physical*

*entity is involved in different random phenomena*³⁰. This, however, is obviously an important case because it can be an extremely frequent one and it can entail subtle explanations for queer behaviours.

For all the above-mentioned reasons it seems clear that the classical theory of probabilities has to be enlarged, and in various directions³¹. This conclusion is strongly reinforced below.

Logical point of view.

Up to now logical considerations concerning the description of microstates have been developed only in terms of a lattice-structure, on the basis of – directly – the mathematical Hilbert-von Neumann-Dirac formulation of quantum mechanics. The concept of a probability tree of a microstate offers a much more deeply set and more general ground on which to place a logical examination³².

A fundamental question *What happens if, for factual or consensual reasons, no sort of relative mutual incompatibility does arise in the considered circumstance?*

In this case the space-time domain covered by the involved operation of generation G is continued by only one ‘branch’ that is common to all the considered mechanical quantities X , which amounts to saying that this common space-time domain acquires the role of a common ‘trunk’ of the tree, that is crowned by a set of probability spaces - one for each quantity X - *that in (3.4) - are only conceptually distinguished from one another and meta-correlated to one another.*

³⁰ K.J. Jung has introduced a concept of ‘synchronicity’ that seemed rather mysterious and has much struck Pauli, possibly because quantum mechanics had suggested to him an explanation, and this has been discussed in the correspondence Jung-Pauli (MMS [2002B], note pp. 279-281).

³¹ This enlargement, in fact, has already been explicitly worked out in (MMS [2002A], [2002B], [2006], [2014]), in quite general terms, not only for the case of microstates. And in the second part of this work (cf. section 7.2.5 it will appear that – implicitly – it is asserted also inside nowadays quantum mechanics (by Dirac’s calculus of transformations) since some 60 years, but via mathematical writings and denominations to which only an algorithmic significance is assigned, while their significance of another nature is simply not noticed.

³²This has been done already but in an only primitive way in (MMS [1992C]). Much later a quite general relativized reconstruction of the logical and the probabilistic conceptualization has been accomplished (MMS [2002A], [2002B], [2006]) that leads to a unification of these two most basic approaches of the human thought. While for the particular case of microstates an improved but not yet achieved version of the concept of a probability tree has been worked out for the first time only recently (in MMS [2009]).

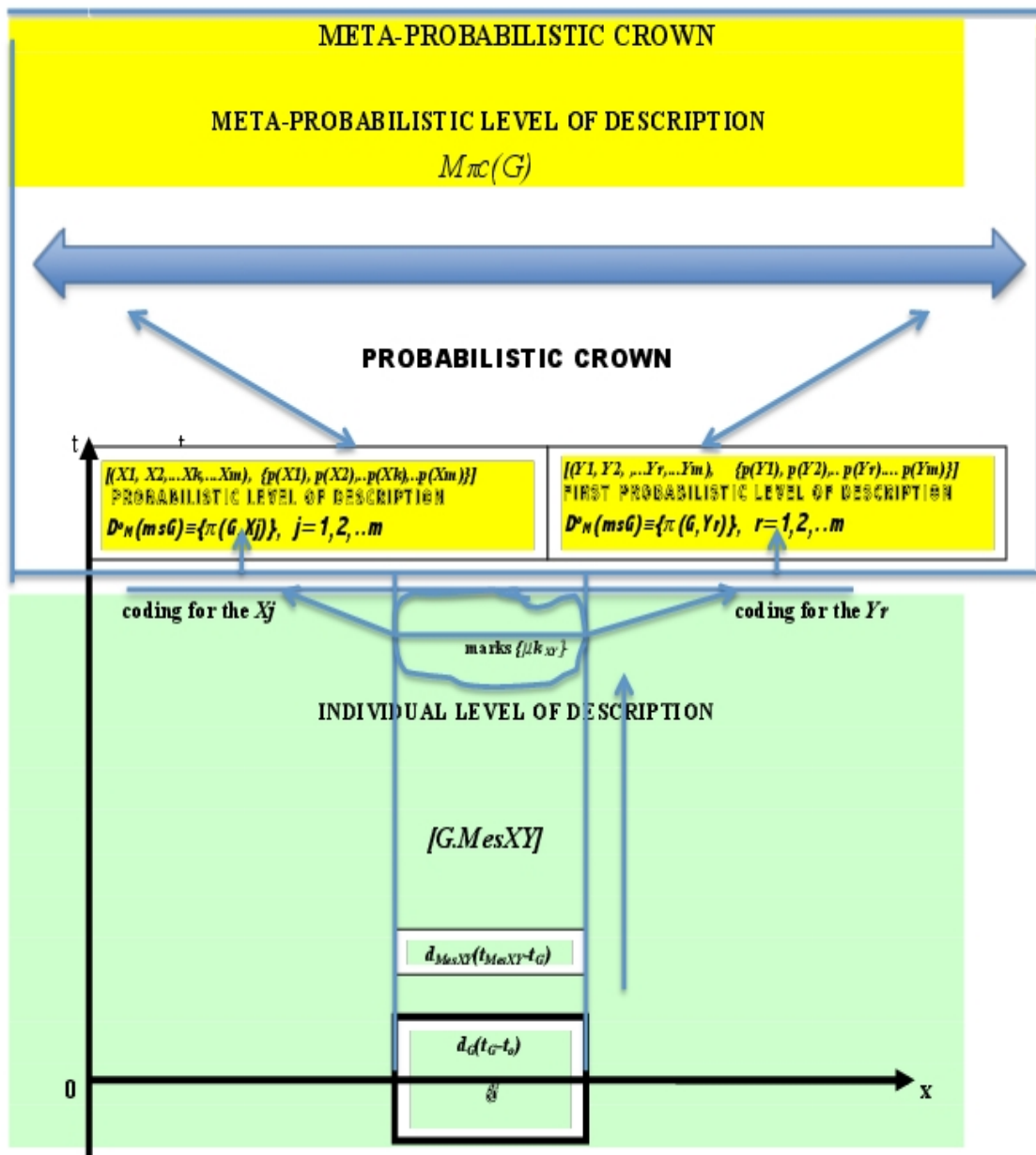



Figure 3.2: The probability-tree $T(G, (X, Y))$ of two relatively compatible observables (in a relative sense)

Connection with the 'scission' remarked in section 2.2. The probability-tree $T(G, V_{Mec})$ of the primordial description (3.4) of a microstate ms_G embodies strikingly the 'scission' on which we have drawn attention at the end of section 2.2: The tree-like a-temporal, geometrized structure of the tree is a consequence of – exclusively – space-time features of the human physical operations accomplished in order to construct the description (3.4). These human operations do entail – quite essentially – certain space-time mutual exclusions inside the set of all such *successive* operations (these are the source of the compatibility or incompatibility of two *given* dynamical quantities X and Y and with respect to a *given sort of microstate*). But the factual and conceptual contents of the global final description (3.4) *itself* are devoid of an *own* space-time structure. On the graphic representation of the tree $T(G, V_{Mec})$ however the final contents of the description (3.4) appear displayed on mutually disjoint spatial zones of the globalized and geometrized structure of the space-time support *of this whole temporal genesis*, wherefrom, at the end of the global process, the temporal aspects have ceased being actual, they have disappeared, evaporated in 'the air of time'. The geometrical mutual disjunction of the different branches of the tree are just representational *vestiges* of the temporal features of the genesis of (3.4) *via human operations*: just traces of a revolved time of descriptive aims. 

3.2 Probability tree of *one* progressive micro-state of *two* or *more* micro-systems: the most non-trivial class of probability trees

Consider now one progressive micro-state $ms_{G[S_1, S_2]}$ of two micro-systems S_1 and S_2 , in the sense of the definitions from section 2.2). Such a microstate is generated by an operation of generation $G[S_1, S_2]$, to which it is tied in the sense of (1.1). So in this case one complete operation of measurement-interaction on one outcome of the microstate $ms_{G[S_1, S_2]}$ involves two partial measurement-interactions, one partial measurement-interaction $Mes(X[S_1])$ with S_1 and one partial measurement-interaction $Mes(Y[S_2])$ with S_2 (in particular the quantities X and Y can identify but in general they are different). So a complete act of measurement will be denoted $Mes(X[S_1]Y[S_2])$. Since $G[S_1, S_2]$ generates one micro-state of two micro-systems and in consequence of the reasons that led to the tree-like space-time structure from the figure 8.1, the two partial measurements $Mes(X[S_1])$ and $Mes(Y[S_2])$ from any one *complete measurement* $Mes(X[S_1]Y[S_2])$ operated upon this one micro-state are lodged both inside *one* branch of the probability tree of which the trunk lodges the space time domain covered by the operation of generation $G[S_1, S_2]$. Another branch of this tree will have to be assigned to the complete measurements that involve another pair of quantities (W, Z) where at least either W is *in*-compatible with X or Z is *in*-compatible with Y – in the sense defined in section 3.2 - or both these possibilities are realized – while concerning W and Z there is no restriction of mutual compatibility. So a two-branches tree founded upon the operation of generation $G[S_1, S_2]$ can be denoted $T(G[S_1, S_2], (X[S_1]Y[S_2], W[S_1]Z[S_2]))$.

Let us focus now upon the following fact: For one micro-state of two micro-systems, the two dynamical quantities X and Y that are involved in a complete act of measurement $Mes(X[S_1]Y[S_2])$ are **always compatible** in the sense defined in section 3.2, since they act via two measurements $Mes(X[S_1])$ and $Mes(Y[S_2])$ that are realized upon, respectively, the two mutually distinct systems S_1 and S_2 that are involved in any one outcome of the microstate $ms_{G[S_1, S_2]}$ ³³. Since the pair $(Mes(X[S_1]), Mes(Y[S_2]))$ belongs to one complete act of measurement $Mes(X[S_1]Y[S_2])$, the corresponding pair of observable marks $(\{\mu_{k_{X[S_1]}}\}, \{\mu_{k_{Y[S_2]}}\})$ – let us denote it $\{\mu_{k_{X[S_1]Y[S_2]}}\}$ – once it has been coded in terms of a pair of values $X[S_1]_j Y[S_2]_r$, $j = 1, 2, \dots, J$, $r = 1, \dots, R$ – constitutes **one elementary event from the universe of elementary events** $U = \{X[S_1]_j Y[S_2]_r\}$, $j = 1, 2, \dots, J$, $r = 1, 2, \dots, R$, from the probability-space that crowns the branch of the complete measurements $Mes(X[S_1]Y[S_2])$; while the factual probability distribution on the universe of elementary events from this probability space, that consists of the transferred description (2.7) with respect to the pair of quantities (X, Y) of $ms_{G[S_1, S_2]}$,

³³We recall that inside the approach developed here the compatibility or incompatibility of two dynamical quantities is relative to both the nature of these quantities and to the type of considered microstate, in the sense of the definitions from section 2.2.

has to be denoted

$$D_{Mec}(ms_{G[S_1, S_2]}) \equiv \{\{\pi(X[S_1]_j Y[S_2]_r)\}_{G[S_1, S_2]}, j = 1, 2, \dots, J, r = 1, 2, \dots, R, \}$$

So the pair (X, Y) of two quantities of which one qualifies the system S_1 and the other one the system S_2 , is everywhere involved as **one whole**. And nevertheless, as by now it is so well known:

The *here-now's* of the corresponding two registered *physical* events, namely [the observation by a human observer of a value $X[S_1]_j$ that qualifies S_1] and [the observation by a human observer, of the value $Y[S_2]_r$ that qualifies S_2], can be separated by an arbitrarily big *space-time distance*. While the corresponding description (3.4) itself is devoid of space-time structure.

We find ourselves face-to-face with the ‘problem’ of non-locality; more, face-to-face with a most explicit analysis of its conceptual inner structure³⁴. This way of reaching the problem brings clearly into evidence the up to now neglected feature that what is called ‘non-locality’ is tied with the fact that any very first, any ‘primordial’ transferred description (2.7) of a microstate, is itself radically **void** of any inner space-time structure, so *it cannot* as yet include explicit space-time specifications, even if these were in principle definable³⁵.

The non-locality problem emerges here in a particularly striking way because it is explicitly and essentially lodged inside the space-time frame of the human observers with their apparatuses. One complete act of measurement $Mes[X[S_1], Y[S_2]]$ involves two macroscopic apparatuses $\mathcal{A}(X, S_1)$ and $\mathcal{A}(Y, S_2)$ that are endowed with perceptible delimited volumes and with perceptible registering devices that pre-structure classes of possible space-time locations of observable results and mark perceptibly the spatial distance between them and the space-time distance between the observable results coded $X[S_1]_j$ and $Y[S_2]_r$. Moreover in the nowadays state of absence inside quantum mechanics of an explicit use of a model of a microstate, the systems ‘ S_1 ’ and ‘ S_2 ’ are implicitly imagined more or less like two small balls, which rises strongly and intuitively the question of *what* ‘exists’ and ‘happens’ ‘between’ them (cf. Appendix I).

The conceptual situation that is represented is also unintelligible from a very basic probabilistic point of view. The questions mentioned above point toward the **inner** features of what is symbolized by ‘ $ms_{G[S_1, S_2]}$ ’, but they emerge in relation with a *one branch*-probability distribution $(\{\epsilon, \delta, N_0\} - \pi(X[S_1]_j Y[S_2]_r))_{G[S_1, S_2]}$, $j = 1, 2, \dots, J, r = 1, 2, \dots, R$, not only inside the meta-probabilistic correlation ($Mpc(G)$). Thereby they appear as tied with a sort of probabilistic ‘dependence’ that is *internal* not only to the elementary observable events $\{X[S_1]_j Y[S_2]_r, j = 1, 2, \dots, J, r = 1, 2, \dots, R, \}$ but also to the studied microstate $ms_{G[S_1, S_2]}$ to which both S_1 and S_2 belong by definition (cf. the definitions from section 2.1); whereas the classical concept of probabilistic dependence cannot deal with such a situation. But on the other hand, as long as one makes conceptual-formal use of the operation of generation $G[S_1, S_2]$ and the successions $[G[S_1, S_2].Mes(X[S_1]Y[S_2])]$ for generating the events $\{X[S_1]_j Y[S_2]_r, j = 1, 2, \dots, J, r = 1, 2, \dots, R, \}$, one is *locked* inside the description (2.7) of **one** micro-state of *two* micro-systems. So trying – in this case – to think of each events $X[S_1]_j$ ‘separately of any event $Y[S_2]_r$ ’ – as it has been very insistently tried – is *devoid* of any defined meaning³⁶. Thereby the whole classical probabilistic conceptualization is strongly perturbed.

Finally, the two branches of the tree $T(G[S_1, S_2], (X[S_1]Y[S_2], W[S_1]Z[S_2]))$, considered together, introduce a meta-probabilistic correlation $(Mpc)(G[S_1, S_2])_{(X[S_1], Y[S_2]), (W[S_1], Z[S_2])}$. This also might

³⁴ Is it not surprising that an approach like that developed here, so general, brings forth so rapidly this face-to-face, in a way so deeply tied with the basic tree-like representation of a microstate and *independently of any mathematical formulation*?

³⁵ A model of a microstate could at least partially compensate this void by offering support to some explanation. But this cannot be offered inside barely a reference structure. This, with respect to the degree of generality desired here, would be too specifically assertive. Therefore we shall come back to the problem of non-locality at the end of the action of reconstruction of the theory of quantum measurements.

³⁶ While trying to conceive S_1 and the qualifications $\{X[S_1]_j\}$, in-dependently of S_2 and the qualifications $\{Y[S_2]_r\}$ (or vice-versa), or trying to conceive them independently of any operation of generation – which still is a quite general and strong tendency – amounts to surreptitiously transmute the initially considered problem, into another problem, and an impossible problem because it is a non-defined problem, as it is explicitly shown in chapter 1

deserve some future examination, in order to identify the specificities with respect to the meta-probabilistic correlations in the simpler case of one microstate of one microsystem³⁷.

The content of this section can be generalized in an obvious way to the case of one progressive microstate of several microsystems.

The probability tree of one microstate of several microsystems illustrates with a particular force *the basic and major role of the general concept of operation of generation G* in a study of microstates. It also illustrates the general clarifying power entailed by an explicit and systematic consideration of all the *defined* possibilities of descriptive relativities entailed by the descriptive cell (G, V_{Mec}) where a probability-tree is confined by construction.

The considerations from this section might open up a constructed door toward unification of the quantum theory with the theory of fields.

3.3 Probability tree of one progressive microstate with *composed* operation of generation

Consider now a *composed* operation of generation $\mathbf{G}(G_1, G_2)$ (chapter 1, section 2.1) of a microstate in which only two simple operations of generation G_1 and G_2 are involved, like in the two-slits experiment of Young. The construction of the primordial transferred description (3.8) for the corresponding microstate $ms_{\mathbf{G}(G_1, G_2)}$ will be found in the second part of this work to raise a central coding-problem. The discussion of this problem and the proposed solution bring strongly into evidence the essential importance of the fact that *the probability-tree $T(\mathbf{G}(G_1, G_2), X)$ is by construction a **one-microstate-tree***. This however cannot be discussed here in detail because it requires a model of a microstate inside the framework of a definite theory of microstates. So concerning this case we shall restrict ourselves to only bring into evidence a striking experimental-conceptual-formal specificity.

Consider an effectively realized microstate $ms_{\mathbf{G}(G_1, G_2)}$. Let us compare its description (3.4) with the descriptions (3.4) of the two microstates ms_{G_1} and ms_{G_2} that *would* be obtained, respectively, if the two operations of generation G_1 and G_2 were each one fully realized *separately*. Not surprisingly, such a comparison brings forth the physical fact that in general, between the probability $\pi_{\mathbf{G}(G_1, G_2)}(X_k)$ of obtaining the value X_k for the microstate $ms_{\mathbf{G}(G_1, G_2)}$ (and given by the k -th element of $\{\{\pi(X_j)\}_{\mathbf{G}(G_1, G_2)}\}$, $j = 1, \dots, J$), and the probabilities $\pi_{G_1}(X_k)$ and $\pi_{G_2}(X_k)$ of obtaining the value X_k for, analogously, the microstates ms_{G_1} and ms_{G_2} , there holds an inequality

$$\pi_{\mathbf{G}(G_1, G_2)}(X_k) \neq \pi_{G_1}(X_k) + \pi_{G_2}(X_k) \quad (3.5)$$

In this sense, the microstate $ms_{\mathbf{G}(G_1, G_2)}$ cannot be regarded as the ‘sum’ of the two microstates ms_{G_1} and ms_{G_2} .

This is indeed a noticeable circumstance. But this *fact* has then been re-expressed in *positive* verbal terms by saying that ‘ ms_{G_1} and ms_{G_2} *interfere* inside $ms_{\mathbf{G}(G_1, G_2)}$ ’. Now, according to (1.1) this re-expression is misleading from a conceptual point of view. Indeed only the *one* microstate $ms_{\mathbf{G}(G_1, G_2)}$ is effectively generated by the unique operation of generation $\mathbf{G}(G_1, G_2)$ that has been performed; and $\mathbf{G}(G_1, G_2)$ is posited to be in a one-to-one relation with its result denoted $ms_{\mathbf{G}(G_1, G_2)}$. So – according to this approach at least – $\mathbf{G}(G_1, G_2)$ *cannot be coherently conceived to generate also* ms_{G_1} and ms_{G_2} . Inside the only *one* realized microstate $ms_{\mathbf{G}(G_1, G_2)}$ the microstates ms_{G_1} and ms_{G_2} have to be conceived as *non-achieved* physically, non-singularized mutually, they possess by construction the status of just two revolved potentialities of separated full operational individualization that have not been actualized. As *such* they are indeed suggested by the structure of the symbol $\mathbf{G}(G_1, G_2)$ because they offer the possibility to *refer* $ms_{\mathbf{G}(G_1, G_2)}$ to ms_{G_1} and ms_{G_2} , if this seems useful. But since ms_{G_1} and ms_{G_2} have not been both and separately effectively realized by $\mathbf{G}(G_1, G_2)$ they do not ‘exist’ inside $ms_{\mathbf{G}(G_1, G_2)}$ and a fortiori they cannot ‘interfere’ inside $ms_{\mathbf{G}(G_1, G_2)}$. Only the tree $T(\mathbf{G}(G_1, G_2), X)$ is factually realized; the trees $T(G_1, X)$ and $T(G_2, X)$ are only reference trees, ghost

³⁷ Again all this stresses the specific powers of a rigorously defined construction of the operational-conceptual-methodological features of the description of a microstate, such that this description emerges when it starts at a local zero of knowledge concerning that microstate and is then developed down-top (fig 1.1).

trees, only virtualities conceived for comparison. That is why the comparison made in (3.5) is very misleading indeed. Language is rich, magic, but also tricky.

The preceding considerations can be generalized in an obvious way to the case of an operation of generation $\mathbf{G}(G_1, G_2, \dots, G_m)$ that composes several operations of generation.

This section closes our exploration on probability trees of progressive microstates. Indeed, for the reasons expressed at the end of section 3.1 the concept of probability tree is not useful for bounded microstates. Therefore in what follows we only add a brief remark on the evolution of a free microstate.

3.4 On the evolution of any free microstate

Is it possible to assert something concerning the evolution of a progressive microstate inside this only qualitative and semantically ‘open’ approach for constructing a general reference-structure for how to create knowledge on microstates? The answer is yes, and again it brings into evidence the crucial role of the concept of operation G of generation of a microstate.

Imagine the *final* moment t_0 assigned to an operation of generation G from (1.1) that introduces initially the microstate to be studied, ms_G . In contradistinction to what has been assumed before, let us admit that during some time interval $\Delta t_1 = t_1 - t_0$ the human observer does *not* act upon the microstate ms_G . But during Δt_1 the initial microstate ms_G can be *posited* to ‘evolve’ in the exterior conditions EC that it encounters (exterior macroscopic fields, obstacles). Indeed it would seem weird to posit that it remains immobilized from any conceivable point of view. Now, this evolution *can be integrated* in (1.1): Nothing interdicts to posit, in full logical coherence with the preceding development, that the association of the initial operation of generation G and what happens to ms_G during $\Delta t_1 = t_1 - t_0$, act together like *another* operation of generation – let us denote it $G_1 = F(G, EC, (t_1 - t_0))$ (F : some function) that generates another, corresponding microstate ms_{G_1} , in the sense of (1.1). This other microstate ms_{G_1} can be studied via sequences of successions $[G_1.MesX]$, $\forall X \in V_{Mec}$ as specified before for *any* microstate ms_G . The time interval $t_1 - t_0$ can be chosen with any desired value, the external conditions EC being kept unchanged. So one can study successively a set of mutually ‘distinct’ microstates ms_{G_k} (accordingly to the language imposed by (1.1)) that correspond respectively to the set of successive operations of generation:

$$G, G_1 = F(G, EC, (t_1 - t_0)), \dots, G_k = F(G, EC, (t_k - t_{k-1})), \dots, G_K = F(G, EC, (t_K - t_{K-1})). \quad (3.6)$$

(K : an integer). For each operation of generation G_k from this set one can construct the corresponding probability tree $T(G_k, X)$, $\forall X \in V_{Mec}$, and so the corresponding description (3.4). This description itself, however, is *at any time* devoid of any definite inner space-time structure: The scission between the observer’s cognitive actions organized inside his space-time framework, and the obtained final description that is devoid of any inner space-time organization, *subsists fully*.

For the sake of generality, from now on we refer to $G_k, k = 1, \dots, K$ in (3.6) by $G^{(t)}$ (i.e. assuming $t = t_k - t_0$). Therefore, we reserve the symbol G for the initially considered microstate, and incorporate a super-index t to express a new operation of generation derived from its free evolution. So, when the operation G from (1.1) is followed by an evolution we can adequately indicate this fact by writing

$$[G^{(t)} = F(G, EC, (t - t_0)), G^t \Leftrightarrow ms_{G^{(t)}}] \quad (3.7)$$

The relation (3.7) absorbs the concept of ‘evolution’ of a microstate into the general concept of operation of generation G , while the concept of ‘one act of measurement $MesX$ ’ is absorbed into the concept of one realization of a succession $[G^{(t)}.MesX]$ in the sense of (3.6) where the particular possibility $G^{(t)} \equiv G$ is left open for being employed for the initial microstate ms_G .

This will come out to be important. And here it permits to re-write the core-result (3.4) of *IQM* in the form

$$D_{Mec}(ms_{G^{(t)}}) \equiv [\{\pi(X_j)\}_{G^{(t)}}, j = 1, \dots, J], (Mpc(G^{(t)}))_{(X,Y)}, \forall (X, Y) \in V_{Mec}^2. \quad (3.8)$$

So, when this is convenient, we can re-write G as $G^{(t)}$ and ms_G as $ms_{G^{(t)}}$. However in general we continue to make use of the basic writing (3.4).

The considerations from this point close the announced construction of a reference structure for estimating a theory of microstates. So let us examine the final result.

4 INFRA-(QUANTUM-MECHANICS)

The result of the approach developed here has been a priori named Infra-(Quantum Mechanics) and is denoted *IQM*. This denomination is an ellipsis for ‘*the organization beneath the quantum theory, of a procedural global structure of reference for constructing a fully intelligible mathematical theory of a ‘mechanics of microstates’*’. The mentioned organization has indeed been constructed independently of any mathematical formalism. It has been subjected to the choice of a *strategy*: To start on the lowest level of conceptualization that can be attained (1.1) – the level of zero pre-accepted knowledge on the physical, individual and *fully* singular outcomes of any microstate-to-be-studied – so as there-from to be able to *control* explicitly the progressive elaboration of mutually connected moulds for optimally receiving in them the semantic elements (concepts, physical operations, methodological choices) out of which can be drawn intelligible scientific³⁸ knowledge concerning ‘microstates’. Doing this we have tried to bring into evidence and to incorporate all the decisive constraints that have to be obeyed. The final result is a qualitative but formal structure that can be characterized as follows.

1. The core of *IQM* consists of the form of a *primordially probabilistic transferred description* developed inside the conceptual cell delimited by an a priori chosen epistemic referential (G, V_{Mec}) . This sort of description has been symbolized by the writing of Eq. (3.8):

$$D_{Mec}(ms_{G(t)}) \equiv [\{ \{ (\epsilon, \delta, N_0) - \pi(X_j) \}_{G(t)}, j = 1, \dots, J \}, (Mpc(G^{(t)}))_{(X,Y)}, \forall (X, Y) \in V_{Mec}^2].$$

The descriptonal structure (3.8) has never before been identified and characterized in explicit terms. It is marked by very remarkable peculiarities:

- It is **devoid of inner space-time organization**.
- It is strongly **relative** to three genetic elements (2.4) $[G, ms_G, V_{Mec}]$ where the pair (G, V_{Mec}) can be formed in strict adequacy with the particular cognitive aim.
 - *the physical operation G of generation of the individual specimens of the microstate to be studied* has never been noticed before, while here it reveals a ubiquitous and central role.
 - The global genetic process (2.5) $\{[G.MesX]\}$, $\forall X \in V_{Mec}$ that brings forth a description (3.8) involves a characteristic that is new with respect to the classical performance of ‘measurements’, namely the fact that *each* act of measurement *MesX* requires in general *the previous realization of also the operation of generation G* of the entity on which the measurement is realized, because in general a measurement-interaction on a specimen of the studied micro-state ms_G destroys this specimen (even if the micro-system involved by the specimen does persist).
 - The brute observable result (2.6) of each one genetic succession $[G.MesX]$ from (2.5) – namely a group $\{\mu_{k_X}\}$, $k_X = 1, 2, \dots, K_X$, of observable physical marks – is *entirely meaningless by itself, it carries no perceivable ‘qualities’ (qualia) associable with that what it signifies, and therefore it does not directly ‘mean’ as soon as it is perceived*.

*And in order to gain for the observable marks $\{\mu_{k_X}\}$, $k_X = 1, 2, \dots, K_X$, a meaning in terms of a value X_j of the measured quantity X assignable to the involved specimen of a microstate, a **coding procedure is necessary** that connect these marks to previously established meanings. This in its turn requires unavoidably, in order to be *definable*, a general *model* of a microstate.*

So:

1. Such a model – regarded as just a methodological artefact, not as the assertion of factually ‘true’ description – *must* be specified inside any acceptable theory of microstates, as well as a corresponding

³⁸Which means: communicable and intelligible, consensual procedures for generating microstates, for predicting concerning microstates and for verifying the predictions.

coding procedure for *each* quantity X and *each* sort of microstate (in the sense of section 2.1). These are essential, *sine qua non* conditions.

2. In contradistinction to the basic descriptonal structure (3.8) itself, the genetic human successions of operations $[G.MesX]$ from (2.5) are endowed – quite essentially – with a specific space-time structure; and the graphic representation (8.1) of the final global geometrized result of all the genetic processes $[G.MesX]$ from (2.5) has a tree-like character that brings forth intuitively *non-classical probabilistic features* of (3.8) that:

- require extensions of the concept of probabilistic dependence;
- these extensions *vary* according to whether one micro-*state* of *one* micro-*system* is involved, or *one* micro-*state* of several micro-*systems*, and in the second case the extensions violate brutally the classical ways of thinking;
- these extensions require a basic extension of also the classical *logical* conceptualization.

The mentioned probabilistic and logical *extensions* from *IQM* have been shown elsewhere (MMS [2002A], [2002B], [2006]) to lead to a *unification of the logical and the probabilistic approaches*. It furthermore is unified with Shannon’s theory of information and permits to define simple, relativized measures of complexity (MMS [2006]).

3. The genetic process (2.5) of a primordially probabilistic transferred description (3.8), and this description itself, constitute together a *whole*, in the quite remarkable sense that the descriptonal structure simply ceases to be clearly intelligible when it is separated from its genesis. This is why we have endowed the whole [genesis+result] with an own name – ‘*representation*’ of the studied microstate – and with a specific symbolization (2.8) $D_M/G, ms_G, V_{Mec}/$.

4. The concept of *probability-tree of an operation of generation of a microstate* embodies and summarizes graphically the whole complex and unexpected structure of a transferred description (3.8) of a microstate.

Considered globally now, *IQM* illustrates with particular power two essential methodological facts, and it raises a major problem of scientific conceptualization.

- The two essential methodological facts are the following ones.

- *Systematic descriptonal relativities restrict and thereby specify, thus entailing precision*. This is strictly opposed to the usual meaning of the misleading word ‘relativism’.
- *The geneses are the vehicles of the semantic contents poured into the description* (3.4) and (3.8).

Both these facts are hugely under-estimated.

- This raises a major problem of scientific conceptualization :

- *How can mathematics be optimally allied with representations of factual reality where geneses and the semantics carried by these determine a pre-mathematical formal structure marked by humans constrains and aims?*

Two specific and intimately related deliberate absences mark *IQM*: The absence of a general model of microstate, and the absence of definite coding rules for assigning meaning to the observable result of a measurement succession $[G.MesX]$ from (2.5). Inside a *general* reference structure able to guide the construction of any representation of microstates these absences are conditions of the generality, because they can stem only from definite and particularising postulations that can be introduced only inside a definite theory of microstates. By contrast, their absence inside *IQM* brings into evidence that:

- without a *model* of a microstate that shall permit to conceive ‘appropriate’ modalities for measuring this or given quantity X on a given sort of a microstate,
- without explicit *coding procedures* for translating the observable result of an act of measurement, into meaning in terms of a definite value of definite mechanical quantity, the primordial transferred

descriptions (3.8) are just a heap of inert puppets. Indeed the strings that can bring these puppets to work and to create this potent impalpable thing that here we call ‘procedural knowledge on microstates’, are precisely a general model of the concept of microstate that permit to state explicit coding-measurement-interactions.

So, out of nearly a nothingness of explicit previously available knowledge on *how knowledge on microstates can emerge*, there has been explicitly defined a rather non-trivial reference structure for constructing such knowledge and for estimating the adequacy of any theory of microstates; so in particular also of the Hilbert-Dirac quantum mechanics. The epistemological-operational-methodological pre-mathematical structure of any acceptable theory of microstates is now represented by a **formal construct of void conceptual loci for semantic data** of which only the nature and the denotation are specified. As soon as these void loci will be charged with data that characterize a particular factual situation and will be associated with a mathematical representation, this construct will set into action and will generate acceptance or refusal, or reveal lacunae, inadequate restrictions, etc. A new, structured Universe of *referred* actions and events will emerge and cohere.

And furthermore, *IQM* can be generalized into a method for constructing *any* consensual, predictive and verifiable relative knowledge on physical entities ³⁹.

~~4.1~~ CONCLUSION ON PART I

When one watches the way in which *IQM* emerges the naïvely realistic view that scientific knowledge is discovery of pre-existing truth collapses into dust. And in its place one sees, one *feels* in what a sense conceptual-operational *procedures* – pointing toward physical operations or abstract ones – can progressively be assembled into a *method* born from the free human curiosity and inventiveness and from explicit aims chosen by men. What has been obtained here is no more than just a *particular* such method. But it is a global coherent method for constructing a *definite* particular piece of procedural knowledge directed by a *definite* specific project. It is not in the least a ‘discovery’ of pre-existing ‘intrinsic truths’ about how physical reality ‘is’ absolutely, ‘in itself’. Such discoveries are mere illusions, genuine Fata Morgana.

We are trapped in a cage where intrinsic truth is irrepressibly felt to pre-exist but to constantly stay out of reach, frustratingly, definitively hidden beyond a non-organized swarm of unending approximations toward ideal targets. One feels assaulted by a sort of impotence, of inefficiency, of enslavement.

I perceive only one attitude that preserves from this sort of fail: With a blindfold deliberately fixed on our metaphysical eye and on the basis of entirely declared data and posits, to **construct**, humbly, hypothetically, **relatively**, but from the maximal possible depth. Thereby an only restricted, finite and methodological knowledge can emerge; but a fully definite knowledge endowed with an entirely exposed genesis left open to constant return and optimization. Such knowledge can be optimized indefinitely, precisely because it is hypothetical and finite and relative and because its genesis, with the aims and constraints that restrict it, are entirely exposed.

³⁹ This has already been proved by construction (MMS [2002A], [2002B], [2006])

Part II

CRITICAL-CONSTRUCTIVE GLOBAL EXAMINATION OF THE HILBERT-DIRAC QUANTUM MECHANICS BY REFERENCE TO *IQM*

THE SPECIFIC AIM OF PART II

The second part of this work is devoted to a global preliminary examination of the Hilbert-Dirac formulation of Quantum Mechanics QM_{HD} , by reference to *IQM*. This examination is intended to yield a perspective on the general structural features of QM_{HD} , from outside QM_{HD} , to identify the model of a microstate that certainly works somehow inside QM_{HD} – because in the absence of any model this theory would be radically impossible –, to establish the main necessary global clarifications, and to identify the reason why the the theory of measurements from QM_{HD} raises so stubbornly various problems since tenth of years.

5 GLOBAL COMPARISON BETWEEN QM_{HD} AND THE IQM REPRESENTATION

5.1 The compared representations

The basic QM_{HD} way of representing a microstate. We reduce to the strict essence the recalling of the QM_{HD} -representation of a microstate of this representation. This essence consists of four *purely formal* problems and the correlative procedures for obtaining the solution, and a fifth **factual**-formal problem with its own solution ⁴⁰:

- **Problem 1:** Determine the state-ket $|\psi(x, t)\rangle$ that represents the microstate to be studied inside the generalized Hilbert space \mathcal{H} of the state-ket of the studied microstate, enlarged to also the eigenket from the various bases introduced by the observable-operators).

Solution to problem 1: Write the Schrödinger equation of the problem, solve it, and *introduce the limiting conditions in order to identify the initial state-ket $|\psi(t_0)\rangle$* . Therefrom the Schrödinger equation determines $|\psi(t)\rangle$ for any desired value t of time.

- **Problem 2.** For any mechanical quantity $A(x, p_x)$ redefined for microstates, determine the predictive probability law $\{\pi(a_j) \equiv |c_j|^2\}$, $j = 1, \dots, J$, $\forall \mathbf{A}$, concerning the possible outcomes of the eigenvalues a_j of the QM_{HD} -observable \mathbf{A} that represents it.

Solution to the problem 2:

- Construct the QM_{HD} -observable \mathbf{A} from the classical definition $A(x, p_x)$ of the quantity A , as a symmetrized function $\mathbf{A}(\mathbf{X}, \mathbf{P}_x)$ of the two basic observables \mathbf{X} and \mathbf{P}_x associated to the two basic classical dynamical quantities x and p_x .

- Write the equation $\mathbf{A}|u(x, a_j)\rangle = a_j|u(x, a_j)\rangle$, $j = 1, \dots, J$ and calculate from it the basis of eigenket⁴¹ $\{|u(x, a_j)\rangle\}$ introduced by \mathbf{A} in \mathcal{H} . Each eigenvalue a_j of the quantum mechanical observable \mathbf{A} is tied in this equation to a corresponding eigenvector $|u(x, a_j)\rangle$ from this basis. **By postulate** any a_j – and *only* this – is a possible outcome of a measurement of \mathbf{A} upon the studied microstate. So the spectrum of \mathbf{A} is $\{a_j\}$, $j = 1, \dots, J$.

- Write now the spectral decomposition of $|\psi(x, t)\rangle$ with respect to the basis $\{|u(x, a_j)\rangle\}$, $j = 1, \dots, J$: $|\psi(x, t)\rangle/\mathbf{A} = \sum_j c(a_j, t)|u_{a_j}(x)\rangle$, $j = 1, 2, \dots, J$. This is the representation of the studied microstate in \mathcal{H} and relatively to \mathbf{A} . Form the set of squared absolute values $|c(a_j, t)|^2$, $j = 1, \dots, J$, drawn from $|\psi_A(x, t)\rangle/\mathbf{A}$ and write the researched predictive probability $\pi^{(t)}(a_j) = |c(a_j, t)|^2$ and probability law $\{\pi^{(t)}(a_j) \equiv |c(a_j, t)|^2\}$, $j = 1, \dots, J$, *accordingly to Born's probability postulate* (confirmed by Gleason's theorem).

- **Problem 3.** Specify the way in which you can transform the representation of the studied microstate in \mathcal{H} and relatively to \mathbf{A} , into the representation in \mathcal{H} of the same microstate but relatively to another observable \mathbf{B} with eigenvalues b_k and eigenvectors $|v_{b_k}(x)\rangle$, $k = 1, \dots, K$, $k = 1, 2, \dots, K$.

Solution to the problem 3: Apply Dirac's 'theory of transformations':

$$d(b_k, t) = \langle v_{b_k} | \psi(x, t) \rangle = \sum_j \tau_{b_k, a_j} c(a_j, t) \quad \text{or} \quad \tau_{b_k, a_j} = \langle v_{b_k} | u_{a_j} \rangle, \quad j = 1, \dots, J, \quad k = 1, \dots, K$$

⁴⁰When the vector representation \mathbf{r} is not specifically necessary, we write in only one spatial dimension; when no spatial dimension is specifically relevant we write $|\psi(t)\rangle$; and when sufficient, we write only $|\psi(t)\rangle$. And, though according to the current formalism the spectra are in general continuous and infinite, we represent them always by finite writings.

⁴¹As usual we write 'ket' without plural.

- **Problem 4.** Represent mathematically the measurement processes by which is verified the predictive probability law $\{\pi^{(t)}(a_j) \equiv c(a_j, t)^2\}$, $j = 1, \dots, J$, drawn from $|\psi(x, t)\rangle/\mathbf{A}$.

Solution to the problem 4: Apply ‘the quantum theory of measurement’.

- **Problem 5:** Verify the statistical predictions of the formalism.

Solution to the problem 5: Accordingly to the quantum theory of measurements, ‘prepare *the measurement-evolution state-vector*’ and operate the measurements.

Concerning this point nothing is clearly specified. The term ‘prepare’ applied to a formal descriptor creates confusion. Some authors seem to consider that *the microstate has to be ‘prepared’* – or to be *also* ‘prepared’ –; ***the coding problem is not formulated***, nor, a fortiori, treated explicitly. The implicit treatment, in so far that it can be identified, raises questions. The factual and conceptual connections with the problem 4 are not worked out. Precisely this, as a whole, is the ‘measurement problem’.

Everything inside QM_{HD} is expressed in absolute terms and via continuous unlimited mathematical analysis or algebra that allows continuity and infinities.

The basic IQM way of representing a microstate. This consist of the whole Part I of this work.

Everything inside IQM is expressed in entirely relativized terms and via finite representations

5.2 The comparison

When the two representations recalled above are compared ⁴², the most striking conclusions are the following ones, readable of Fig. 5.1

⁴²From now on, for notational uniformity, the classical dynamical qualifying quantities will be indicated by A, B, \dots (instead of X, Y, Z, \dots) even if they are conceived to belong to IQM . Correlatively, the corresponding eigenvalues will be indicated by a_j, b_k , etc. (we shall write, for instance: $[(D_A(ms_G)) \equiv \{\pi(G, a_j)\}, j = 1, 2, \dots, J]$ or $[(D_{Mec}(ms_G) \equiv \{\pi(G, a_j)\}, j = 1, 2, \dots, J, \forall A \in V_{Mec}, Mpc(ms_G)]$; etc.).

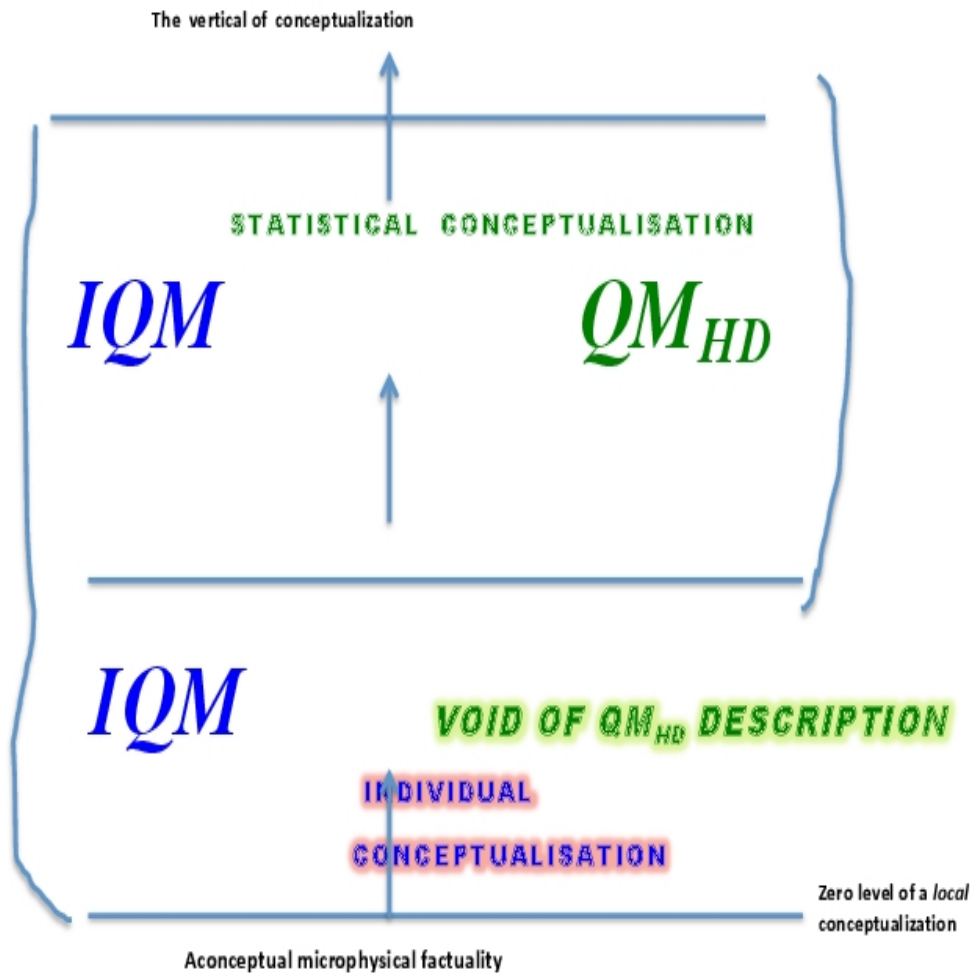


Figure 5.1:

* Inside QM_{HD} all that is explicitly ruled is *defined in a purely mathematical and algorithmic way*. The main descriptive element is the concept of a state-ket $|\psi\rangle$ from the Hilbert-space assigned to the studied microstate and this is obtained exclusively via mathematical procedures. *No factual or even only qualitatively represented procedure of any sort comes in*. Even the way to ‘give’ the initial state-vector $|\psi(t_0)\rangle$ is purely mathematical (‘initial conditions’).

This restricts the domain of rigorous applicability of QM_{HD} , to the domain of problems that introduce calculable state-ket.

Whereas inside IQM the description (3.8) is directly rooted into the factual microscopic physical reality and is constructed on the basis of individual definitions and operations, via **factual** procedures or conceptual-methodological posits.

* The concept of ‘microstate’ – that indicates what the whole formalism represents – is not defined inside QM_{HD} and it is even left devoid of symbolization (so the question of its definability is not even considered). A fortiori there is no concept of operation of generation of a microstate, and no model of a microstate is defined. While inside IQM the specification of a model of a microstate has appeared as a basic necessity inside any theory of microstates. Correlatively, the concept of an individual and *physical* operation G of generation of a microstate is devoid of definition and of symbolization inside QM_{HD} while inside IQM it manifests a quite determining role, namely via (1.1) it leads to:

* The classification of the sorts of operation of generation (simple, composed, revolved inside the past (in the case of bound states)).

* In consequence of (1.1) $G \Leftrightarrow ms_G$ the operation of generation G entails also a corresponding basic classification of the microstates from section 3.1.

* The basic tree-like structure from the figure 8.1 and 3.2 that summarize graphically the whole IQM stems from one operation of generation.

We take now over in general terms. QM_{HD} contains no explicit representation of practically **none** of **all** the individual physical operations, concepts and entities that inside the reference-structure IQM have been shown to be basically necessary for an intelligible theory of microstates. The concepts $[G, ms_G, \text{model of a microstate, individual succession of operations } [G.MesA], \text{ coding procedures for translating the observable physical marks produced by one succession } [G.MesA] \text{ in terms of one definite value } a_j \text{ of the measured quantity } A]$, *all* these fundamental individual descriptive elements are devoid of factual definition inside QM_{HD} and often they are even devoid of any formal representation.

Nowhere inside QM_{HD} does one find a clear *distinction* between individual and statistical representations, so neither between representations of *physical* entities or operations, and abstract constructs that point toward *global and purely abstract numerical results* of manipulations of physical entities and operations.

In fact QM_{HD} begins directly with the construction of abstract statistical descriptors because, advancing top-down, it has *encountered* first only statistical manifestations and, after having dwelt with these, it has stopped trying to advance more downward into the factual source of the statistical manifestations. Lacking of orientation it botched up the problem of representing this factual source also, it closed the approach by hasty postulations.

In short: When compared with IQM the Hilbert-Dirac formulation QM_{HD} appears as a conceptual bas-relief, not as a conceptual statue. The representation of its physical-operational support is lost undone inside an amorphous substrate ⁴³.

Whereas the IQM representation is explicitly constructed starting from a level of zero local knowledge concerning the studied microstate, and *therefrom* the main features of all the successive levels of conceptualization – individual, probabilistic, meta-probabilistic – are *necessarily encountered* and are clearly distinguished from one another, characterized and mutually connected. As a formal reference-construct IQM stands upright on its conceptual and factual feet and these are *rooted into* – not only placed upon – a-conceptual factuality; while the essential concepts left undefined in consequence of

⁴³A more violent metaphor would be to crudely say that QM_{HD} appears as a genetically malformed conceptual being of which the legs are hidden in its belly.

the role of only a reference-structure assigned to IQM – a general model of a microstate and the coding-problem – are overtly declared and are required as a necessity for a theory of microstates. The figure 5.1 represents the mutual ‘position’ of IQM and QM_{HD} with respect to the vertical of conceptualization.

The above comparison brings into evidence a radical opposition. As long as this opposition subsists it is an obstacle in the way of intelligibility. It did not need to be spelled out in order to have constantly worked and created conceptual unease during many years.

6 BASIC CLARIFICATIONS: A GENERAL MODEL OF A MICROSTATE; USEFULNESS OF ‘G’; REFUSAL OF VON NEUMANN’S REPRESENTATION OF MEASUREMENTS; CRITIQUE OF THE QM_{HD} THEORY OF MEASUREMENTS

The main aim of this chapter is to identify:

- the model of a microstate that – necessarily – exists and works inside QM_{HD} , since this theory introduces the concept of ‘measurements’ operated upon microstates.
- the connections between this model and the operations G of generation that draw specimens of this model into the domain of potential observability.
- The reasons why the theory of quantum measurements raises stubbornly various questions since tenths of years.

6.1 The $[IQM-QM_{HD}]$ meaning of an eigenfunction of a quantum observable and consequences

Digging out the detailed meaning of an eigenfunction. Let us place ourselves inside QM_{HD} . Consider the equation $\mathbf{A}|u_j(x, a_j)\rangle = a_j|u_j(x, a_j)\rangle$, $j = 1, 2, \dots, J$ that determines the eigenfunctions $\{u_j(x, a_j)\}$ from the basis of eigenket introduced by \mathbf{A} in the generalized Hilbert space \mathcal{H} of the studied microstate. In general such an eigenfunction is not square integrable. This is considered to be a ‘problem’, in the following sense. A state-function $|\psi(x, t)$ from a state-ket $|\psi(x, t)\rangle$ is *required* to be square-integrable, since it represents a set of distributions of probability. But an eigenfunction in general is not square-integrable and furthermore it is *not required such*. Why, exactly? That is the ‘problem’.

Bohm ((1954) p. 210-211) writes:

«...We obtain $|\psi = e^{ipx/\hbar}$ Strictly speaking, the above eigenfunctions cannot, in general, be normalized to unity...Let us recall, however, ...that in any real problem the wave function must take the form of a packet, since the ‘particle’ is known to exist somewhere within a definite region, such as in the space surrounded by the apparatus. To obtain a bounded and therefore normalizable packet, we can integrate over momenta with an appropriate weighing factor»⁴⁴.

So Bohm adopts an exclusively mathematical point of view. Not a moment does he focus on the meaning. He does not even make use of a specific notation for distinguishing between eigenfunction and state-function. And in order to deal with the mathematical situation he accepts approximations without any hesitation, notwithstanding that the considered question seems to be a question of principle.

Dirac (1958, p. 48), on the contrary, writes:

«It may be that the infinite length of the ket-vectors corresponding to these eigenstates is connected with their unrealizability, and that all realizable states correspond to ket vectors that can be normalized so that they form a Hilbert space.»

⁴⁴ Note that a ‘packet’ of eigenfunctions belonging to a formalism that introduces an axiom of superposition, is not the unique possible way for representing mathematically a delimited spatial support (solitons, etc.).

(What does «‘connected with their unrealizability’ » mean here? That the ket-vectors represent ideal *limits* of something? or that the corresponding – presupposed – state cannot be generated? or both?).

As for the most outstanding didactic exposition of QM_{HD} , that by CTDL⁴⁵, it proposes «*a physical solution to the difficulties*» (proposed also by Bohm (1954, p. 212)), namely to replace the eigenfunction by a δ -distribution centred upon the corresponding eigenvalue.

Nobody has conceived that an ‘eigen-state’ might simply not be ‘a state’.

However recourse to history reveals that the ‘problem’ of non-integrability of an eigenfunction is a *false* problem because an eigenfunction has a specific *meaning* that is radically different from that of a state-function. So the problem is not mathematical, it is conceptual. The meaning of an eigenket stems from Louis de Broglie’s Thesis (1924, 1963). Louis de Broglie has derived his famous relation $p = h/\lambda$ from his well-known model of a microstate, erroneously named the wave-‘particle’ model. The model itself stems from the usage made of Fourier decompositions inside classical electromagnetism. In a Fourier decomposition of an electromagnetic wave each constant value λ of a monochromatic wavelength is associated with a corresponding plane wave. By analogy, to each value p_{x_j} of the classical mechanical fundamental quantity of momentum p_x of a free electron de Broglie has associated a plane wave with a ‘*corpuscular phase-function*’ $\Phi(x, t) = ae^{(i/\hbar)\beta(x, t)}$ where a denotes an arbitrary and *constant* amplitude of vibration and the ‘*corpuscular phase*’ is written as $\beta(x, t) = (Wt - p_{x_j} \cdot x)$, where $W = m_0c^2/\sqrt{1 - v^2/c^2}$ is the – relativistic – energy of the ‘*corpuscular-like aspect of the corpuscular wave*’ while p_{x_j} denotes the constant value posited for the momentum of this ‘*corpuscular-like aspect*’ (in one spatial dimension)⁴⁶.

The ‘*corpuscular-like aspect of the corpuscular wave*’ remained devoid of representation inside the mathematical expressions that Louis de Broglie associated to his model. This has been a huge mistake because in mathematical physics only what possesses a definite mathematical expression does subsist. The rest does not strike all the attentions and so at last it evaporates into the air of history. But verbally de Broglie has clearly specified in his writings that the ‘*corpuscular aspect*’ consists of a ‘*singularity of the amplitude of the corpuscular wave*’; namely a very localized region where this amplitude is so much bigger than its surrounding constant value, that it concentrates in it practically the whole energy $W = mc^2/\sqrt{1 - v^2/c^2}$ of vibration of the corpuscular wave. This singularity was conceived to *glide* inside the wave *like* ‘a small classical mobile’ that – in consequence of its strong spatial localization and its relatively very high energy – admits at any time the *mechanical* qualifications of ‘*position*’ and ‘*momentum*’, from which – in classical mechanics – all the other mechanical qualifications can be constructed (whereas the rest of the wave, of course, does *not* accept mechanical qualifications). In short, de Broglie’s model does not introduce any ‘*particle*’ whatsoever.

In the course of the construction of the relation $p = h/\lambda$ de Broglie has proved the ‘theorem of concordance of the phases’⁴⁷ according to which:

The model of a microstate of a free electron can be *stable* if and only if the corpuscular-like singularity of the amplitude of its corpuscular wave glides inside the wave in a way such that the phase of the up-down-up vibration of the amplitude of the *localized* singularity – a *clock-like* phase in the sense of Einstein relativity at any given location x – is at *any* moment t – identical to the phase-function $\beta(x, t)$ of the oscillation of the x -extended amplitude of the portion of wave that surrounds the singularity at that time t , though $\beta(x, t)$ designates a *wave-like* phase in the sense of Einstein relativity⁴⁸.

This theorem is crucial for understanding the meaning of the QM_{HD} -concept of eigenket. Indeed inside the Hilbert-Dirac formalism Louis de Broglie’s wave-function $\Phi(x, t) = ae^{(i/\hbar)\beta(x, t)}$ satisfies the equation $\mathbf{P}_x\Phi(x, t) = p_{x_j} \cdot \Phi(x, t)$ for eigenket and eigenvalues of the momentum observable. And

⁴⁵Cohen-Tannoudji C., Diu B. and Laloë, F., 1973.

⁴⁶ We introduce the notations ‘ Φ ’ and ‘ β ’ in order to distinguish from the start the representation of a physical phase of a physical wave introduced by Louis de Broglie, from the phase $\varphi(x, t)$ of a mathematical ‘state-function’ $|\psi(x, t)\rangle$ introduced inside a QM_{HD} state-ket $|\psi(x, t)\rangle$ that represents a formal tool for statistical predictions on results of measurements on a microstate.

⁴⁷ The conceptual content of this proof of only several lines is a jewel of human thought.

⁴⁸ So de Broglie’s model is a relativistic model in the sense of Einstein’s Relativity. Why, then, is QM_{HD} not formally compatible with Einstein’s relativity, is a basic mystery that remains to be solved before any attempt at unification of quantum mechanics with relativity.

the equation $\mathbf{A}|u_j(x, a_j)\rangle = a_j|u_j(x, a_j)\rangle$, $j = 1, 2, \dots, J$ generalizes this particular mathematical fact to any QM_{HD} -observable and introduces it in the ket-bra expressions of the formalism. This leads immediately to the following identification of the general meaning of the equation:

The eigenfunction $u_j(x, a_j)$ from the eigenket $|u_j(x, a_j)\rangle$ associated with the eigenvalue a_j of the observable \mathbf{A} , plays the role of a mathematical representation of ***a sample of a definite sort of wave-movement around the spatial location of the corpuscular-like singularity in the amplitude of the involved corpuscular wave***: If the wave-movement that surrounds the singularity is constantly represented by the eigenfunction $u_j(x, a_j)$, then – and only then – the value a_j of the mechanical quantity A that qualifies in mechanical terms the displacement inside the wave, of the location of the corpuscular-like singularity from the amplitude of the wave, stays constant.

As soon as this is spelled out it leaps to one's eyes that the form of the equation itself simply cries it out on the roofs. So – no offense to Bohr – de Broglie's model of a microstate is quite basically and massively present inside the whole formalism of QM_{HD} . It defines the physical-conceptual meaning of all the bases in the Hilbert-space of any microstate, as well as all the spectral decompositions of any state-ket. No more, no less. The whole formalism of QM_{HD} is an undeclared infusion from de Broglie's model, wherefrom the physical significances are drawn. In any decomposition $|\psi(x, t)\rangle/\mathbf{A} = \sum_j c(a_j, t)|u_j(x, a_j)\rangle$, $j = 1, 2, \dots, J$, of a state-ket $|\psi(x, t)\rangle$ with respect to the basis $\{|u_j(x, a_j)\rangle\}$, $j = 1, 2, \dots, J$, introduced in \mathcal{H} by an observable \mathbf{A} , the eigenket $|u_j(x, a_j)\rangle$ from the term $c(a_j, t)|u_j(x, a_j)\rangle$ is the symbol of the *sample of that what is counted by the real squared modulus* $|c(a_j, t)|^2$ of the *numerical complex coefficient* $c(a_j, t)$ (in an analogous way, in the expression 34m the symbol 'm' means that the length that is measured is 34 times the length of the sample of a meter from the National Bureau of Standards of Weights and Measures).

Consequences of the identification of the meaning of an eigenfunction. The preceding conclusion has noteworthy consequences.

- It evaporates the false 'problem' why an eigenfunction is in general⁴⁹ not required to be square-integrable: if it *were* required square-integrable *that* would be a real problem.

- In classical thinking a unique semantic dimension (for instance 'color') suffices for carrying *all* its 'values' ('red', 'green', etc.). But when a microstate is considered it obviously is very useful – if not even necessary – to analyse the representation more, namely so as to compensate for the absence of any perception of a quale for assigning meaning to the brute result of one act of measurement. The Hilbert-Dirac formalism realizes this analysis by a formal splitting: An observable-operator \mathbf{A} represents – separately – the considered semantic dimension and exclusively this ('a momentum' 'a total energy', etc.). And on the other hand – like in a catalogue joined to \mathbf{A} – inside the set $\{|u_j(x, a_j)\rangle, a_j\}$, $j = 1, 2, \dots, J$, are represented separately each one of the 'values' carried by the semantic dimension \mathbf{A} , and each 'value' is specified by a **pair** $(|u_j(x, a_j)\rangle, a_j)$, $j = 1, 2, \dots, J$, because the wave-movement of a corpuscular wave *and* a mechanical qualification of the 'corpuscular aspect' of that wave are both involved and are tied in a one-to-one connection⁵⁰. This is marvellously expressive and it is also effective when it is discretized via the adjunction of a corresponding unit for measuring the considered quantity $A(\mathbf{A})$.

- This also explains the high adequacy of the use of a Hilbert space \mathcal{H} for representing mathematically the predictions on issues of measurements on a microstate: Each 'value' $(|u_j(x, a_j)\rangle, a_j)$, $j = 1, 2, \dots, J$ of \mathbf{A} can be placed on a separate axis reserved to it, on which the state-ket $|\psi(x, t)\rangle$, when projected onto that axis, determines the complex number $c(a_j, t)$, so also the probability $|c(a_j, t)|^2$ predicted for the emergence of the pair $(|u_j(x, a_j)\rangle, a_j)$ if a measurement of A is performed upon the microstate with state-ket $|\psi(x, t)\rangle$ (which mimics geometrically the expansion $|\psi(x, t)\rangle/\mathbf{A} = \sum_j c(a_j, t)|u_j(x, a_j)\rangle$, $j = 1, 2, \dots, J$, of $|\psi(x, t)\rangle$).

⁴⁹ In a bound state of a microsystem the eigenket of the total energy has the same mathematical expression as the state-ket, it is confounded with the state-ket, and the eigenket and the state-ket are both required to be square-integrable and they are such.

⁵⁰ Degenerate spectra are not considered here.

- The preceding remarks specify that Dirac's theory of transformations expresses mathematically passages from a given 'semantic space', to another one: The semantic consists of the pairs $(|u_j(x, a_j)\rangle, a_j)$: *Dirac's calculus is a calculus endowed potentially with semantic specifications*⁵¹.

6.2 From the hidden presence of de Broglie's model inside QM_{HD} to its explicit, general, physical-operational incorporation into $[IQM-QM_{HD}]$

We shall now draw de Broglie's model into $[IQM-QM_{HD}]$, endowing it with an explicit operational meaning. We want to gain unrestricted access to *all* its conceptual potentialities, instead of keeping it limited to the meaning extracted from this model for, the concept of eigenstate of an observable.

Association of the general concept of microstate defined in Part I with de Broglie's model. After the cure of positivistic purity suffered by microphysics since nearly a century, according to which models were interdicted, what follows might be perceived as a shocking regression into intellectual primitivism. But I hold that we cannot indefinitely submit to arbitrary diktats and fashions, even if they have induced a reflex passive acceptance. Modern microphysics *oblige*s to penetrate now into the never as yet conceptualized before and to optimally conceptualize out of *that*. And this *oblige*s to specify in an explicit way a model for the considered microstates and to bring it to an expression that permits to freely work with it operationally whenever this is necessary.

So Louis de Broglie's model of a microstate, though inside QM_{HD} it remained hidden, has instilled meaning into the concept of eigenstate of an observable. Inside $[IQM-QM_{HD}]$ we will now extend the possibility to make free use of the potentialities of de Broglie's model by connecting it with the definition (1.1) of a microstate as well as with the definitions from section 2.1.

(a) We want to reconstruct a *mechanics* of microstates. According to de Broglie's model – that is essentially involved in QM_{HD} – only the corpuscular aspects from a corpuscular wave do admit mechanical qualifications. Consider now the definitions from section 2.1 of various sorts of microstates.

It is clear that what is called there 'system' has to be identified with de Broglie's 'corpuscular-like singularity' (in the amplitude of a corpuscular wave).

So we posit that the operation of generation G of 'one micro-state of one micro-system' introduces one de Broglie singularity into the domain of what can be qualified by a human observer, whereas an operation of generation $G(S_1, \dots, S_n)$ (where S_1, \dots, S_n are n systems), of one micro-state of n micro-systems introduces n de Broglie-singularities.

(b) It would be arbitrary – and also conceptually inconsistent – to conceive that an operation of generation G defined factually by the use of macroscopic apparatuses and conceptually by the use of only macroscopically controlled parameters, cuts radically the microsystem ms_G generated by it, from the indefinitely extended 'corpuscular wave' that incorporated it before the action of G . So we posit that G just captures for some time into the domain of what can be operated upon by human observers, a portion of a corpuscular wave that carries one de Broglie singularity if one microstate of one microsystem is generated, or carries n such singularities when one microstate of n microsystems is generated; while the main part of the wave-like phenomenon to which this portion of corpuscular wave was incorporated before the action of G , remains in the physical substratum, even though it is *connected* with the generated microstate via the captured corpuscular-like singularity(ies). Indeed what we are thinking about in this moment is *just* the frontier between the still a-conceptual physical reality, and that – from this substratum – that can be subjected to a very *first* process of conceptualization. The classical concept of a classical 'object' with 'definite' spatial volume is still so far that it is out of view⁵².

⁵¹ In Dirac's mind might have worked aims that they he did not care to communicate.

⁵² Nowadays not even the macroscopic classical 'objects' (a living body, a chair, etc.) are conceived to be radically cut from the surrounding 'physical waves', in some absolute sense. Though nobody knows what is vibrating, any 'object' in the classical sense is admitted to emit and to absorb waves of various natures, or to be traversed by such waves.

We also posit that:

(c) The *location* of the de Broglie singularity inside the corpuscular wave of the studied microstate ms_G in general *varies* arbitrarily from one individual specimen of ms_G to another one (this is an essential element from de Broglie’s own view, that led him first to his fundamental relation $p = h/\lambda$ and (much later) to his theory of a ‘double solution’ (de Broglie 1956)).

(d) The other characters of the corpuscular-wave trapped by the operation of generation G into the domain of possibility of interaction with it accordingly to human aims, constitute the ‘class of similarities’ that justifies their a priori common designation in (1.1) as ‘the *one* microstate ms_G generated by G ’, even though these characters are unknown as yet (they remain to be specified more in the future).

(e) Finally, in agreement with de Broglie’s works and with those of the nowadays physicists from Bohm’s school (in particular Peter Holland 1993), we also posit the famous *guidance relation* according to which the phase of the corpuscular wave in the neighbourhood of the singularity, ‘guides’ the singularity by determining its momentum.

On the basis of the assumptions $\{(a),(b),(c),(d),(e)\}$ we can now introduce the following new steps:

* **Model of a microstate.** The general model of a microstate that is specified by the assumptions $\{(a),(b),(c),(d),(e)\}$ is called the *G-(corpuscular-wave)-model* and will be denoted $ms_{G,cw}$ (*cw*: corpuscular wave).

* We introduce the following *modelling postulate*:

MP($\mathbf{ms}_{G,cw}$). Any one realization of G generates a particular instance of the model $ms_{G,cw}$ of a microstate accordingly to the assumptions $\{(a),(b),(c),(d),(e)\}$ and the definitions section 2.1.

* **Definition of inner specificities of a specimen of a microstate.** So – via the above points (a) and (d) – the modelling postulate $MP(ms_{G,cw})$ specifies a *defined* difference between, on the one hand

- the concept of ‘the microstate ‘ ms_G ’ that corresponds to G ’, introduced in (1.1) by just sticking on the result of G the label ‘of how this result has been generated, which is *exterior* to the result itself; and on the other hand

- the result of G , re-noted $ms_{G,cw}$ because via the modelling postulate $MP(ms_{G,cw})$ it is now endowed with *own* characters *interior to it*, drawn from de Broglie’s ideal and general model of a microstate.

From now on – but only when useful – a specimen of the microstate ms_G can be denoted $\sigma(ms_{G,cw})$.

We can also, when useful, re-write (1.1) $G \Leftrightarrow ms_G$ as

$$G \Leftrightarrow ms_{G,cw} \tag{6.1}$$

where

$$ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\} \tag{6.2}$$

(the notation $ms_G \equiv \{\sigma(ms_G)\}$ can also be used if the model of a microstate plays no role).

Louis de Broglie’s model of a microstate has been initially introduced with an only mental status. It has been conceived to be part of an only postulated, but unobservable, fundamental and universal wave-like physical factuality⁵³. But it has also been expressed mathematically, namely by an ideal, non-realizable plane wave. This however did not last. Louis de Broglie himself has quasi immediately replaced (only a couple of pages later) his initial representation by an ideal plane-wave, by the *operationally constructible* concept of a wave-packet. This apparently minimal step has been in fact a very misleading mathematical cover thrown over the *abyss of nature* that separates a mental, subjective, *individual* representation, from a consensual *statistical* representation of sets of numerical results of acts of measurement. Indeed while a *plane* ‘wave’ can be used as an indefinitely extended model of a definite kind of local wave-movement, a wave packet cannot be endowed with such a meaning, and nobody even tried to employ it for this aim. From the start it has been introduced in order to represent – by an *integrable* squared-amplitude – the *probability* of ‘presence’ in space, at any given time, of

⁵³ An instantiation of Spinoza’s concept of ‘substance’.

the distribution of the locations with respect to a common system of reference, of the corpuscular-like de Broglie singularities inside a big set of corpuscular waves. This surreptitious but radical switch of meaning tied with the passage from de Broglie's initial plane wave, to a wave-packet, has prolonged the plane-wave descriptor into a new domain of meaning, while the initial – and quite essential – domain of meaning, that of a model of wave-movement, has been abandoned *undeclared* and devoid of a mathematical representation recognized to be its own representation. This is the process that has generated the illusion that quantum mechanics manages to predict on microstates without involving any model of a microstate. But the in-homogeneity of meaning between state-ket and eigenket subsisted inside QM_{HD} . And the in-homogeneities of meaning, when enclosed in a mathematical representation, are not tolerated there because they hinder the writing of equalities and a fortiori of identities. Indeed, in the case of QM_{HD} they mixed up the results of sheer imagination about individual physical phenomena, with the results of merely abstract operations on sets of numbers placed on a statistical level of representation⁵⁴; this generated also the false problem of the non-integrability, in general, of the eigenket. But:

Inside the framework [$IQM-QM_{HD}$], via the modelling postulate $MP(ms_{G,cw})$ and (1.1), (6.2), de Broglie's initial mental model is incorporated to the result ms_G of the clear-cut operational and consensual concept of operation G of generation of a microstate. This endows now the result of G – re-noted $ms_{G,cw}$ – with qualifications posited to concern the *own* nature of the result ms_G **itself**, while initially, in (1.1), it was provisionally qualified exclusively by the label of the way in which it has been produced.

So the modelling postulate $MP(ms_{G,cw})$ is synergetic. It enriches the basic concept of microstate while de Broglie's model, via the operational and consensual concept of operation G of generation of a microstate, is drawn into an inter-subjective consensual representation of micro-phenomena.

Let us immediately add the following important remark:

In consequence of the content assigned to $MP(ms_{G,cw})$, the definition in section 2.1 of 'one microstate of one micro-system with *composed* operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$ ' – endowed with a *non-null* quantum potential so with possibility of quantum fields – *involves only one singularity in the amplitude of the corpuscular wave of the corresponding microstate $ms_{\mathbf{G}(G_1, G_2, \dots, G_k), cw}$* .

Even if walking on such a denuded edge between physics and metaphysics might instill uneasiness and vertigo, these results are a noteworthy advance: They offer from now on clearly defined concepts and words for conceiving specified investigations placed strictly on the frontier between the still a-conceptual factuality and what is extracted therefrom for a primary conceptualization able to lead to communicable, consensual and verifiable 'scientific' knowledge', and to subsequent developments⁵⁵. And mainly, it brings into evidence that on the very first level of conceptualization of microscopic physical entities, the spatial delimitation of perceptions of *individual* 'objects' in the classical sense, very likely are just human constructs. Individuality, like also stable space-time inner structure, might stem exclusively from the human way of conceiving and of characterizing results of the human cognitive actions.

6.3 Clarifications inside QM_{HD} via the concept of operation G of generation of a microstate

Use of the concept G of generation of a microstate would have economized the false problem of why eigenkets in general are not square-integrable. Indeed a state-ket $|\psi(x, t)\rangle$ is introduced in QM_{HD} as

⁵⁴ Such mixtures always entail uncontrollable consequences.

⁵⁵ For instance: a microstate of a 'system' (or a 'particle') with electric charge or magnetic moment can be drawn into the realm of the observable by use of macroscopic fields. But how could be manipulated the result of an operation G if this operation generates (for instance by a nuclear reaction) a 'particle' that is sensitive *exclusively* to a gravitational field? (Which probably means a maximally 'simple' de Broglie singularity, a 'pure quantum of *de Broglie-mass*' (with non-null spin)? a 'graviton'?). Such questions touch as much the most modern researches, as the *dBB* representation of the sub-quantic substance: In de Broglie [1956] the chapter XI, the pp. 119-131, are fascinating in relation with gravitation, teleportation, etc. And $MP(ms_{G,cw})$ and (1.1) offer legal scientific access to the *dBB* representation.

the ‘representation of the microstate ms_G to be studied’. So inside $[IQM-QM_{HD}]$ a state-vector $|\psi(x, t)\rangle$ is certainly connected with the operation of generation G from the trunk of the probability-tree of that microstate (namely it represents mathematically the set of all the predictive probability laws that in the *figure* 8.1 crown, on two levels, the branches of the probability tree of G). Whereas the concept of an eigenket $|u_j(x, a_j)\rangle$ appeared in section 6.2 to have been constructed as just a model of a possible wave-movement inside a specimen of the microstate to be studied, which in general does *not* depend of any operation of generation. Availability inside QM_{HD} of the concept ‘ G ’ would have detected this basic difference ⁵⁶.

Below we bring into evidence other three fundamental sorts of circumstances where the $[IQM-QM_{HD}]$ concept ‘ G ’ entails clarification of ambiguities or problems from QM_{HD} .

The concept of operation G of generation, spectral decompositions, and superposition-state-ket. Inside QM_{HD} works a mathematical principle of spectral decomposability of any state-ket $|\psi(x, t)\rangle$, i.e. the posit that for any state-ket it is justified to assert the equality ⁵⁷:

$$|\psi(x, t)\rangle / \mathbf{A} = \sum_j c_j(t) |u_j(x, a_j)\rangle, j = 1, \dots \quad (6.3)$$

Furthermore, the general choice of a vector-space-representation permits to write the state-ket associated to a microstate $ms_{\mathbf{G}(G_1, G_2, \dots, G_n)}$ generated by a composed operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$ (section 1.1), as a mathematical superposition

$$|\psi_{12\dots k}(x, t)\rangle = \lambda_1 |\psi_1(x, t)\rangle + \lambda_2 |\psi_2(x, t)\rangle + \dots \lambda_k |\psi_k(x, t)\rangle \quad (6.4)$$

of the state-ket of the microstates $ms_{G_1}, ms_{G_2}, \dots, ms_{G_k}$ that **would** have been obtained **if** each one of the operations of generation G_1, G_2, \dots, G_k that have been composed inside $\mathbf{G}(G_1, G_2, \dots, G_k)$ **would** have been realized *separately*.

In QM_{HD} the Hilbert space of a state-ket is extended into a ‘generalized-Hilbert space \mathcal{H} ’ where the eigenket are included as a limiting sort of vectors. This entails that from a strictly mathematical point of view both writings (6.3) and (6.4) are just superpositions of vectors inside \mathcal{H} , permitted by the mathematical *axiom* – included in the definition of the algebraic structure called a vector-space – that any two or more elements from a given vector-space admit an additive composition; which is expressed by saying that they can be ‘superposed’. This installed a purely mathematical language that calls *indistinctly* ‘superposition’ *any* additive combination of ket, whether only state-ket like in (6.4) or state-ket and eigenket like in (6.3), or only eigenket as in Dirac’s theory of transformations. *No physical criteria, nor conceptual ones, are made use of in order to make mutual specifications inside the general category of additive compositions in \mathcal{H} .* In section 6.2 we have seen an illustration of the consequences of precisely this blindness that illustrates strikingly the dangers of the intimate relation between physics and mathematics that emerges inside mathematical physics, and how, under the protection of this intimacy, mathematics can chase the intelligibility out of physics. In the case of the writing (6.3) this same sort of blindness induces into the minds the more or less explicit interpretation that all the eigenket $c_j(t) |u_j(x, a_j)\rangle$ from the second member are of the same nature as the state-ket $|\psi(x, t)\rangle$ from the first member. Which has been shown to be false. And in (6.4) it suggests that the state-ket $|\psi_{12\dots k}\rangle$ points toward a superposition of all the microstates $ms_{G_1}, ms_{G_2}, \dots, ms_{G_k}$, themselves that would ‘coexist inside $|\psi_{12\dots k}\rangle$ ’. Here we only draw attention upon this formally induced suggestion, because in the chapter 7 it will play a key-role.

And we just add that inside IQM , so also in $[IQM-QM_{HD}]$, any possibility of ambiguities of this sort is avoided by construction. Indeed :

⁵⁶ Dirac’s “impossibility to be realized” (of an eigenket) quoted in section 6.1 appears to be in fact only a mathematical manifestation of a *non-necessity* and even an *inadequacy* to be *expressed* as an integrable ‘quantity’ because *it is not that*. This sort of mathematical control of a mathematical formalism, over a *semantic* feature, is highly interesting and the mechanism that brings it about deserves being elucidated and utilized as a deliberate tool.

⁵⁷We recall that adaptation to a finite representation and the correlative finiteness of the domain of investigation – as required by our choice of effectiveness – will have to be introduced by an a posteriori conceptual-mathematical adjustment.

- In (6.3) only the state-ket $|\psi(x, t)\rangle$ from the first member corresponds – on the statistical level of conceptualization – to the studied microstate ms_G while all the terms $c_j(t)|u_j(x, a_j)\rangle$ from the right-hand expansion of $|\psi(x, t)\rangle$ are symbols of a product of a *number* $c_j(t)$ and a *model* $|u_j(x, a_j)\rangle$ of a possible corpuscular-wave-movement.

- In (6.4) the resulting *one* microstate $ms_{\mathbf{G}(G_1, G_2, \dots, G_k)}$ that is effectively generated by the *one* composed operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$, cannot be coherently conceived as a coexistence of all the microstates $ms_{G_1}, ms_{G_2}, \dots, ms_{G_k}$ that would have been obtained via the separate realizations of G_1, G_2, \dots, G_k . Indeed – by definition – these microstates have *not* been all generated separately via $\mathbf{G}(G_1, G_2, \dots, G_k)$ while according to (1.1) any actual microstate is in a one-one relation with a separately realized operation of generation; while furthermore, according to the modelling postulate $MP(ms_{G,cw})$, *one* microstate of *one* micro-system with a *composed* operation of generation introduces only *one* singularity in the amplitude of the corresponding corpuscular wave, whereas a *non-composed* operation of generation of one microstate of *two or more* micro-systems introduces, respectively, *two or more* singularities in the amplitude of the corpuscular wave. So inside $[IQM-QM_{HD}]$ the state-ket from the second member of (6.4) have to be regarded as *virtual* representational elements. Inside QM_{HD} these representational elements are useful precisely in consequence of the possibility to represent mathematically the state-ket $|\psi_{12\dots k}(x, t)\rangle$ by the additive expression (6.4). Indeed this possibility permits to deal mathematically with the observable factual *in-equality*

$$\pi_{\mathbf{G}(G_1, G_2, \dots, G_k)}(Xj) \neq \pi_{G_1}(Xj) + \pi_{G_2}(Xj) + \dots + \pi_{G_k}(Xj) \quad (6.5)$$

(cf. Part I): via the spectral decomposition (6.3) of $|\psi_{12\dots k}(x, t)\rangle$ on the basis of eigenket of an observable \mathbf{X} and application of Born's postulate of probability to the complex expansion coefficients $c_j(t)$, between the expansion coefficients there emerge *mathematical* 'interference'-terms that entail the inequality (3.5).

So QM_{HD} and IQM involve two distinct views concerning the significance of writings of the form (6.4): This is doomed to come to some factual confrontation.

We are aware of this, and attentive.

Quite generally now, *when a state-ket seems to be 'absent', this means that the microstate that corresponds to this state-ket is not generated separately*, so it has not been brought into factual existence. The most striking case of such 'absence' of a state-ket is that of *one* micro-state of *two or several* micro-systems, tied with the problem of locality. The formalism of QM_{HD} – rightly – represents such a microstate by only one state-ket. But for each involved system it introduces a distinct representation-space, and the mathematical relation between these spaces is specified in a way that is indicated by the now current words 'intrication' and 'non-separability'. Certain authors speak of «absence of an 'own' state-ket for each 'system'»; other authors speak of «absence of 'information'» (in what a sense, exactly?); as if a state-ket were a planet or a lake, something that 'is' somewhere outside but nobody knows how to go and see where and how it 'is'. In the textbooks it is written that «often a micro-system 'is' represented by a state-ket and if so the state is 'pure'»; while if it 'is' not pure, then it 'is' a 'mixture', but in such a case (happily) one can nevertheless dispose of a statistical operator (how, exactly, one should act in order to construct it, is *not defined*). But in the case of the 'problem of locality' not even a true statistical operator does 'exist', only 'a partial-trace' operator; but this *cannot change the fact that* there is non-separability because a statistical correlation is observed even when the spatial distance between the involved systems is very big. All the mentioned ways of speaking suggest that the state-ket, the statistical operators, etc., are conceived to possess an existence quite independently of the representational choices, decisions, elaborations *of humans, of physicists*. As soon as there is a 'system', 'its' state-ket should also 'be', and nevertheless sometimes it is absent and we do not know why, nor where it is gone. The special case of a microstate of two microsystems in the sense of section 2.1 is a particularly strong discloser of how the whole mathematical formalism of QM_{HD} is currently conceived:

We are in presence of a reification of the mathematical formalism of QM_{HD} , considered to constitute the whole of QM_{HD} , by itself alone.

The fact that QM_{HD} is just a human construction achieved by men in order to represent physical microstates, has receded out of the minds. This situation produces a sort of consternation. It even produces a sort of religious admiration for QM_{HD} because the experiments on locality have confirmed the predictions of the formalism⁵⁸.

But inside $[IQM-QM_{HD}]$ one understands that, and how, these attitudes stem from the following circumstances::

* Notwithstanding that, in general, the mathematical writings from QM_{HD} are in agreement with the definitions from section 2.1 *these definitions are not spelled out inside QM_{HD}* and furthermore, in the case of one microstate of two microsystems, in the current language that accompanies the use of the formalism one speaks of two or several ‘systems’ – never of one micro-state of two micro-systems – and so the *one-one indirect and non-explicated connection*

$$G \Leftrightarrow |\psi_G(x, t)\rangle \quad (6.6)$$

between G and the state-ket of the studied microstate ms_G generated by G – a connection that via (1.1) $G \Leftrightarrow ms_G$ is logically *entailed* inside $[IQM-QM_{HD}]$ – is simply out of perceptibility, even though it is generally accepted that always «a state-ket represents the studied ‘system’».

* Going now to the roots, all the preceding examples illustrate how inside QM_{HD} unintelligibility is entailed by the fact that no clear and systematic distinction is made between, on the one hand *individual* concepts (ms_G , or $|u_j(x, a_j)\rangle$) or physical entities (operations G , acts of measurement $MesA$, or specimens $\sigma(ms_G)$ of a microstate ms_G), and on the other hand the statistical descriptors like $|\psi_G(x, t)\rangle$.

In these conditions, inside the minds used to QM_{HD} like a New-York boy is used to Manhattan, an explanation is badly needed indeed, why sometimes some of the two or several state-ket that would be so ‘necessary’, nevertheless are stubbornly ‘absent’.

We close this point by the following convention:

Notational convention 1. Inside $[IQM-QM_{HD}]$ any state-ket $|\psi(x, t)\rangle$ that corresponds to a physically generated micro-state, will be re-noted as $|\psi_G(x, t)\rangle$, and the sort of operation G that indexes it will be explicitly stated, and when necessary it will be distinguished graphically.

For instance, (6.3) will be written as

$$|\psi(x, t)_G\rangle/\mathbf{A} = \sum_j c_j(t)|u_j(x, a_j)\rangle, j = 1..... \quad (6.7)$$

and (6.4) will be written as

$$|\psi(x, t)_{\mathbf{G}(G_1, G_2, \dots, G_k)}\rangle = \lambda_1|\psi_{G_1}(x, t)\rangle + \lambda_2|\psi_{G_2}(x, t)\rangle + \dots \lambda_k|\psi_{G_k}(x, t)\rangle \quad (6.8)$$

where the unique operation of generation $\mathbf{G}(G_1, G_2, \dots, G_k)$ that has been accomplished is written in bold font, while the only a priori possible but not separately realized operations of generation that in the second member are involved by the state-ket of virtual microstates-of-reference, will be written in non-bold font.

These specifications will entail clarification.

⁵⁸ It is true that it *does* seem amazing to find out to what a degree the mathematical formalism is ‘observant’ of (compatible with) the involvement or not, in a given state-ket symbol, of an operation of generation G of a corresponding micro-state and with the significance of the involved state-ket from the viewpoint of the definitions from section 2.1; notwithstanding that inside the formalism the concept ‘ G ’ is neither defined nor represented and the definitions from section 2.1 are not stated while the specific meaning of an eigenket has not been recognized either. Indeed: (a) a spectral decomposition (6.2) is usually conceived to involve an infinite number of terms, the coefficients from these terms are complex numbers dependent on time, and the eigenfunctions – models of wave-movement – are correctly written as independent on time; whereas (b) a superposition (6.3) of state-ket tied with a composed operation of generation, is written as a finite number of terms, the coefficients are usually constant real numbers, and the ket from the superposition are dependent on time. Everything in the mathematical writings is fully concordant with the analyses made here inside $[IQM-QM_{HD}]$. This raises strongly a very interesting question concerning something that could be called ‘*the semantic expressivity of mathematical internal syntactic coherence*’.

6.4 Refusal of von Neumann's representation of quantum measurements

Here we definitely walk into Absurdland, and so abruptly and totally that I do not dare to immediately make use of my own voice, by fear of being considered subjective and malevolent. So I first offer a look at the following extract from Wikipedia.

«The **measurement problem** in quantum mechanics is the problem of how (or *whether*) wave-function collapse occurs. The inability to observe this process directly has given rise to different interpretations of quantum mechanics, and poses a key set of questions that each interpretation must answer. The wave-function in quantum mechanics evolves deterministically according to the Schrödinger equation as a linear superposition of different states, but actual measurements always find the physical system in a definite state. Any future evolution is based on the state the system was discovered to be in when the measurement was made, meaning that the measurement “did something” to the system that is not obviously a consequence of Schrödinger evolution.

To express matters differently (to paraphrase Steven Weinberg[1][2]), the Schrödinger wave equation determines the wavefunction at any later time. If observers and their measuring apparatus are themselves described by a deterministic wave function, why can we not predict precise results for measurements, but only probabilities? As a general question: How can one establish a correspondence between quantum and classical reality?[3].

Schrödinger's cat

The best known example is the “paradox” of the Schrödinger's cat. A mechanism is arranged to kill a cat if a quantum event, such as the decay of a radioactive atom, occurs. Thus the fate of a large scale object, the cat, is entangled with the fate of a quantum object, the atom. Prior to observation, according to the Schrödinger equation, the cat is apparently evolving into a linear combination of states that can be characterized as an “alive cat” and states that can be characterized as a “dead cat”. Each of these possibilities is associated with a specific non-zero probability amplitude; the cat seems to be in some kind of “combination” state called a “quantum superposition”. However, *a single, particular observation* of the cat does not measure the probabilities: it always finds either a living cat, or a dead cat. After the measurement the cat is definitively alive or dead. The question is: *How are the probabilities converted into an actual, sharply well-defined outcome?*

Interpretations (Main article: Interpretations of quantum mechanics)

Hugh Everett's many-worlds interpretation attempts to solve the problem by suggesting there is only one wavefunction, the superposition of the entire universe, and it never collapses—so there is no measurement problem. Instead, the act of measurement is simply an interaction between quantum entities, e.g. observer, measuring instrument, electron/positron etc., which entangle to form a single larger entity, for instance *living cat/happy scientist*. Everett also attempted to demonstrate the way that in measurements the probabilistic nature of quantum mechanics would appear; work later extended by Bryce DeWitt.

De Broglie–Bohm theory tries to solve the measurement problem very differently: this interpretation contains not only the wavefunction, but also the information about the position of the particle(s). The role of the wavefunction is to generate the velocity field for the particles. These velocities are such that the probability distribution for the particle remains consistent with the predictions of the orthodox quantum mechanics. According to de Broglie–Bohm theory, interaction with the environment during a measurement procedure separates the wave packets in configuration space which is where apparent wavefunction collapse comes from even though there is no actual collapse.

Erich Joos and Heinz-Dieter Zeh claim that the phenomenon of quantum decoherence, which was put on firm ground in the 1980s, resolves the problem.[4] The idea is that the environment causes the classical appearance of macroscopic objects. Zeh further claims that decoherence makes it possible to identify the fuzzy boundary between the quantum microworld and the world where the classical intuition is applicable.[5][6] Quantum decoherence was proposed in the context of the many-worlds interpretation[citation needed], but it has also become an important part of some modern updates of the Copenhagen interpretation based on consistent histories.[7][8] Quantum decoherence does not describe the actual process of the wavefunction collapse, but it explains the conversion of the quantum probabilities (that exhibit interference effects) to the ordinary classical probabilities. See, for example, Zurek,[3] Zeh[5] and Schlosshauer.[9]

The present situation is slowly clarifying, as described in a recent paper by Schlosshauer as follows:[10]

Several decoherence-unrelated proposals have been put forward in the past to elucidate the meaning of probabilities and arrive at the Born rule ... It is fair to say that no decisive conclusion appears to have been reached as to the success of these derivations. ...

As it is well known, [many papers by **Bohr** insist upon] the fundamental role of classical concepts. The experimental evidence for superpositions of macroscopically distinct states on increasingly large length scales counters such a dictum. Superpositions appear to be novel and individually existing states, often without any classical counterparts. Only the physical interactions between systems then determine a particular decomposition into classical states from

the view of each particular system. Thus classical concepts are to be understood as locally emergent in a relative-state sense and should no longer claim a fundamental role in the physical theory.

A fourth approach is given by objective collapse models. In such models, the Schrödinger equation is modified and obtains nonlinear terms. These nonlinear modifications are of stochastic nature and lead to a behaviour which for microscopic quantum objects, e.g. electrons or atoms, is unmeasurably close to that given by the usual Schrödinger equation. For macroscopic objects, however, the nonlinear modification becomes important and induces the collapse of the wavefunction. Objective collapse models are effective theories. The stochastic modification is thought of to stem from some external non-quantum field, but the nature of this field is unknown. One possible candidate is the gravitational interaction as in the models of Diósi and Penrose. The main difference of objective collapse models compared to the other approaches is that they make falsifiable predictions that differ from standard quantum mechanics. Experiments are already getting close to the parameter regime where these predictions can be tested.[11]

An interesting solution to the measurement problem is also provided by the hidden-measurements interpretation of quantum mechanics. The hypothesis at the basis of this approach is that in a typical quantum measurement there is a condition of lack of knowledge about which interaction between the measured entity and the measuring apparatus is actualized at each run of the experiment. One can then show that the Born rule can be derived by considering a uniform average over all these possible measurement-interactions. [12][13]. >>

I now dare to continue by my own summary of the situation. In what follows immediately we are inside QM_{HD} alone, not inside $[IQM-QM_{HD}]$. So I just reproduce the current nowadays language and reasoning about quantum measurements:

The representation of measurements on microsystems is that one proposed by von Neumann in 1932: The Schrödinger equation of the problem endows us with the state-ket of the problem, $|\psi(x, t)\rangle$. So this state-ket is given mathematically, we dispose of it from the start in consequence of purely mathematical operations. We want now to *represent* the measurements. Therefore we have to write the state-ket for the measurement-interaction. For this we proceed as follows: Let $t = t_0$ be the initial moment given in $|\psi(x, t)\rangle$. At a time $t_1 > t_0$ we want to measure the observable \mathbf{A} on the ‘system’ represented by $|\psi(x, t)\rangle$. We take now into account that for $t \geq t_1$ there is interaction between the studied system and the measurement-apparatus. So:

For $t \geq t_1$ the measurement-evolution must represent also the apparatus “because” the apparatus is also constituted of microsystems.

So the measurement-evolution is to be represented by a state-ket of [(the studied system S)+(the apparatus for measuring \mathbf{A})]. Let us then write, say, $S + app(\mathbf{A})$ and $|\psi_{S+app(\mathbf{A})}(x, t)\rangle$. Since we measure the observable \mathbf{A} , the expansion of $|\psi_{S+app(\mathbf{A})}(x, t)\rangle$ with respect to the basis of \mathbf{A} comes in. Accordingly to the well-known quantum theory of a ‘system composed of two systems’ we write the tensor-product expansion:

$$|\psi_{S+app(\mathbf{A})}(x, t)\rangle = \sum_k \sum_j c_j(t) d_k(t) |u_j(x, a_j)\rangle |q_k(x, a_k)\rangle, \quad j = 1, 2, \dots, \quad k = 1, 2, \dots \quad (6.9)$$

where $|q_k(x, a_k)\rangle, k = 1, 2, \dots$ are the eigenket of the observable called the ‘needle-position of the $app(\mathbf{A})$, that can be denoted $\chi(\mathbf{A})$, with eigenvalues, say $(\nu(a_k)), k = 1, 2, \dots$, that express, respectively, ‘the needle-positions of $app(\mathbf{A})$ that correspond to the eigenvalues a_k of \mathbf{A} ’. Furthermore – by the definition of the concept of ‘apparatus for measuring \mathbf{A} ’ – the set $\{c_j(t) d_k(t)\}$ of the global, product-expansion coefficients $(c_j(t) d_k(t))$ from (6.9) reduces to a set $\{\alpha_{jj}(t)\}$ (with $\alpha_{jj} = c_j d_j$) of only the coefficients with non-crossed indexation, because the needle position $\nu(a_j)$ of the $app(\mathbf{A})$ is what – **alone** – indicates the obtained eigenvalue a_j of \mathbf{A} ⁵⁹. So in fact in this case we have only

$$|\psi_{S+app(\mathbf{A})}(x, t)\rangle = \sum_j \sum_j \alpha_{jj}(t) |u_j(x, a_j)\rangle |v_j(x, a_j)\rangle, \quad j = 1, 2, \dots \quad (6.10)$$

⁵⁹ So no coding problem arises according to this ‘measurement-theory’: One is protected from this problem, the apparatus will know where to settle its needle, since it is conceived for this aim.

The measurement evolution is produced accordingly to a measurement-Schrödinger equation *where the Hamiltonian operator $\mathbf{H}(\mathbf{A})$ commutes with \mathbf{A}* . And it is *posited* that this evolution finishes with a definite needle position $\chi(a_j)$ that indicates *one* definite result a_j ⁶⁰.

Now, the above-mentioned representation is considered to raise two ‘problems’.

- The *reduction problem*: what happened to all the terms from (6.10) with index $k \neq j$ that accordingly to a linear formalism should subsist? Where have they disappeared?
- The problem of ‘decoherence’: how can we *prove* that after the realization of the position $\chi(a_j)$ of the apparatus-needle that announces the result a_j , the measurement interaction really ceases⁶¹?

Here finishes my own summary of the general framework accepted for the representation of measurements. In what follows I go now back inside [IQM-QM_{HD}] and I speak again for myself and by use of the language introduced up to now in this work.

Bertrand Russell has written somewhere that aims are induced by temperament while the choice of a method is induced by intelligence. With respect to the aim to represent the measurements on microstates, von Neumann’s choice of a method is stunning. If we followed his argument, in order to measure the position of a star by use of a telescope, given that the telescope and the star are both made of microsystems, we should represent [(the telescope)+(the star)+(the measurement interaction between these two entities)]; and we should prove in terms of the theory thus conceived, that the star and the telescope do really separate physically once the star’s position has been established. Such an argument manifests luminously a total blindness with respect to the rather obvious fact that in science what decides the optimality of a representation is the *cognitive situation* of the observer-conceptualiser with respect to that on what he wants to obtain some knowledge, etc. The inner constitution of that what has to be qualified, or of the instruments that are made use of, has nothing to do with the criteria for generating the desired knowledge. The functionality of a construct is not in a one-to-one relation with its material (or abstract) structure (those who have realized the aeroplanes have thoroughly understood that). Moreover, in the case of microstates, most often what *can* be registered is just marks on a sensitive registering device and/or durations determined by chronometers. From these data one has to *construct conceptually* the researched ‘value’ of the measured ‘quantity’ that, in its turn, is constructed beforehand on the basis of conceptual-mathematical operations. And finally, von Neumann’s representation of measurements dodges the crucial coding problem. It simply makes it disappear behind an amorphous heap of words and symbolic writings void of definition, so of meaning. Indeed:

Von Neumann’s representation of measurements even *transgresses* QM_{HD}: ‘The observable’ $\chi(\mathbf{A})$ called the ‘needle-position’ of the *app*(\mathbf{A}) is not a quantum mechanical observable, it cannot be constructed formally in a definite way from one definite classical mechanical quantity, and so its eigenfunctions cannot be *calculated*.

An apparatus that is made use of in a scientific description of something else than this apparatus itself is introduced as a *primary datum*, if not one enters indefinite regression⁶². This is a logical interdiction. And so on. So I declare without shades that *I quite radically reject von Neumann’s framework for representing quantum measurements*.

⁶⁰ As far as I know, this has never been proved inside QM_{HD} to be generally insured by the condition imposed upon the measurement evolution.

⁶¹The locality-problem incites to think that it might not do this, but so what?

⁶² Wittgenstein has written somewhere: «There is one thing of which one can say neither that it is one metre long, nor that it is not one metre long, and that is the standard metre in Paris». I dare to complete: At least one class of things cannot be absorbed into the quantum mechanical representation of measurements: The class of the measurement apparatuses.

6.5 Investigation on the implicit assumptions of the conceptual essence of the QM_{HD} theory of measurements

Preliminaries: questions, a fundamental distinction, notational convention. We shall now concentrate on the essence of the representation of quantum measurements because the core of the unintelligibility of QM_{HD} is hidden there. The developments from chapter 6 and the refusal of von Neumann's 'theory' of quantum measurements leave us with real, crucial problems of intelligibility that must be stated independently before any attempt is made at perceiving a way of solution.

Questions.

Let us dare to pour out (with more or less feigned naivety) childish questions that might come in mind concerning quite generally the *verification* of statistical predictions.

One constructs statistics in order to be able to predict statistically. Sometimes the construction can be realized by purely mathematical operations (via some equation, algorithm, etc.) and this has initially happened indeed in microphysics for bound states of microsystems. But in most current circumstances a predictive statistic has to be constructed via individual acts of measurement.

Consider now the operation of *verification* of the predictions drawn from a given statistic. It is not impossible to imagine directly and exclusively statistical verifications of the predictions, via collective acts of measurement (this also can be done in certain cases concerning the total energy of bound microstates (absorption or emission of radiation versus registration of the intensity of spectral lines of emission or absorption of radiation)). This possibility is precious when individual verifications seem out of question or are awkward. But:

1. Can at least one example be found, of *verification* of a statistic conceived to basically concern some given category of individual measurement-operations on free microstates, via – exclusively – *individual modifications of this statistic itself*?

This seems very difficultly conceivable because a statistic is just a set of abstract signs – a set of numbers –, while verification of the statistical predictions expressed by such a set of numbers involves some *factual* use of that about what the statistic predicts. A set of numbers cannot scratch marks on registering devices, nor trigger registrations by a chronometer. For *this* sort of effects the presence of something material is required. Nevertheless the reduction problem raised by von Neuman's representation of quantum measurements seems to presuppose a possibility of this sort. Thereby one is led to ask:

2. Does a state-ket $|\psi(x, t)\rangle$ (or $|\psi_{S+app(\mathbf{A})}(x, t)\rangle$, no matter) lodge inside itself something physical? Then *what*, exactly, is the physical content of this or that sort of 'wave-function' from a state-ket? What *coding*-procedure is involved in a measurement-evolution of a state-ket, that can effectively lead to the consensual *identification* of the result of one measurement-evolution *of that state-ket*?

3. If there exists an answer to the question 2, how has it been *possible* to identify it – even if only on the basis of implicit reasoning – without making use of a general model of a microstate? Has de Broglie's model of a microstate been made use of, in fact, via the concept of eigenstate that involves this model? If this is so, *how*, exactly, has it been made use of? It is usually admitted that QM_{HD} offers statistical predictions that only mirror in the mathematical tools the involved physical facts and operations. This is certainly so. But at least it should be stipulated also of what, exactly, *that* what is mirrored does consist.

These are the genuine problems with which we are now left. Let us try to examine them inside $[IQM-QM_{HD}]$ where the individual level of conceptualization and the statistical one, are mutually distinguished.

6.5.1 A fundamental distinction: individual physical wave-functions versus abstract statistically predictive 'state'-functions.

We have already noted that the wave function initially introduced by Louis de Broglie was conceived to represent a *physical* 'corpuscular wave' $\Phi(x, t) = a(x, t)e^{(i/\hbar)\beta(x, t)}$, but of which the mathematical

representation was ideal, a plane wave with constant amplitude where the ‘corpuscular-like singularity’ remained non-specified formally; while very rapidly afterward, this initial representation has been replaced by an operationally realizable ‘wave-packet’ of which the amplitude was used to express the ‘presence *probability*’ of the singularity. Thereby in de Broglie’s mind the content of the initial descriptor Φ became more ‘complex’:

- On the one hand, by its amplitude Φ pointed toward a statistical-probabilistic prediction for results of repeated *position*-registrations of the corpuscular-like singularity posited to be involved by the amplitude of the specimens of the considered sort of physical wave, which is a piece of statistical predictive knowledge.

- And on the other hand, via its phase β , Φ was conceived to continue to point also toward a physical individual entity, namely the physical ‘corpuscular’ singularity from the amplitude of each specimen of the considered sort of physical wave, but that remained non-represented in the mathematical function ‘ Φ ’.

So, in fact, *de Broglie’s physical individual corpuscular wave has never been fully represented mathematically*⁶³. In this way the history of quantum mechanics has *started* directly with an only *statistical* mathematical representation of a microstate in the sense of de Broglie. The in-distinction inside the mathematical formalism of QM_{HD} between the individual level of conceptualization, and the statistic-probabilistic level, stems from de Broglie’s Thesis itself. Let us begin our investigation of the measurement problem by finally suppressing this in-distinction via a second notational convention:

Notational convention 2. The *physical individual wave-like* phenomenon introduced in the domain of scientific conceptualization by one realization of the operation G of generation of one specimen $\sigma(ms_{G,cw})$ of the studied microstate $ms_{G,mw}$ (in the sense of (6.1) and (6.2)) will be systematically denoted by a completed *wave-function* $\Phi_{G,cw}(x,t)$ ⁶⁴ (in short Φ_G) of which the mathematical form is posited to include the representation of also the ‘corpuscular singularity(ies)’ from the amplitude. While the *state-function* from the QM_{HD} -state-ket $|\psi_G(x,t)\rangle$ associated with $ms_{G,cw}$ will represent *exclusively* a mathematical tool for statistical predictions concerning results of measurements on individual specimens $\sigma(ms_{G,cw})$.

The distinction introduced above does not in the least interdict any possible degree of similitude between the global mathematical forms of Φ_G and $|\psi_G\rangle$ ⁶⁵.

6.6 Implications of the QM_{HD} -representation of measurements and critique

The coding rule implied by the QM_{HD} -formalism. Since we refuse the von Neumann representation of quantum measurements we go back to the initial representation of these⁶⁶. The notations,

⁶³ Louis de Broglie has tried, but very late after his thesis (de Broglie 1956), to finally introduce a clear separation between a physical individual corpuscular wave – renamed *u-function* – and statistics concerning results of measurements for mechanical qualifications of this sort of entity.

⁶⁴ We maintain the notation Φ .

⁶⁵ For a free microstate there certainly exists a strong similitude between the mathematical function $\Phi_G(x,t)$ appropriate for representing $\sigma(ms_{G,cw})$, and the state-function $\psi_G(x,t) = a(x,t)e^{(i/\hbar)\varphi(x,t)}$ from the state-ket of ms_G . But in consequence of the predictive task assigned to $\psi_G(x,t) = a(x,t)e^{(i/\hbar)\varphi(x,t)}$ – also certainly – there is no identity (this is now clear by definition for the amplitude function $a(x,t)$; but even the phase $\varphi(x,t)$ from $\psi_G(x,t)$ might indicate only some sort of mean-phase with respect to the unknown individual phase functions $\beta(x,t)$ that are involved in the set of specimens $\{\sigma(ms_{G,cw})\}$ of the studied microstate ms_G (cf. (6.2)). Anyhow, for now the fact is that in general we do not know what equation can yield as solutions the functional representations of the individual specimens $\sigma(ms_{G,cw})$, and nothing insures a priori that the Schrödinger equation of the problem would always be adequate (de Broglie asserts such an equation and has characterized it in detail (de Broglie [[1956],[1957]]); nevertheless, of coarse, the features that in this work stem from *IQM* are entirely absent from his approach).

⁶⁶ Our aim here is – exclusively – to identify *how* inside QM_{HD} it has been supposed that one can translate the result of one act of measurement, in terms of a definite value of the measured quantity. To those who want to understand clearly *all* the aspects of the problem I recommend strongly to read the outstanding analysis of Bohm (1954, pp. 588-608). His Stern-Gerlach illustration brings forth the essence. And a patient reader will explicate from this analysis that for the aim spelled out above the von Neumann representation is of no use whatever while it permits to complicate the reasoning as

as well as the point of view are those from [IQM-QM_{HD}].

We admit for the moment the current supposition that it has been possible to generate the state-ket $|\psi_{G,\mathbf{H}}(t)\rangle$ of the microstate ms_G to be studied via the Schrödinger equation of the problem, acted by a Hamiltonian operator \mathbf{H} . The general solution of this equation involves an infinite number of definite solutions and in order to select the time dependent solution $|\psi_{G,\mathbf{H}}(t)\rangle$ that corresponds to the considered problem one has to give that solution at the initial moment t_0 , $|\psi_{G,\mathbf{H}}(t_0)\rangle$. If at a time $t_1 \geq t_0$ one wants to measure the observable \mathbf{A} on a specimen of ms_G , the procedure is as follows. Write the expansion $|\psi_{G,\mathbf{H}}(t_1)\rangle/\mathbf{A}$ of $|\psi_{G,\mathbf{H}}(t_1)\rangle$ on the basis $\{|u(x, a_j,)\rangle\}$, $j = 1, 2, \dots, J$ of eigenket of \mathbf{A} :

$$|\psi_{G,\mathbf{H}}(t_1)\rangle/\mathbf{A} = \sum_j c_j(a_j, t_1)|u(x, a_j,)\rangle, \quad j = 1, 2, \dots, J \quad (6.11)$$

Starting from t_1 the action of \mathbf{H} is stopped and the state-ket $|\psi_{G,\mathbf{H}}(t_1)\rangle$ is subjected to a new evolution, namely a ‘measurement-evolution’ defined by a new Schrödinger equation where acts a measurement-Hamiltonian $\mathbf{H}(\mathbf{A})$ that commutes with \mathbf{A} . So for the duration $t_f - t_1$ that separates the time t_1 when the measurement evolution begins, from the time t_f when it finishes, a ‘measurement-state-ket’ $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle$ is introduced. The spectral decomposition of this new state-ket on the basis $\{|u(x, a_j,)\rangle\}$, $j = 1, 2, \dots, J$ of eigenket of \mathbf{A} is then performed:

$$|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A} = \sum_j c_j(t_1)|u(x, a_j,)\rangle, \quad j = 1, 2, \dots, J \quad (6.12)$$

The evolution 6.12 is supposed *not* to change the square moduli $|c_j(t_1)|^2$ of the coefficients $c_j(a_j, t_1)$ during the time interval $t_f - t_1$, while the eigenket $|u(x, a_j,)\rangle$ are time-independent. So the effect of the evolution can only consist in the way in which the coefficients $c_j(a_j, t_1)$ change. We want to specify this change.

Remember Gottfried’s presentation of quantum measurements (1966), de Broglie’s analyses (1957), and quite especially the Stern-Gerlach method for spin measurement (Bohm 1954) and Feynman’s time-of-flight method for measuring the momentum observable, thoroughly studied by Park and Margenau (1968). When these last two methods are examined it appears quite clearly that what is supposed is that: *a measurement-evolution (6.12) generates, for any index j , statistical correlation between the presence of observable marks inside a more or less extended space-time domain $(\Delta x, \Delta t)_{k_A}$, and a corresponding eigenvalue a_{k_A} of \mathbf{A} , to be considered as the result of the performed measurement-evolution (in particular Δx or Δt can be null) ⁶⁷.*

much as one likes. It is worth noting that this brilliant mathematician has initiated long-lasting conceptual catastrophes both times that he touched to quantum mechanics (the assertion of the absolute impossibility of hidden parameters, and the representation of measurements).

⁶⁷ The method ‘time of flight’ is complete and paradigmatic. Therefore we summarize it:

Let $\delta E(G)$ be the space-domain covered by one realization of the operation G of generation of one specimen of the microstate ms_G to be studied. Place a very extended detection-screen S sufficiently far from the space domain $\delta E(G)$ for permitting to assimilate S to a point relatively to the distance OS between S and $\delta E(G)$ measured along an axis O_x that starts on $\delta E(G)$ and is perpendicular on S . (The duration $\delta t(G)$ of the operation of generation G does not come in, only the time elapsed between the moment t_0 when the operation of generation G ends and time t when an impact is recorded on S , does matter, as it appears below). Then we act as follows :

(a) We effectively carry out an operation G_n and we denote by t_0 the time when G_n ends (the index ‘ n ’ individualizes the considered realization of G and it will also mark all the data that are specifically tied with this realization)

(b) If the result of G_n included fields then at the time t_0 we turn them off. If between the space-time support $\delta E, \delta t(G_n)$ and the screen S there pre-exist external fields or material obstacles, we remove them.

On the basis of these precautions the measurement-evolution assigned to the specimen of ms_G created by the realized operation G_n is posited to be a ‘free’ evolution, i.e. without acceleration (Notice immediately that this precaution can only stem from the presumption that ms_G is such that it does involve a localized ‘momentum’ $\mathbf{p} = m\mathbf{v}$ for which any acceleration would modify the vector value, that is, it can only stem from a presupposed model of a microstate).

(c) After some time an impact is produced on a spot of the screen S that we indicate by a coordinate x_n . When this happens the needle of a chronometer connected to S moves to a non-zero position, say ch_n , that marks the time t_n when this event occurs. This is indicated by saying that “the ‘time of flight’ has been registered to have been $\Delta t_n = t_n - t_0$. (But ‘flight’ of what? Again the involved model manifests its hidden presence).

(d) The vector-value of the distance \mathbf{d}_n covered between $\delta E(G)$ and x_n is denoted $\mathbf{d}_n = \mathbf{0}_{x_n}$. The square of the absolute value of this distance is $|\mathbf{d}_n| = \sqrt{d_{xn}^2 + d_{yn}^2 + d_{zn}^2}$ where $d_{xn} \equiv OS$ is measured on O_x and d_{ny} , d_{nz} are measured on the two axes from the plane of S that, with O_x , determine an orthogonal Cartesian referential.

Let us keep the denotation *dB* for the approach developed by de Broglie and Bohm. We denote by *BBGPM* the view on quantum measurements developed by de Broglie, Bohm, Gottfried, Park and Margenau. If this last view, explicated above, is indeed factually true, then – via the *presence* probability $\pi(X(a_{j(\mathbf{A})}))$ in the physical space – a *measurement-evolution* (6.12) entails a coding rule :

$$[(\text{a group of marks } \{\mu_j\}_{\mathbf{A}}, j_{\mathbf{A}} = 1, 2, \dots, J_{\mathbf{A}}) \text{ on } (\Delta_x \cdot \Delta_t)_{j(\mathbf{A})}] \equiv (\text{outcome of the eigenvalue } a_{j(\mathbf{A})} \text{ of } \mathbf{A})] \quad (6.13)$$

where Δ_x or Δ_t can be null.

However as far as I know no general proof of such a rule has been worked out inside QM_{HD} . So it seems likely that a general proof is not possible inside QM_{HD} . But this is not a crucial circumstance. Indeed Park and Margenau (1968) *did* prove the correlation 6.13 inside QM_{HD} for the particular case of the basic momentum observable \mathbf{P}_x ; for the basic position observable \mathbf{X} the correlation is tautologically valid, by definition ; and the QM_{HD} ‘postulate of representation’ of the dynamical observables permits to form by calculus any other observable $\mathbf{A}(\mathbf{X}, \mathbf{P}_x)$ from $(\mathbf{X}, \mathbf{P}_x)$, so also its eigenvalues $a_{j(\mathbf{A})}$ from the eigenvalues (X_j, p_{xj}) of, respectively, \mathbf{X} and \mathbf{P} (indirectly if \mathbf{A} is a vector-observable). Nevertheless we want to *understand*, just to understand on what grounds and for which category(ies) of microstates – inside $[IQM-QM_{HD}]$ where we are now constructing – a one-to-one coding-rule of the essence of 6.13 could be genuinely founded and expressed explicitly. For this goal – an important goal – we remember that the method time-of-flight for coding the observable marks registered by one act of measurement of the momentum observable, is intimately founded upon undeclared *classical* assumptions concerning a ‘trajectory’ of that what produces the marks. So we shall try to insure as much as possible comparability with the behaviour asserted in classical mechanics for a classical mobile. We proceed as follows.

We go back to (6.12). The \mathbf{A} -measurement-state-ket $|\psi_{G, \mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle / \mathbf{A}$ from (6.12) is a statistical descriptor. In section 6.6 we have introduced a radical distinction between a statistical state-ket $|\psi_G(x, t)\rangle$ that is a mathematical tool for statistical predictions, and an individual physical wave-function Φ_G – in general *unknown*, that accordingly to the modelling postulate $MP(ms_G, cw)$

(e) The vector-eigenvalue \mathbf{p}_n measured for the quantum mechanical momentum-operator, and its absolute value $|\mathbf{p}_n|$, are calculated according to the formulas

$$\mathbf{p}_n = m(\mathbf{d}_n / \Delta t_n, |\mathbf{p}_n| = m(\sqrt{d_{x_n}^2 + d_{y_n}^2 + d_{z_n}^2}) / \Delta t_n$$

where m is the ‘mass’ associated with the microsystem involved by the microstate ms_G such as this mass is defined in atomic physics or in the theory of elementary particles (here again models come in: The mass in de Broglie’s sense has a different definition, and this difference will certainly come out one day to be very important).

This completes the considered act of momentum-measurement. Now note what follows.

In the case described above the observable physical manifestations produced by the act of measurement are : the position x_n of a point-like mark and the position ch_n of the needle of a chronometer connected to the screen S . These manifestations are not directly numerical values, nor do they ‘possess’ any quale. They are only perceptible physical effects, say μ_{1n} and μ_{2n} , respectively, produced by the act of measurement on two ‘recorders’ of the utilized measurement-apparatus. (The apparatus being made up of a chronometer associated with the operation G , the suppressor of external field, the screen S , and the chronometer connected to the screen).

The meanings associated with the recorded observable marks as well as the numerical values associated with these are defined simultaneously by:

- the way of conceiving an act of measurement of, specifically, the momentum \mathbf{p} assigned by postulate to a microstate;
- the posited relations $\mathbf{p} = m(\mathbf{d} / \Delta t, |\mathbf{d}| = \sqrt{d_{x_n}^2 + d_{y_n}^2 + d_{z_n}^2})$ and $\Delta t = t - t_0$ of which the first two ones are imported from the classical mechanics and the classical atomic physics.

It seems utterly clear that: (1) the concept of momentum \mathbf{p} involved by the studied microstate ms_G is defined in the classical manner and that the whole coding procedure called ‘time of flight’ seems reasonable precisely because and only because it changes the micro-state in such a way that it does not also alter the value of \mathbf{p} that is to be measured. This procedure would be completely arbitrary — and even inconceivable — in the absence of the classical macroscopic model from which it stems; (2) the vector-value registered via the observable marks (x_n, ch_n) where the location of x_n is in fact permitted to vary inside some space-domain Δx because the origin of the flight cannot be defined strictly, while ch_n can be endowed with a minimal in-definition Δt entailed by the chosen unit of time). So the method for constructing from the registered ‘marks’ (x_n, ch_n) the measured vector-value \mathbf{p}_j of \mathbf{p} , consists indeed of generating via this measurement-evolution a statistical correlation between the presence of the registered ‘marks’ (x_n, ch_n) inside a space-time domain $(\Delta x, \Delta t)_j$, and a corresponding value \mathbf{p}_j of \mathbf{p} .

Similar conclusions hold concerning the Stern-Gerlach method for measuring the spin (in which case Δt is null and Δx is a whole semi-plan).

from section 6.2) represents a *physical* specimen $\sigma(ms_{G,cw})$ of the studied microstate ms_G , able to produce observable marks via its singularity in the amplitude of the physical corpuscular wave of $\sigma(ms_{G,cw})$. We keep this in mind.

Let us consider the most simple case, that of a free microstate ms_G of one microsystem and with *non*-composed operation G of generation. Indeed according to the *dBB* representation the existence of a non-null quantum potential internal to ms_G , even if it is constant, could generate uncontrollable quantum fields when an obstacle acts (think of ‘walls’, ‘barriers’, ‘wells’) and so act as a new operation of generation, namely of a microstate of one microsystem with a composed operation of generation. So a microstate with *non*-composed operation G of generation is much protected from the emergence of quantum fields. Then, insofar that moreover the external obstacles are also expressly suppressed, all the possible sources of uncontrolled inner instability are eliminated. So, in such conditions and from a mechanical point of view, the unique corpuscular singularity from the corpuscular wave of any given specimen $\sigma(ms_{G,cw})$ of the studied microstate ms_G should behave exactly as a classical mobile would behave in the same external conditions.

Notwithstanding all these precautions, there persists a problem in the way of a clear comparability with classical mechanics. The formalism from QM_{HD} does not distinguish between statistical description and individual description. Correlatively, in (6.12) the measurement-evolution Hamiltonian $\mathbf{H}(\mathbf{A})$ conserves only the *mean* value of the eigenvalues of \mathbf{A} . Whereas a coding relation is quite essentially required to involve a one-one relation that specifies *one* definite eigenvalue a_j of \mathbf{A} . So in order to be able to obtain a sharp comparability with a classical mobile, we should focus the mathematical representation upon one eigenvalue a_j . To achieve this, let us go to the legal limit of the concept ‘ $\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f) \rangle / \mathbf{A}$ ’ and consider that the expansion (6.12) contains only one term:

$$c_{j'}(t_1) = 0 \text{ for } \forall j' \neq j, |c_j(t_1)| = 1, c_j(t_1) = 1e^{i\alpha(x,t_1)}, |\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f) \rangle / \mathbf{A} = (e^{i\alpha(x,t_1)} |u(x, a_j)\rangle) / \mathbf{A} \quad (6.14)$$

where $\alpha(x, t)$ is an arbitrary phase-function. In the particular case 6.14 according to QM_{HD} the unique eigenvalue a_j that is involved coincides with the mean value of the possible eigenvalues, so it is itself conserved by the measurement-evolution generated by $\mathbf{H}(\mathbf{A})$.

Now, the Hilbert-space *statistical* representation in ‘physical’ space-time, of the limit-(measurement-state-vector) $(e^{i\alpha(x,t_1)} |u(x, a_j)\rangle) / \mathbf{A}$ from 6.14, is a *wave-packet* of which the unique maximum has a *dynamics* and is tied at any moment with the ‘presence’-*probability* of the unique corpuscular-like singularity that is involved. And the location of this maximum is what changes in time. But from a conceptual point of view the mathematical representation of an eigenket – that is an individual model of wave-movement – by a wave-packet – that is an essentially statistical descriptor – is a *quintessentially misrepresentation*. Nevertheless, such a sit has been chosen, shows clearly that what has been conceived to change during the measurement-evolution from (6.12) and 6.14, is the location of the corpuscular singularity from the specimen of the studied microstate that is involved in a given measurement-interaction. But this is not perceptible inside the *exclusively statistical* representation of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f) \rangle / \mathbf{A}$. The *dynamical* features of such a change lie entirely outside formalism from QM_{HD} because it concerns an *entity* that *itself* is exterior to the QM_{HD} formalism. However the dynamical features in the physical space-time of the singularity from de Broglie’s individual wave-function $\Phi_G(x, t) = a(x, t)e^{\beta(x, t)}$ of the involved specimen of the studied microstate are indirectly mimed inside QM_{HD} by the dynamics of the maximum of a corresponding wave-packet that indicates, like in a dispersing mirror, a maximal *probability* of ‘presence’. We have hit here the extreme limit opposed by the perfectly statistical QM_{HD} , to the representation of individual features of the studied microstate. When we place ourselves inside QM_{HD} we enclose ourselves captive in a sort of conceptual tangent plane that – in the case identified here, becomes punctually *common* to classical mechanics and to QM_{HD} . Our examination can not escape from this plane without quitting rigor, or quitting QM_{HD} . But this fundamental divergence does not interfere with our present aim, namely to understand and to construct conditions of emergence of *coding* observable marks. Indeed the parameters of a statistical wave-packet representation of an eigenket – around any chosen initial space-location of the involved corpuscular singularity – are very adjustable. This permits to approach quite satisfactorily all the essential features researched for the coding-aim, namely conservation of the wave-movement

model from the eigenket in the surrounding of the initial location of the singularity – that corresponds to the maximum of the initial statistical wave-packet so to maximal ‘presence’-probability – as well as the direction and degree of stability of the spatial concentration of the subsequent dynamics of the initial maximum, which permits to adjust also the optimal choice for the locations of the registering devices. So, on the basis of the meaning of an eigenket identified in section 6.1, the parameters can be chosen such as to insure that the initial *individual* value a_j of \mathbf{A} stay constant, with *controlled* approximation, on the direction of displacement that reaches a ‘corresponding’ registering device where any impact means ‘ a_j ’. Indeed inside classical mechanics it is obvious that a mechanical displacement of a mobile throughout which the value a_j of a given mechanical quantity A keeps constant, leads the mobile into a predictable spatial domain Δ_{x_j} if the displacement lasts sufficiently, and that this domain can become as distant as one wants with respect to any other domain $\Delta_{x_{j'}}$ that corresponds to another value $a_{j'}$ of \mathbf{A} , with $j' \neq j$ (the source-domain being the same). Which permits to mutually singularise these spatial domains Together, all the preceding considerations permit to think that inside QM_{HD} the *BBGPM* approach (6.12) was indeed founded upon the presupposition that a coding rule of the form 6.13 is always possible:

The state-ket (6.12) $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ is founded on this presupposition, and it is conceived as a descriptor of the statistic of *all* the individual coding-measurement-evolutions involving the observable \mathbf{A} and a given microstate ms_G , for *any* category of micro-states.

This answers with rather strong likelihood the question why the representation (6.12) has been introduced and stayed more or less accepted throughout so many years ⁶⁸.

The arguments explicated above lead us, in agreement with *BBGPM*, to admit the possibility of principle to realize coding-measurement-evolutions (6.12). But the above examination draws attention upon the conditions that *restrict* this conclusion to free microstates of one microsystem and with *simple* operation of generation, that evolve in absence of external fields and obstacles. Outside the domain of validity of these conditions the question of measurement-evolutions that shall insure the possibility to code observable marks in terms of a definite eigenvalue of the measured quantity, remains unexplored.

The major confusions from the QMHD representation of a measurement-evolution.

The question of aim and reasonability being answered, we shall now concentrate upon the mathematical expression. The statistical ***predictions*** concerning a studied microstate ms_G and any observable \mathbf{A} are obtained from the state-ket $|\psi_{G,\mathbf{H}}(t)\rangle$ associated to ms_G . The representation (6.12) of the measurement-state-ket tied with $|\psi_{G,\mathbf{H}}(t)\rangle$ concerns the ***verification*** of the statistical predictions drawn from the state-ket $|\psi_{G,\mathbf{H}}(t)\rangle$. This essential distinction between the operational aims of the two descriptors $|\psi_{G,\mathbf{H}}(t)\rangle$ and $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ – a sort of opposition relative to the temporal succession “I construct a tool for predicting statistically”, “I want to verify whether what I predicted statistically is factually true”, is very little stressed inside the mathematical QM_{HD} -writings, and even in the ways of speaking that accompany the use of the formalism. Taken together

- [the more or less explicit assumption that the state-ket $|\psi_{G,\mathbf{H}}(t)\rangle$ can *always* be obtained via purely mathematical operations (the quite essential requirement to ‘give’ the initial state-ket $\psi_{G,\mathbf{H}}(t_0)\rangle$ being sidestepped)]

and

- [the *statistical* representation (6.12) of the verification-measurements]

form a sort of mock-passage that, like a bridge, permits to pass in apparent continuity from the construction and expression of statistical predictions, to the activity of verifying these predictions. While in fact these two sorts of actions, though related, are of radically distinct natures:

⁶⁸What a prowess! The only way to find answer to a *basic question of physics*, inside a *basic theory of physics* called ‘quantum mechanics’, has been to unmask an intruder hidden in a statistical cloth inside the homogeneous whole of only abstract numerical potential statistics and abstract names of which QM_{HD} in fact consists, and, with the faint reflection from a physical thing that this intruder emits, to draw at distance a virtual trace on the initial representation of the mechanics of real physical things. Let us stop and incorporate this experience.

* It is usually said that \ll at t_1 “the *system* is prepared for measurement, and – correspondingly – is also ‘prepared’ the measurement-evolution-state-ket $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ \gg . What happens meanwhile, factually, on the individual level, in order to effectively verify via $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ the predictions drawn from $|\psi_{G,\mathbf{H}}(t_1)\rangle$, is neither represented, nor explicitly stated. It is *postulated* that each act of measurement of \mathbf{A} produces an eigenvalue a_j of the measured observable \mathbf{A} indicated by the needle of the registering device. But how this does come about is not examined. Everything hovers calmly in the high mathematical-verbal atmosphere. No stress whatever is placed upon the obvious fact that in order for a_j to be ‘produced’ into knowledge, a *physical* interaction between an *individual physical* specimen of the studied microstate ms_G and a registering-device *must* somehow take place, and *must* be such as to permit to induce from its result the eigenvalue a_j . Since we ‘have’ an apparatus for measuring \mathbf{A} , the needle of the apparatus knows how to achieve all this. And how we come to ‘have’ the apparatus is not questioned any more, this lies on the edge of the questionings.

* It is added that when a_j emerges this is accompanied by a ‘reduction’ of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ to only one of its terms. What happens meanwhile to the involved specimen of micro-*state* – namely that it usually is destroyed while a_j appears, even if the involved *system* subsists – is well-known but it is only allusively mentioned from time to time. So the necessity, in general, to generate another specimen of ms_G before entering upon a new measurement-evolution, does not trouble the attention, and the operation of generation G of ms_G remains entirely hidden in the shadow that surrounds \ll the ‘preparation’ of the ‘system’ for measurement \gg . As for the mathematical \ll preparation of the statistical measurement-state-ket $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ \gg , this is placed far above all, and the whole attention goes to it like to a big statue that dominates the surroundings on a pedestal, in the middle of a square ⁶⁹.

This is how the organization of individual descriptors came to remain out of grasp inside QM_{HD} ⁷⁰. But here, on the basis of IQM , we are supported to firmly ask precise questions:

* Why are the coefficients from (6.12) $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ posited a priori to be identical to those from the corresponding expansion (6.11) of $|\psi_{G,\mathbf{H}}(t_1)\rangle$ when the aim of the descriptor $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ is precisely to verify the coefficients from (6.11)? This, even if it is not detrimental, manifests conceptual confusion.

* Consider now the ‘reduction’ of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ at the moment t_f . From a general conceptual viewpoint this ‘reduction’ is not at all unacceptable when one thinks of the general formal representation from the calculus of probabilities: Each one realization of the ‘experiment’ that, by repetition, generates the whole universe of possible outcomes, produces only one outcome from this previously identified universe, which can be indicated by saying that the universe is thereby ‘reduced to only one event from it’. But from a purely mathematical point of view the ‘reduction’ of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ draws attention upon the fact that a *linear mathematical formalism is not perfectly ‘consistent’ with the way prescribed for making use of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$* by mixing inside one same descriptor individual physical events, with a statistical representation of numerical results of these. While from a psychological point of view it might suggest that possibly it is conceived

⁶⁹ Concerning QM_2 anything seems conceivable. Think of Schrödinger’s cat or of Everett’s *infinity of parallel universes generated by each ‘reduction’ of a mathematical writing on a sheet of paper: in this case the mixture between formal descriptors written on paper or screens, and physical facts, reaches perfection.*

⁷⁰ Here and there, since the measurement-‘problem’ came into being with Schrödinger’s cat, questions about this \ll preparation of the measurement-state-ket \gg have emerged in some minds. There was a time when every year there were congresses on the problems of quantum measurements. (What is one supposed to conceive concerning the temporal existence of $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$? Should it be thought to be constructed only *once* for a given pair (ms_G, \mathbf{A}) and to subsist notwithstanding that it is conceived to be ‘reduced’ by each registration of an eigenvalue? Or should we conceive that $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ must be reconstructed after each ‘reduction’ such as it was initially?) But nowadays the *statu quo* has gained. This is how things are. Quantum mechanics is marvellous, it works, so *basta!* In the privacy of the minds such questions probably still emerge, but they are blown away by a sort of vague uneasiness, because, for instance, one feels that even-though quantum mechanics is marvellous, *the time of a statistic cannot be placed on the same time-dimension that is conceived for individual actions and events, and subjected there to a common order of succession; one feels that it requires another dimension of time, and a meta-dimension for connecting temporally inside it times of individual descriptors with times of statistical descriptors.* All this is likely to traverse the minds, but like the obscurity entailed by the passage of a cloud. And then one has to go back reasonably to the serious work of calculating.

sometimes that the statistic *itself* achieves the individual acts of measurement and thereby it is each time ‘perturbed’?

- And quite radically now:

Why should the process of *verification* of a *statistical* prediction concerning the numerical outcomes of individual acts of measurement, *be represented itself statistically*?

When here the aim is not to verify the degree of stability of a statistic, like when one wants to establish in pure mathematics the existence of a probabilistic convergence? when we are in a *physical* theory where, more or less explicitly, the basic entities to be studied are definitely asserted to possess an individual character? *even* if the predictions that can be established concerning results of measurements on these individual entities, are indeed only statistical in general, even so, *why*?

When one stops a moment to consider globally the remarks and the questions accumulated above, all of a sudden a unifying explanation leaps to one’s eyes.

The descriptor (6.12) $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ of a measurement-evolution is just an aborted attempt at a representation of, not **only predictive statistics** of all the numerical *results* of all the individual acts of measurement on the studied microstate – which requires indeed to maintain the coefficients from (6.12) for each given observable \mathbf{A} – but **both** these statistics **and** all the factual individual successions $[G^{(t)}.Mes\mathbf{A}], \forall \mathbf{A}$, that lead not only to the verification of these predictive statistics, but also to these statistics **themselves**.

The descriptor from (6.12) is an attempt to crowd all this, together, inside **one** descriptor that consists of exclusively potential sets of numbers, and no matter how times are manipulated and distorted and mixed up in order to achieve the exploit. Indeed:

◇ The measurement-evolution involved by $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ consists precisely of that what inside one *IQM*-succession $[G^{(t)}.MesA]$ is denoted ‘*MesA*’ and finishes with the registration of a group of observable marks posited to code for one value of the measured quantity A , while it appeared that inside *QM_{HD}* one act of ‘*MesA*’ is conceived precisely such as to **insure** by each measurement-evolution that the result *shall* code for one definite eigenvalue of the measured operator.

But the descriptor (6.12) is statistical because the *QM_{HD}* conceptualization, that progresses top-down, begins directly on the statistical level. It simply did not reach as yet the individual level of conceptualization.

While *IMQ*, that begins upon absolute local zeros of knowledge on individual specimens of microstates, and therefrom progresses down-top, traverses first the whole individual level of conceptualization and out of it has to construct explicitly the statistical level.

So the meeting between the *QM_{HD}* approach, and the *IQM* approach still remains to be organized.

◇ *What is denoted $G^{(t)}$ in (3.8) is not visible in $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$.* It remains hidden in the mist of confusion between what is placed on the physical individual level of conceptualization – namely $[G, \sigma(ms_{G,cw}), \text{the registered groups}]$ (2.6) $\{\mu_{kA}$ of physical observable marks, the translation of a group $\{\mu_{kA}\}$ into one value a_j of an observable \mathbf{A} – and on the other hand the statistical, mathematical *QM_{HD}*-concepts $|\psi_{G,\mathbf{H}}(t_1)\rangle$ and $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$.

◇ As for the time-parameters, the statistical ones from ‘ $|\psi_{G,\mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle/\mathbf{A}$ ’ simply cannot be directly connected with the individual times from ‘ $[G^{(t)}.MesA]$ ’ where $G^{(t)} = F(G, EC, (t - t_0))$. These two sets of time-parameters *are not defined for descriptive elements placed on the same level of conceptualization*. When a G -time-value acts, any statistics to which this operation G brings some contribution is either not yet constituted, or already entirely established. Only a meta-representation might introduce an a posteriori worked out temporal connection between a whole set of individual measurement-successions and a simultaneous synthesizing statistical connection of this whole set. Absolute, universal temporal specifications are meaningless. Only temporal specifications of definite events can possess power of intelligible organization, if and only if they do not mix up the distinct

levels of conceptualization relatively to which – exclusively – they do *exist* ⁷¹.

◇ And let us stress that for a *free* microstate – in contradistinction to what happens for a bound microstate – each measurement evolution that has finished with a registration that (in general) has destroyed one specimen of the studied microstate, requires *inescapably* some way for introducing somewhere another specimen of the studied microstate, able to produce new observable marks.

To all this must be added the fact that the coding rule 6.13 – that inside QM_{HD} seems to be presupposed for *any* measurement on *any* microstate – in fact can be understood more or less *clearly* only in the *particular* case of absence of quantum fields, *which is a huge restriction*.

In short: The mathematical formalism from QM_{HD} *itself* rejects its own in-distinction between the individual level of conceptualization and the statistical one, that – both – are irrepressibly involved but cannot be coherently stuffed together into only one common formal expression (cf. the preceding note). In consequence of all these answers we conclude as follows.

QM_{HD} is devoid of an acceptable representation of measurements.

It is not surprising that finally such a stubbornly persistent effervescence of ‘interpretations of quantum mechanics’ is developing.

Nevertheless QM_{HD} has worked and it continues to work. This theory has achieved remarkable successes and it still could achieve other successes, even if it is left just such as it now stands. Indeed the enormous genius of human mind invents local and individual implicit understandings that permit to act adequately there where and when one actually does act. It seems that for the experimenters it suffices to believe that a quantum theory of measurements exists, in order to measure adequately and to make progress. This makes humble those who try to construct theories. Theories serve much as highways that enhance the traffic. This also *proves that a fully satisfactory theory of quantum measurements is possible*, since no doubt it is quite often ‘applied’ without being known. So there is no pragmatic urgency.

But conceptually there is urgency. Indeed, what value of principle – as *a theory* – does a representation of *non*-perceptible microstates possess, if it predicts via purely mathematically constructed predictive descriptors and does not state in a clear and generally valid way how to conceive-and-perform measurements for *verifying* the predictions?

6.7 CONCLUSION ON PART II

- The eigenstates $|u_j(x, a_j)\rangle$ and eigenvalues a_j introduced by an ‘observable’ \mathbf{A} have been found to have a very special status inside QM_{HD} : They are represented inside QM_{HD} even though they possess a quite essentially *individual* significance. But this significance has not been identified. Consequently it has introduced false problems instead of contributing to intelligibility.

While the concept of an ‘eigenvalue a_j of an observable \mathbf{A} ’ is both represented and recognized to possess an individual significance, but the emergence of an eigenvalue as the result of any act of measurement is *just* postulated without explanation, and it is tied with the ‘reduction problem’. So inside QM_{HD} *there is no worked out semantic-operational coherence between the concepts ‘ $|u_j(x, a_j)\rangle$ ’ and ‘ a_j ’, though their intimate mathematical relation is overtly introduced by postulation*.

We have identified that an eigenket has the meaning of a mathematically expressed possible wave-movement around a singularity in the amplitude of de Broglie’s general corpuscular-wave model. This has triggered a succession of constructive steps that has led inside $[IQM-QM_{HD}]$ to the definition of a general ‘ \mathbf{G} -corpuscular-wave model’ of a microstate – denoted $m_{SG,cw}$ – and has been associated to

⁷¹ Human conceptualization is dominated by a host of still hidden ‘methodological laws’ marvellously tied with a sort of incorporated a priori exclusion of false problems and paradoxes (MMS [2002A], [2002B], [2006]). Whether they are known or not these laws do work. And when they are violated, false problems and paradoxes burst out and ring the bell.

a modelling postulate $MP(ms_{G,cw})$. Thereby de Broglie's general model – an ideal model – has been incorporated to an operational approach apt to be made use of consensually. At the same time this has entailed a transformation of the initial, purely methodological relation (1.1), into a relation (1.1) that transforms the mere labelling by 'G' of the result ' ms_G ' of one realization of the operation G of generation of a microstate, into a qualification of this result itself, that concerns its inner structure. *Thereby from now on the framework [IQM-QM_{HD}] is connected in an operational way with the dBB approach.*

- We have then brought into evidence the *general* power of clarification entailed by a systematic specification of: the existence – *or not* – of a connection between a ket from a mathematical QM_{HD} -expression, with an operation of generation G of a microstate, and with the character of this operation of generation (simple or composed). This led to a new notation that indicates explicitly the relation between a state-ket that represents a physically realized microstate, with the operation of generation G of this microstate.

Finally we have examined the quantum theory of measurements from QM_{HD} . We have refused von Neumann's representation, we have identified the coding rule that is implicitly assumed and we have found that it is devoid of a general validity, and we have then brought into evidence that the very *essence* of the QMHD representation of measurement is unacceptable. Indeed, the representation of microstates offered by QM_{HD} is founded upon the belief that it is possible to always obtain the statistical predictions concerning non-perceptible microstates by a practically *exclusive* use of mathematical means, as if independently of *physical* entities and operations. The unique but radical violation of this belief, namely the fact that one has to 'give' the initial state-ket in order to come in possession of a representation of the statistical predictions via the general solution of Schrödinger's equation of the problem – is introduced in a very inconspicuous way, and moreover, in its turn, this way *also* is concealed under mathematical clothes: one is asked to just 'introduce' the 'initial conditions' in mathematical writings; but in fact this is possible only in idealized didactical cases or in bound states, while in general adequate mathematically expressed initial conditions are far from being specifiable. So in general the problem of *constructing* the predictive statistics subsists, but it is occulted. Thereby the individual physical-operational level of conceptualization represented inside IQM with all the own, specific problems that it involves – in particular the coding-problem – is quasi imperceptible inside QM_{HD} .

And when finally one comes face-to-face with the problem of the *verification* of the (supposedly always possible) mathematically elaborated statistical predictions, *it is tried to represent the verification also in only statistical terms*, namely by the use of a statistical descriptor, a 'measurement-evolution state-ket', supplemented by all the necessary *postulates* for coming down upon the individual level of conceptualization where – unavoidably – at last, the individual result has to be dropped off.

In short, it appeared that nowadays quantum mechanics is devoid of a conceptually acceptable and factually applicable representation of measurements.

Thereby the preliminary global critical examination of QM_{HD} by reference to IQM has come to its end.

Part III

THE PRINCIPLES OF A SECOND QUANTUM MECHANICS factually rooted and computationally assisted

THE SPECIFIC AIM OF PART II

The third part of this work is resolutely constructive. Its specific aim is to bring forth an acceptable theory of quantum measurements, as the core for subsequently integrating around it a new, generally valid representation of the microstates.

INTRODUCTION TO PART III

Only a new construction can ruin and replace a previously achieved construction.

Author of which I have forgotten the name

The aim of the third part of this work is to define the main lines – only – of a fully *intelligible* mathematical representation of microstates, in the sense already defined of a procedural piece of communicable and consensual way of constructing predictions about microstates and of verifying these predictions. Thereby the third part of this work will naturally keep continuity with respect to the result of the first and second parts.

In order to avoid inertial attitudes of mind, we stress that:

What follows is *not* a new *interpretation* of quantum mechanics; nor an achieved new theory of microstates; nor a didactic itemization of something that already exists. It is a first outline of a newly conceived representation of microstates required to be general, scientific and *intelligible*.

Obviously at its start such an attempt can concern only *foundational* aspects. In order to express these foundational aspects we shall make use of the two most utilized mathematical formulations, the Hilbert-Dirac formulation denoted QM_{HD} and the de Broglie's seminal Ph.D. thesis.

Throughout what follows QM_{HD} as well as the essence of dB's thesis are supposed to be well known.

For the sake of effectiveness we consider a priori only *finite* concepts (spatial or abstract extensions, spectra, number of repetition of operations, mathematical procedures, etc.). In particular:

(a) Any grid for qualification of a dynamical quantity is required to introduce definite units for measuring the considered qualifying quantity. This confers it a well delimited domain of validity, in particular inside space-time.

(b) The space-time domain itself of any investigation is always presupposed to be finite. (Continuity and infinities can be reached afterward via specified and conceptually surveyed processes of extension)⁷².

⁷² The possibility of arbitrarily small units has to be controlled from a conceptual point of view: Usually (and possibly always) one finds a lower limit for the domain of conceptual definability of meaning (for instance, is Plank's constant compatible with a periodic time-unit that approaches zero without a lower-bound restriction?). If this work is accepted,

Together, the above specifications characterize what we name ‘principles of a second quantum mechanics’ and we denote $QM2$.

$QM2$ will emerge as a new sort of association between mathematically constructed representations, and *factually* constructed data that, together, cover larger domains of facts and of questionings than those accessible to QM_{HD} alone. The core-result will consist of: The full elucidation of the ‘reduction-problem’; a *factual-formal duplication* of the QM_{HD} basic mathematical representations; the revelation of a whole domain of *non*-validity of the QM_{HD} representation with respect to the vital requirement of verifiability of the asserted predictions. This last result entails a crucial question that can be solved only experimentally and that *has* to be answered in order to come in possession of a definitive basis for asserting a *generally* valid new theory of quantum measurements.

But whatever the experimental answer will be, we shall be left with a thoroughly reorganized view concerning an intelligible and generally valid representation of a mechanics of microstates.

the condition of finiteness of the domain of investigation will have to be treated later with mathematical rigor, for each case separately. The whole question of effectiveness brings face-to-face, on the one hand, the requirement of a modern Physics compatible by construction with informatics, and on the other hand the classical mathematical analysis founded on continuity and infinity. Here we just announce a choice of principle and a goal, but the corresponding elaboration will remain absent. However the mentioned goal will be permanently manifest in the notations that are not compatible with continuous and non-finite mathematics. This is not an error, but a deliberate choice.

7 EMERGENCE OF THE FIRST LINES OF $QM2$

To reach the point that you do not know
you must take the way that you do not know.

San Juan de la Cruz

7.1 The provisional framework [IQM - QM_{HD}] for the construction of $QM2$

Let us now enter upon the constructive attempt. The general features of the approach are announced below.

Inside QM_{HD} the statistical predictions are obtained exclusively by mathematical operations. This, for real physical situations (not for didactical idealizations) is in general difficult to accomplish (think also of Schrödinger's treatment for solving the 'simplest' real cases of the one electron in a atom of hydrogen and of a linear harmonic oscillator). Moreover, in order to dispose of the predictive state-ket (6.11) $|\psi_{G,\mathbf{H}}(t)\rangle$ of the problem one has to 'give' the initial state-ket $|\psi_{G,\mathbf{H}}(t_0)\rangle$ which might often be impossible (think of an electron-microstate that would encounter from the start some irregular macroscopic material obstacle). Even the writing of a Schrödinger equation of the problem might be impossible (in non-Hamiltonian situations); and when this equation can be written, nearly always the mathematical *general* solution involves already approximations, quite basically, and the factual effect of these cannot be controlled mathematically.

In these conditions, factual verifiability of the predictions is essential.

While in fact inside QM_{HD} the problem of the factual verification is not treated clearly, taking into account both specificities and the aim of generality.

We shall now try to fill this lacuna by uniting progressively IQM with QM_{HD} .

7.1.1 Conditions of semantic compatibility

IQM has been constructed like only a reference-structure for understanding QM_{HD} and estimating its adequacy. As such IQM has been endowed with the maximal generality compatible with its status. This led to deliberately leave *undefined* the model of a microstate. In consequence of the absence of a model the measurement operations ' $MesA$ ' remained equally undefined inside IQM as well as the 'external conditions' EC from the generalized definition 3.7 of an operation of generation $G^{(t)}$.

The conclusions from section 6.6 entail that the conditions of comparability between the semantic contents of QM_{HD} and IQM are not defined as yet. Only structural comparability between IQM and QM_{HD} has been possible up to now. But just below we shall state now also the semantic conditions that, added to the structural ones, insure full comparability between IQM and QM_{HD} , in particular concerning the essential questions of prediction and verification on microstates.

Let us admit that the situation is Hamiltonian and that it has been possible to define the Schrödinger equation of evolution of the problem and to establish the general solution of this equation. The basic remark is that in general the statistical predictions drawn from a state-ket (6.11) $|\psi_{G,\mathbf{H}}(t_1)\rangle$ concerning an observable \mathbf{A} cannot be verified experimentally *otherwise* than via a very big number of repetitions of whole successions $[G^{(t)}.Mes\mathbf{A}]$, $\forall \mathbf{A} \in V_{Mes}$, in the sense of IQM . And in order for this to be possible, the merely structural conditions imposed by IQM have to be completed as follows.

(a) We make use of the model $MP(ms_{G,cw})$ posited for a microstate as expressed by (6.1) $G \Leftrightarrow ms_{G,cw}$ and (6.2) $ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\}$

(b) We posit that the external conditions ‘*EC*’ from the expression (3.7) $G^{(t)} = F(G, EC, (t - t_0))$ are those expressed in QM_{HD} by the Hamiltonian operator \mathbf{H} that in the Schrödinger equation of evolution of the problem acts abstractly upon state-ket $|\psi_{G,\mathbf{H}}(t)\rangle$, ($t_0 \leq t \leq t_1$) thus determining it⁷³ :

$$G^{(t)} = F(G, \mathbf{H}, (t - t_0)) \quad (7.1)$$

(c) According to *IQM* ‘one act of measurement’ consists of a sequence of individual operations $[G.Mes\mathbf{A}]$ as defined in (1.3) and section 2.2. And according to QM_{HD} a sequence $[G^{(t_1)}.Mes\mathbf{A}]$ from (3.7) has to be determined by a measurement-Hamiltonian $\mathbf{H}(\mathbf{A})$ that throughout the duration ($t_1 \leq t \leq t_f$) of *MesA* acts physically upon the (unknown) individual physical wave-function $\Phi(x, t) = ae^{(i/\hbar)\varphi(x,t)}$ of the specimen $\sigma(ms_G)$ of the studied microstate ms_G that is involved⁷⁴ (cf. section 6.5).

7.1.2 A basic assertion of prediction-verification compatibility between *IQM* and QM_{HD}

We now formulate explicitly the following ‘assertion’ *Ass.1* supported by an ‘argument’ *Arg(Ass.1)*⁷⁵:
Ass.1. Consider the factually constructed *IQM*-descriptor

$$(D_{Mec}(ms_{G^{(t)}})) \equiv [\{(\epsilon, \delta, N_0) - \pi(a_j)\}_{G^{(t)}}, (Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}], \forall (A, B) \in V_{Mec}^2, j = 1, 2, \dots, J \quad (7.2)$$

If $(D_{Mec}(ms_{G^{(t)}}))$ is *constructed* by use of the same successions $[G^{(t)}.Mes\mathbf{A}]$, $\forall A \in V_{Mec}$ specified in (a) and (b) that are performed for *verifying* the statistical predictions drawn from the mathematically constructed QM_{HD} -state-ket $|\psi_G(x, t)\rangle$, ($t_0 \leq t \leq t_1$), then the predictions of $|\psi_G(x, t)\rangle$ can be found to be verified experimentally only if they identify⁷⁶ with the statistical predictions entailed by the factual description $(D_{Mec}(ms_{G^{(t)}}))$.

Arg(Ass.1). Obvious. Since inside *IQM* the description $(D_{Mec}(ms_{G^{(t)}}))$ is always constructed *factually*, in order to verify the predictions of $(D_{Mec}(ms_G))$ one is obliged to just repeat its construction. No other way is conceivable. So **the verification of $(D_{Mec}(ms_{G^{(t)}}))$ is certain a priori**⁷⁷. So, if the – *necessarily factual* – verification of $|\psi_G(x, t)\rangle$ is accomplished in the same way as the factual construction of $(D_{Mec}(ms_{G^{(t)}}))$, then, if it verifies $|\psi_G(x, t)\rangle$ it must also reconstruct $(D_{Mec}(ms_{G^{(t)}}))$.
 ■

At a first sight it might seem that the pair $(Ass.1, Arg(Ass.1))$ expresses a circularity. But in fact this is not at all the case because the semantic connection realized between *IQM* and QM_{HD} is ‘vertical’ in this sense that it ties the individual level of conceptualization of the microstates from *IQM*, with the mathematical representation of the statistical level of conceptualization from QM_{HD} (fig. 8.1, fig. 8.1). And this connection suggests a very remarkable fact, namely *the possibility to circumvent the Schrödinger equation of evolution when this is convenient, via the performance of a big number of factual coding-measurement-successions $[G.Mes\mathbf{A}]$ or $[G^{(t)}.Mes\mathbf{A}]$, $\forall A \in V_{Mec}$ that generate the QM_{HD} -state-ket by a convenient factual-mathematical procedure.*

This suffices for understanding that $(Ass.1, Arg(Ass.1))$ is far from being a tautology. On the contrary, it is an essential new element that entails the possibility of a precisely defined aim that includes and *transcends* the aim to only reconstruct the QM_{HD} -representation of quantum measurements.

Thereby all of a sudden *IQM* and QM_{HD} appear to be soldered to one another inside a new sort of theory that twinkles on the horizon.

⁷³ The definition of \mathbf{H} is conceived to include not only the external macroscopic fields but also the involved ‘obstacles’ (walls, barriers, wells).

⁷⁴ This is what happens also in the case of the bound microstate from an atomic or molecular structure where one specimen $\sigma(ms_G)$ subsists for an arbitrarily long time and meanwhile interacts from time to time with test-particles or other devices (Zeeman or Stark effects, etc.).

⁷⁵ Throughout what follows we speak in terms of ‘assertions’ and ‘arguments’ because we are not yet inside a formally stabilized structure.

⁷⁶ Inside the limits permitted by the triad (ϵ, δ, N_0) from (3.4).

⁷⁷ Of course, it is presupposed that $D_{Mec}(ms_G)$ has been considered to have been accomplished only when a convenient choice in (3.4) of the set of parameters (ϵ, δ, N_0) has stabilized the quasi-identical recurrence of $D_{Mec}(ms_G)$ when one reconstructs it inside the correspondingly admitted fluctuations.

7.1.3 Global conceptual structure

A QM_{HD} -state-vector $|\psi_G(x, t)\rangle$, once it has been obtained for a given microstate ms_G –mathematically or otherwise – acts as a permanently available mathematical tool that yields the statistical *predictions* on ms_G and also permits to compare these statistics generated by calculus, with the final count of the *factual* results of a big number of individual achieved for either *verifying* the statistical predictions drawn from $|\psi_G(x, t)\rangle$ by (ϵ, δ, N_0) -identification in the sense of 3.8, or for invalidating these predictions by clear (ϵ, δ, N_0) -non-identification with 3.8. This is always possible because between a *final* count of all the results obtained by many measurement-evolutions, and the permanent set of predictions asserted by the mathematical descriptor $|\psi_G(x, t)\rangle$, there is conceptual homogeneity (in contradistinction with the fact that the realization of individual measurement operations, and a statistical description of the results of these, are mutually heterogeneous from a conceptual point of view). And finally:

A set of very numerous *previously achieved* coding-measurement-evolutions $[G.Mes\mathbf{A}]$ or $[G^{(t)}.Mes\mathbf{A}]$, $\forall \mathbf{A} \in V_{Mec}$ can be represented *a posteriori* on the statistical level of conceptualization by a ‘Schrödinger equation of evolution’ of a ‘coding-measurement-ket’ (6.12) $|\psi_{G, \mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle / \mathbf{A}$, but where no reductions that bring down on the individual level of representation are necessary any more : such a state-ket is just *statistical history*.

So the elimination of the QM_{HD} representation of measurements on exclusively the statistical level via the malformed descriptor (6.12) of $|\psi_{G, \mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle$, associated with the displacement of the representation of the coding-measurement-evolutions, from the statistical level onto the individual level of conceptualization where the basic descriptor is a whole factually achieved succession $[G^{(t)}.Mes\mathbf{A}]$ of first an operation of generation $G^{(t)}$ and afterwards an act of measurement $Mes\mathbf{A}$ determined by the measurement Hamiltonian $\mathbf{H}(\mathbf{A})$, clarify radically the features of principle of what is called ‘measurement’, for any qualifying aspect and any sort of microstate.

In this view, that no doubt any experimenter admits more or less explicitly, everything becomes trivially intelligible.

7.1.4 Precise formulation of the constructive aim

So the pair (*Ass.1*, *Arg(Ass.1)*) indicates the a priori possibility to connect the *whole* essentially statistical mathematical QM_{HD} -representation of microstates, to the general and complete structural representation of microstates constructed inside IQM in only qualitative terms, but that is directly rooted into factuality.

This could be achieved along the following lines.

- The rooting into a-conceptual micro-physical factuality is realized by the model $MP(ms_{G,cw})$ of a microstate and the relations (6.1) $G \Leftrightarrow ms_{G,cw}$, $ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\}$ founded on this model that, in its turn, is founded upon the elucidation in section 6.1 of the significance of the QM_{HD} -concept of eigenstate $|u(x, a_j)\rangle$ of an observable \mathbf{A} .

- The connection between the IQM representations on the individual level of conceptualization, and the QM_{HD} -representation, can be realized via sequences of $[G^{(t_1)}.Mes\mathbf{A}]$ with ‘ $G^{(t_1)}$ ’, defined by: (7.1) $[G^{(t)} = F(G, \mathbf{H}, (t - t_0))]$ and the conditions of semantic compatibility between IQM and QM_{HD} defined in section 7.1.2, and the pair (*Ass.1*, *Arg(Ass.1)*).

- The formal use of the expansion postulate (6.3) associated with Born’s postulate.

This should lead to a sanitized formal representation of the quantum measurements. Furthermore:

This should also permit a **factual** ‘channel’ for constructing the mathematical QM_{HD} -state-ket $|\psi_G(x, t)\rangle$ ‘of the problem’ – inclusively the initial state-ket $|\psi_G(x, t_0)\rangle$ that inside QM_{HD} itself is required to be ‘given’.

This should be possible in any factual situation, Hamiltonian or not, tied or not with a corresponding constructible and solvable Schrödinger equation. The mathematical QM_{HD} construction of the state-ket of *free* states could be entirely wrapped in factuality and rooted in it whenever one does not want to be restricted to only the domain of calculability of the state-ket.

This sort of factual duplication is an aim that exceeds by far a mere reconstruction of the QM_{HD} representation of quantum measurements.

7.2 Construction of a factual-mathematical [IQM- QM_{HD}]-representation of measurements on free microstates of *one* microsystem and *without quantum fields*

In what follows immediately we consider only the particular case of free microstates that do not involve possibility of quantum fields, that is, free microstates of one microsystem and non-composed operation of generation. We denote them ‘ $ms(\text{free}, 1)_{G(n-c)}$ ’ (‘ $G(n-c)$ ’: non-composed operation G of generation).

7.2.1 Conservation of the Hilbert-Dirac representation

In section 6.6 we have found that the sort of measurement-evolution that is presupposed in QM_{HD} by the *BBGPM* approach *can* be conceived – in *coherence* with the Hilbert-Dirac representation – to achieve an implicit coding of the registered observable result in terms of a definite eigenvalue a_j of the measured observable \mathbf{A} . But it also has appeared that the mentioned supposition has a restricted validity in this sense that it can be ‘understood’ only in the absence of quantum fields. On this basis, and supported by the method ‘time-of-flight’ for measuring the momentum observable and by the Stern-Gerlach procedure for measuring spin, we have admitted the efficiency of the mentioned implicit way of coding. So – to begin with – inside [IQM- QM_{HD}] we shall construct a *factual-mathematical* Hilbert-Dirac representation of measurements for microstates $ms(\text{free}, 1)_{G(n-c)}$.

7.2.2 A coding-postulate for microstates $ms(\text{free}, 1)_{G(n-c)}$

In section 6.6 we have been obliged to *start* from the statistical level, namely with the statistical descriptor (6.12) $|\psi_{G, \mathbf{H}(\mathbf{A})}(t_1 \leq t \leq t_f)\rangle / \mathbf{A} = \sum_j c_j(t_1) |u(x, a_j,)\rangle$, $j = 1, 2, \dots, J$ where all the individual ‘state’-descriptors from the second member were eigenket (in fact ideal *models* of corpuscular-wave-movements according to section 6.1). So we had no other choice for starting the examination than (in essence) an eigenket-representation of the measurement-evolution of an *individual* specimen of the studied microstate that in the (space-time)-position QM_{HD} -representation is approximated on the *statistical* level by a wave-packet.

While *IQM* initiates a down-up order of conceptualization instead of the up-down inertial order of conceptualization induced by history (fig. 1.1). So the order of construction is now reversed with respect to that from the analysis from section 6.6 of $|\psi_{G, H(A)}(t \cdot t_1)\rangle$. We start with the operation of generation (7.1) $G^{(t_1)} = F(G, \mathbf{H}, (t_1 - t_0))$ and we continue with the coding-measurement evolution $Mes\mathbf{A}$, staying constantly on the individual level of representation. According to the modelling postulate $MP(ms_{G, cw})$ from section 6.2 and to (7.1) the operation $G^{(t_1)}$ introduces a specimen $\sigma(ms_{G_1^{(t)}, cw})$ of the studied micro-state $ms_{G, cw}$ produced by the initial operation of generation G followed until the moment t_1 by an evolution that, inside QM_{HD} , represented *statistically* by the state-ket $|\psi_{G, \mathbf{H}(\mathbf{A})}(t_1)\rangle$. But according to *IQM* the operation $G^{(t_1)}$ has introduced a physical *individual* corpuscular wave of a specimen $\sigma(ms_{G_1^{(t)}, cw})$ of $ms_{G_1^{(t)}, cw}$ that is represented by an unknown physical wave-function $\Phi(x, t) = a(x, t)e^{(i/\hbar)\beta(x, t)}$. We have admitted the *BBGPM* implication that a Hamiltonian operator $\mathbf{H}(\mathbf{A})$ that commutes with the measured observable \mathbf{A} – if after t_1 it works on $\sigma(ms_{G_1^{(t)}, cw})$ in absence of quantum fields – installs for the corpuscular wave of $\sigma(ms_{G_1^{(t)}, cw})$ a structure of wave-movement represented by an eigenket of \mathbf{A} , while correlatively, for the corpuscular-like singularity in the amplitude of $\sigma(ms_{G_1^{(t)}, cw})$ it generates a dynamic that leads it into a space-domain Δx_j (or a space-time domain $(\Delta x_j \Delta t_j)$)⁷⁸ that is in a one-one relation with a given eigenvalue a_j of \mathbf{A} . So, as stated in section 7.1.1, we have to require a coding-measurement-evolution imposed by fields

⁷⁸ This, very possibly, might be provable somehow with full generality (think of the method ‘time-of-flight’ for measuring the momentum observable). But as long as a proof is not available we only can postulate it.

represented by a Hamiltonian operator $\mathbf{H}(\mathbf{A})$. This leads to introduce the following *coding-postulate*:

$\mathbf{P}(\mathbf{cod})_{\mathbf{G}(n-c)}$. A coding-measurement-evolution $[G^{(t_1)}.Mes\mathbf{A}]$ performed upon a microstate $ms(free, 1)_{\mathbf{G}(n-c)}$ obeys the general representation:

$$[(G^{(t_1)} \rightarrow \sigma_{\Phi}).Mes\mathbf{A}(\sigma_{\Phi})] \rightarrow_{\mathbf{H}(\mathbf{A})} (\text{marks registered in } \Delta x \Delta t)_j \equiv 'a_j' \quad (7.3)$$

where $G^{(t_1)}$ is defined accordingly to (7.1) $G^{(t)} = F(G, \mathbf{H}, (t - t_0))$ and σ_{Φ} is an abbreviation for $\sigma(ms_{Gcw})$.

If in particular it is supposed that the coding-measurement-evolution is performed by starting at the time t_0 when the initial operation of generation G finishes, then we make use of the corresponding particular form of $P(cod)_{\mathbf{G}(n-c)}$ ⁷⁹:

$$[(G \rightarrow \sigma_{\Phi}).Mes\mathbf{A}(\sigma_{\Phi})] \rightarrow_{\mathbf{H}(\mathbf{A})} (\text{marks registered in } \Delta x_j \Delta t_j) \equiv 'a_j' \quad (7.4)$$

The postulate $P(cod)_{\mathbf{G}(n-c)}$ acts in a rigorous conceptual sense, upon an individual specimen of the studied microstate. It replaces from now on the approximate statistical limiting descriptor $|\psi_{G, \mathbf{H}(\mathbf{A})}(t - t_1)\rangle$ from (6.12) that has permitted to identify it. It *also* 'explains' the non-analysed QM_{HD} -postulation of 'emergence of an eigenvalue a_j of \mathbf{A} when \mathbf{A} is measured'.

7.2.3 Gleason's theorem

Since 1954 the Hilbert-Dirac representation is endowed with Gleason's well-known theorem. In the present context the essence of this theorem can be reduced to what follows⁸⁰. Suppose that the generalized Hilbert space \mathcal{H} associated to the studied microstate possesses a dimension of at least 3. Let $\{|u_j(x, a_j)\rangle\rangle$, $j = 1, 2, \dots$ be the basis in \mathcal{H} defined by \mathbf{A} and let us denote by $\{(\psi_G, a_j)\}$, $j = 1, 2, \dots$ the set of events that consist of the registration of an eigenvalue a_j of \mathbf{A} as result of a measurement of \mathbf{A} on a specimen $\sigma(ms_{G,cw})$ of a microstate ms_G that is represented by the state-ket $|\psi_G\rangle$. Suppose that it is possible to associate to the set of events $\{(\psi_G, a_j)\}$, $j = 1, 2, \dots$ a probability law $\{\pi(\psi_G, a_j)\}$, $j = 1, 2, \dots$. Gleason's theorem asserts that:

If a probability law $\{\pi(\psi_G, a_j)\}$, $j = 1, 2, \dots$ is given, the mathematical possibility to represent it inside \mathcal{H} is necessarily subjected to the identity of form

$$\pi(\psi_G, a_j) \equiv_{Gl} |Pr.j|\psi_G\rangle|^2, j = 1, 2, \dots \quad (7.5)$$

where $Pr.j|\psi_G\rangle$ is the projection of $|\psi_G\rangle$ on the eigenket $|u_j\rangle$ from the basis defined in E by \mathbf{A} (the symbol ' \equiv_{Gl} ' is to be read 'identical according to Gleason').

So Gleason's theorem asserts the same mathematical *form* as Born's probability postulate

$$\pi(\psi_G, a_j) = [|Pr.j|\psi_G\rangle|^2 \equiv |c_j|^2] \quad (7.6)$$

However there is *no full conceptual identity* between (7.5) and (7.6). Indeed Born's postulate contains assertions *of facts* (there *exists* a probability law $\{\pi(\psi_G, a_j)\}$, $j = 1, 2, \dots$; the state-ket $|\psi_G\rangle$ is known; so the expression $|Pr.j|\psi_G\rangle|^2 \equiv |c_j|^2$ defines a numerical value *and* this numerical value is tied with the Hilbert-Dirac form (7.6)). Whereas Gleason's theorem has not the status of an assertion

⁷⁹ By now we are so deeply used to the purely mathematical and statistical representation from QM_{HD} that the content of the whole section 7.2.3 might seem unbearable inside a work of theoretical physics. But the reader is asked to remember that we want to root quantum mechanics in factuality, and in a non-perceivable and as yet a-conceptual physical factuality. This requires with necessity to withstand all the inertial psychological reactions induced since nearly a century by feebly intelligible, abstract, purely algorithmic representations, supported by philosophical diktats. Inside the classical disciplines of theoretical physics one accepts quite currently labels and norms that establish *direct* relation with physical facts. These labels and norms are precisely what induces efficiency.

⁸⁰Pitowsky (2008) has drawn this essence in connection with 'quantum logic'.

of (physical or representational) *facts*, it has the pure status of a *logical implication* ('if-then'). It presupposes nothing concerning the existence, or not, of a probability law $\{\pi(\psi_G, a_j)\}$, $j = 1, 2, \dots$, nor – if this law exists – concerning the way in which are calculated the numerical values subjected to this law.

Gleason's theorem brought into light the rather subtle notion that the *mathematical* form postulated by Born long before, is not only possible, but furthermore it is *imposed if one chooses a Hilbert-space representation*. And in this case – *via the expansion postulate of the state-ket and its projection on the axes of the basis introduced by the considered observable \mathbf{A}* – it permits a representation of the predictions accordingly to Born's postulate, in a way that is pragmatically precious because it is intuitive and very economical: that is the main advantage of the Hilbert-space representation of the statistical QM_{HD} -predictions.

Moreover it will turn out that thereby Gleason's theorem is very useful for a *factual-mathematical* reconstruction of the QM_{HD} -concept of a state-ket, because it permits to make use in a very direct and simple way of a mathematical Hilbert-space-like framework where to lodge the results of factual individual measurement evolutions.

7.2.4 Factual-mathematical construct equivalent with respect to prediction-and-verification to the state-ket of a microstate $ms(\text{free}, 1)_{G(n-c)}$

Consider a microstate ms_G of type $ms(\text{free}, 1)_{G(n-c)}$. Let $|\psi_{G(t)}\rangle$ be the symbol of the – *unknown* – state-ket of ms_G at a time t , such as, supposedly, it *would* be obtained via the current mathematical procedures from QM_{HD} . We make the following assertion *Ass.2*:

Ass.2. Inside $[IQM-QM_{HD}]$ all the spectral decompositions of the state-vector $|\psi_{G(t)}\rangle$ can be constructed via a *factual-mathematical* procedure that: **(a)** entails that the set of all the spectral decompositions of $|\psi_{G(t)}\rangle$ constitute a factual-formal equivalent of the state-ket $|\psi_{G(t)}\rangle$ itself, with respect to both prediction and verification; **(b)** the construction makes no use of the Schrödinger equation nor of Born's postulate.

Arg(Ass.2). The assertion *Ass.1* and the postulate $P(\text{cod})_{G(n-c)}$ entail that, if the first-order probabilistic predictions of the unknown state-ket $|\psi_{G(t)}\rangle$ were available and were also *verified* by coding-measurement-evolutions $[G^{(t)}.Mes\mathbf{A}]$, $\forall \mathbf{A}$, $\forall t$, from (7.3), *then* these predictions would necessarily be the same as those from the statistical description (3.8)

$$(D_{Mec}(ms_{G(t)})) \equiv \{[(\epsilon, \delta, N_0) - \pi(a_j)]_{G(t)}, (Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}\}, \forall (A, B) \in V_{Mec}^2, j = 1, 2, \dots, J \quad (7.7)$$

constructed inside IQM via the same set of coding-(measurement-successions). So according to *Ass.1* and Gleason's theorem (7.5), inside the Hilbert space \mathcal{H} of $|\psi_{G(t)}\rangle$ we must have the succession of identities⁸¹:

$$\{\pi[|\psi_{G(t)}\rangle, a_j]\} \equiv_{Gl} \{|c_j(t)|^2\} \equiv_{\text{textit Ass.1}} \{\pi_{G^{(t)}}(a_j)\}, j = 1, 2, \dots, J, \forall \mathbf{A}, \forall t \quad (7.8)$$

where : $\{\pi[|\psi_{G(t)}\rangle, a_j]\}$ designates the whole (unknown) first-order probability law asserted by the state-ket $|\psi_{G(t)}\rangle$ for the eigenvalues $\{a_j\}$ of \mathbf{A} ; ' \equiv_{Gl} ' is to be read 'identical according to Gleason'; the set of real numbers $\{|c_j(t)|^2\}$ designates the set of projections of $|\psi_{G(t)}\rangle$ on the eigenket from the basis of eigenket $\{|u_j(x, a_j)\}$ introduced by \mathbf{A} in the Hilbert-space \mathcal{H} of $|\psi_{G(t)}\rangle$; ' $\equiv_{Ass.1}$ ' is to be read 'identical according to *Ass.1*'; $\{\pi_{G^{(t)}}(a_j)\}$ is the first-order probability law assigned by the IQM description (3.8) to the eigenvalues $\{a_j\}$ of \mathbf{A} , if $G^{(t)}$ obeys the definition (7.1).

The identities (7.8) can be represented inside the Hilbert-space of $|\psi_{G(t)}\rangle$ for any pair $(|\psi_{G(t)}\rangle, \mathbf{A})$, via the following factual-formal procedure:

⁸¹ We recall that accordingly to our choice of effectiveness the spectra are finite in consequence of the finiteness of the investigated space-time domain.

- First are *constructed **factually*** for the studied microstate $ms_{G(t)}$ the probabilities $\{\pi_{G(t)}(a_j)\}$, $\forall \mathbf{A}, \forall t$, from 3.8, accordingly to *IQM* but by use of the *IQM coding-measurement-evolutions* (7.3) [$G^{(t)}.MesA$] that obey the postulate $P(cod_{G(n-c)})$ with $G^{(t)}$ of the form (7.1).

This exhausts the purely factual phase of the construction.

- Then, for a given observable $\mathbf{A} \in V_{Mec}$, one writes the corresponding *form* of the spectral decomposition of the *unknown* state-ket $|\psi_{G(t)}\rangle$, denoted $|\psi_{G(t)}\rangle/\mathbf{A}$:

$$|\psi_{G(t)}\rangle/\mathbf{A} = \sum_j e^{i\alpha(\mathbf{A},j)} |c_j(t, \mathbf{A})| |u_j(x, a_j)\rangle, \quad j = 1, 2 \dots J, \quad \forall t \quad (7.9)$$

The expansion coefficients describe the projections of $|\psi_{G(t)}\rangle$ on the eigenket from the basis $\{|u_j(x, a_j)\rangle\}$ of \mathbf{A} in \mathcal{H} , written in the explicit form

$$c_j(t, \mathbf{A}) = e^{i\alpha(\mathbf{A},j)} |c_j(t, \mathbf{A})| \quad (7.10)$$

of a product of real number $|c_j(t, \mathbf{A})|$ and a *non*-specified complex phase-factor $e^{i\alpha(j)}$. In (7.8) we have

$$|c_j(t, \mathbf{A})|^2 \equiv_{Ass.1} (\pi_{G(t)}(a_j)), \quad j = 1, \dots, J, \quad \forall t,$$

and the numbers $(\pi_{G(t)}(a_j))$ have been determined factually for $j = 1, \dots, J$ and for any chosen time t . So we can write the right-hand member from (7.9) in the ‘factual-mathematical’ form

$$\sum_j e^{i\alpha(\mathbf{A},j)} \sqrt{\pi_{G(t)}(a_j)} |u_j(x, a_j)\rangle, \quad j = 1, 2 \dots J, \quad \forall t \quad (7.11)$$

that makes explicit use of Gleason’s theorem (7.5).

- The same procedure is valid for the spectral decomposition of $|\psi_{G(t)}\rangle$ with respect to all the other dynamical observables. So we have

$$\left\{ \sum_j e^{i\alpha(\mathbf{A},j)} \sqrt{(\pi_{G(t)}(a_j))} |u_j(x, a_j)\rangle \right\}, \quad \forall \mathbf{A}, \quad \forall t \quad (7.12)$$

This settles for any observable the question of the absolute values of the coefficients from (7.10).

In $e^{i\alpha(\mathbf{A},j)}$ the observable \mathbf{A} is a variable. When one passes from \mathbf{A} to another observable \mathbf{B} the concept of state-ket $|\psi_{G(t)}\rangle$ involves conditions of mutual consistency that have to be respected. This can be achieved via a lemma $L(Ass.2)$ formally established inside QM_{HD} .

L(Ass.2). If in (7.9) an arbitrary set $\{e^{i\alpha(\mathbf{A},j)}\}$ of complex factors is introduced for \mathbf{A} , then Dirac’s theory of transformations determines *consistently* with this initial choice, all the complex factors to be introduced in all the other expansions of $|\psi_{G(t)}\rangle$ corresponding to any other dynamical observable \mathbf{B} , that does not commute with \mathbf{A} , so $[\mathbf{A}, \mathbf{B}] \neq 0$.

Proof of L(Ass.2). Consider the expansion

$$|\psi_{G(t)}\rangle/\mathbf{B} = \sum_k e^{i\gamma(\mathbf{B},k)} |d_k(t, \mathbf{B})| |v_k(x, b_k)\rangle, \quad k = 1, 2 \dots K, \quad \forall t \quad (7.13)$$

of $|\psi_{G(t)}\rangle$ on the basis $\{|v_k(x, b_k)\rangle\}$ of eigenket introduced in \mathcal{H} by an observable \mathbf{B} , $[\mathbf{A}, \mathbf{B}] \neq 0$, that does not commute with \mathbf{A} . For any given value of the index k we have inside QM_{HD}

$$\langle v_k(x, b_k) | \psi_{G(t)} \rangle = e^{i\gamma(\mathbf{B},k)} d_k(t, \mathbf{B}) = \sum_j \tau_{kj}(\mathbf{A}, \mathbf{B}) c_j(t, \mathbf{A}), \quad \forall t \quad (7.14)$$

where $\tau_{kj}(\mathbf{A}, \mathbf{B}) = \langle v_k | u_j \rangle$, $j = 1, 2 \dots J$. So for *any* complex factor we have a separate condition

$$e^{i\gamma(\mathbf{B},k)} = \langle v_k | \psi_{G(t)} \rangle / |d_k(t, \mathbf{B})| = \sum_j \tau_{kj}(\mathbf{A}, \mathbf{B}) c_j(t, \mathbf{A}) / |d_k(t, \mathbf{B})|, \quad j = 1, 2 \dots J, \quad \forall t \quad (7.15)$$

(where ‘|’ is to be read: divided by). This proves the lemma and it closes the *formal construction*.

So we finally are in possession of a ‘*factual-mathematical*’ definition of a set

$$\left\{ \sum_j e^{i\alpha(\mathbf{A},j)} |c_j(t, \mathbf{A})||u_j(x, a_j)\rangle, j = 1, 2 \dots J, \forall \mathbf{A}, \forall t \right\} \quad (7.16)$$

that represents with mutual formal coherence all the expansions of the unknown state-ket $|\psi_{G(t)}\rangle$ with respect to all the quantum mechanical dynamical observables from QM_{HD} . And the *factual-mathematical* definition (7.16) is equivalent to the state-ket $|\psi_{G(t)}\rangle$ with respect to prediction-verification of the first order probabilistic assertions :

In what concerns prediction the equivalence is a priori insured *by construction*. In what concerns verification the equivalence follows obviously from the assertion *Ass.1*.

Then the expressions (7.11) and (7.16) permit to write:

$$\left[\left\{ \sum_j e^{i\alpha(\mathbf{A},j)} \sqrt{(\pi_{G(t)}(a_j))} |u_j(x, a_j)\rangle, j = 1, 2 \dots J \right\} \equiv_{\text{pred.-verif.}} |\psi_{G(t)}\rangle / \mathbf{A}, j = 1, \dots, J, \forall \mathbf{A}, \forall t \right] \quad (7.17)$$

where the sign ‘ $\equiv_{\text{pred.-verif}}$ ’ is to be read: identical with respect to prediction-*and*-verification); and, globally, the first member represents the *factual-formal* procedure of construction and the second member represents the QM_{HD} equivalent of the first member. And no use has been made of the Schrödinger equation, nor of Born’s postulate. ■

Comments on the Ass.2

(a) The relation (7.17) reminds of Husserl’s argumentation that a physical classical “object”, very far from being a paradigm of materiality as it is currently considered to be, in fact is just a very useful and quasi unconsciously installed and named conceptual synthesis of a very rich set of mutually distinct perceptual representations of a posited exterior and material invariant that is never perceived entirely, in its full wholeness (an architect’s sketches of a “house” can show it only from above, left, etc.). This suggests the following question:

Could the set of factually-formally constructed representations from the firsts member of (7.17) be proved to be equivalent in some strictly defined way, with a minimal set of transferred descriptions in the sense of (3.8), that acts like a group of transformations with respect to an “objectual” invariant?

(Hermann Weil seems to have considered a Husserl’s view, but did he achieve a mathematical positive answer?).

(b) The preceding question can be continued as follows. Since initially an arbitrary set $\{e^{i\alpha(\mathbf{A},j)}\}$ of complex factors is introduced in (7.9), the argument *Arg(Ass.2)* entails the possibility of an infinite set of writings (7.17) that are all equivalent from a predictive point of view. So $|\psi_{G(t)}\rangle$ is *not* fully defined with respect to its predictive content. Now, inside QM_{HD} is admitted a mathematical ‘principle’ of spectral *de*-composability that, in expressions of the form (6.3), permit to write the sign ‘=’. This sign however is no doubt far from indicating a provable possibility of *strict* mathematical identification of the two members of the asserted equality (think of the conditions of possibility of a Fourier decomposition). This fact, when associated with (7.17), leads to wonder whether the availability of a mathematical state-function is indeed a pragmatic necessity, or just an intellectual comfort offered by the belief in the existence of mathematical guarantee; but much more fundamentally it also suggests a *reversed* mathematical question:

Is it possible to completely define, out of the factually constructed member of (7.17), via a procedure of ‘effective’ computation, a (family of) *functional* representation(s) of a state-ket $|\psi_{G(t)}\rangle$?

A definite answer would be very interesting because, since all the elements from the left member of (7.17) contain predictions of the state-ket $|\psi_{G(t)}\rangle$ each one of which is relative to a given observable \mathbf{A} , the mathematical integration imagined above would emerge from a *beforehand relativized* genesis, which is *not* the case if $|\psi_{G(t)}\rangle$ is obtained via a Schrödinger equation and is *afterward* relativized to a class of predictions. This difference might be quite noteworthy, in a sense that in what follows becomes entirely specified.

(c) For a microstate $ms(\text{free}, 1)_{G(n-c)}$ and concerning first order probabilistic predictions and verifications, the *Ass.2* endows the corresponding state-ket $|\psi_G(t)\rangle$ from QM_{HD} , supposed to have been obtained mathematically via the Schrödinger equation of the problem, with a factual-formal equivalent that, no doubt, nowadays can be currently obtained by a convenient use of computers. And this factual-formal equivalent is directly rooted into the unknown physical factuality. The advantages involved by this duplication are quite noteworthy:

- The Schrödinger equation of a given problem is often difficult to write down and to solve, partially because it might be difficult (or simply impossible) to ‘give’ the initial state-ket $|\psi_G(t_0)\rangle$; while in a non-Hamiltonian situation it cannot be utilized. So it is noteworthy that the procedure from the *Arg(Ass.2)* permits to construct *factually* the predictive contents (7.17) of the ‘unknown state-ket $|\psi_G(t)\rangle$ of a problem’, whether this ket has been calculated, or *not*. Moreover, if $|\psi_{G(t)}\rangle$ has been calculated, (7.17) permits to *verify its predictions and to bring forth by comparison the non-predictable deviations from factual truth that $|\psi_{G(t)}\rangle$ might have inserted in consequence of mathematical approximations*. While if it has not been possible to calculate $|\psi_G(t)\rangle$, then (7.17) permits to *replace* its predictive role, while the verification – the factual truth – is insured by construction.

- Since the *Arg(Ass.2)* is valid for any time, it also is valid at the initial time t_0 . So it specifies for a microstate $ms(\text{free}, 1)_{(G(n-c))}$ a general way for constructing a factually rooted equivalent, in the sense of (7.17), of also the initial state-ket $|\psi_G(t_0)\rangle$ of the problem, in any situation. *This facilitates strongly, extends and optimizes the use of the QM_{HD} -formalism.*

- The genesis of the equivalence (7.17) separates radically the individual, physical level of conceptualization, from the statistical one: Thereby inside $[IQM-QM_{HD}]$ the QM_{HD} mathematical representation of a state-ket is incorporated to *IQM*. The two representations tend toward unification. But let us stress that this unification hinges upon the *formal possibility* to make use of the expansion postulate (2.2) of any state-ket ⁸².

In short:

Inside $[IQM-QM_{HD}]$ where is implied the coding-measurement postulate $P(\text{cod})_{G(n-c)}$, and if the possibility to make use of the expansion-postulate is admitted, the expressions (7.1) and (7.17) endow the QM_{HD} statistical representation of microstates from the category $ms(\text{free}, 1)_{G(n-c)}$ with:

- * A high degree of genetic factual *independence with respect to the mathematical formalism from QM_{HD}* .
- * Control upon only mathematically established predictions.
- * A degree of applicability that is notably enlarged with respect to that of the formalism QM_{HD} itself.

7.2.5 Dirac’s theory of transformations as part of a potential calculus with semantic contents

Consider now also the correlations $(Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}$ between globally considered branch-probability laws. With respect to the first-order level from the *IQM*-description 3.8, these correlations are placed on a meta-probabilistic level of conceptualization ; they are second-order probabilistic qualifications. We make the following new assertion, *Ass.3*:

Ass.3. The relations (3.2) that inside *IQM* assert in qualitative and general terms the meta-probabilistic correlations $(Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}$ can be regarded to outline the contours of a more general conceptual framework inside which is lodged Dirac’s theory of transformations, namely: *The framework for a mathematical Hilbert-space calculus with semantic dimensions and values of these* (MMS [1993]).

Arg(Ass.3). Consider the *IQM* descriptor from 3.8

$$\pi(b_k) = \mathbf{F}_{\mathbf{b}_k, \mathbf{A}} \{ \pi_{G^{(t)}}(a_j) \}, \quad \forall k \in \{k = 1, 2, \dots, K\}, \quad j = 1, 2, \dots, J, \quad \forall (\mathbf{A}, \mathbf{B}) \quad (7.18)$$

⁸²We stress this because in section 7.3 we shall be in a circumstance in which precisely the absence of this possibility raises a serious problem of representation.

that denotes meta-probabilistic correlations $(Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}$ between whole probability laws that crown two distinct branches of a given probability-tree of the operation of generation $G^{(t)}$ of the studied microstate. Inside QM_{HD} the Dirac transformation from the Hilbert-space representation of the state-ket $|\psi_G(t)\rangle$ of the studied microstate with respect to the eigenvalues a_j of an observable \mathbf{A} , to the representation of $|\psi_{G^{(t)}}(t)\rangle$ with respect to the eigenvalues b_k of another observable \mathbf{B} that does not commute with \mathbf{A} , is defined by

$$d_k(t, \mathbf{B}) = \sum_j \tau_{kj}(\mathbf{A}, \mathbf{B}) c_j(t, \mathbf{A}), \quad j = 1, 2, \dots, J, \quad k = 1, 2, \dots, K, \quad \forall(\mathbf{A}, \mathbf{B}), \quad \forall t \quad (7.19)$$

The aim of Dirac's QM_{HD} calculus of transformations is entirely ignorant of the *IQM* operational-semantic categorization of the set of all the considered pairs of observable events $\{(a_j, b_k)\}, \forall(\mathbf{A}, \mathbf{B}), \forall t$ tied with the studied microstates inside a tree-like *probabilistic* whole founded upon the operation of generation G or $G^{(t)}$ that corresponds to the state-ket $|\psi_{G^{(t)}}\rangle$ of the studied microstate. This is so because the individual operations G or $G^{(t)}$ – like also any corresponding coding-measurement-evolution – are not represented inside QM_{HD} . So inside QM_{HD} Dirac's calculus of transformation from one 'representation' of $|\psi_G(t)\rangle$ with respect to an observable \mathbf{A} , to another observable \mathbf{B} , is asserted as just a mathematical algorithm devoid of a more general meaning. Nevertheless the isomorphism between the two writings

$$[\mathbf{F}_{\mathbf{AB}}(G^{(t)}) = \{\mathbf{F}_{\mathbf{b}_k, \mathbf{A}}\{\pi_{G^{(t)}}(a_j)\}\}], \quad k = 1, 2, \dots, K, \quad j = 1, 2, \dots, J, \quad \forall(\mathbf{A}, \mathbf{B}) \quad (7.20)$$

and

$$\{d_k(t, \mathbf{B}) = \sum_j \tau_{kj}(\mathbf{A}, \mathbf{B}) c_j(t, \mathbf{A})\}, \quad j = 1, 2, \dots, J, \quad k = 1, 2, \dots, K, \quad \forall(\mathbf{A}, \mathbf{B})$$

claims that these formulas point toward the possibility of a *much more general calculus, of 'semantic proximities'*, that remains to be exploited: For instance, the scalar product of two distinct state-ket of two different microstates, expressed inside one same representation, might be used as a measure of a concept of '*degree of angular proximity inside this representation, so relatively to qualifications by the observable that determines the representation*' (MMS [1993]). ■

Comment on the Ass.3

The *Arg(Ass.3)* draws attention upon the fact that, at least for a microstate $ms(\text{free}, 1)_{G(-nc)}$, the complex factors from the expressions (7.10) $c_j(t, \mathbf{A}) = e^{i\alpha(\mathbf{A}, j)} |c_j(t, \mathbf{A})|$ of an expansion coefficient are active only in the second-order probabilistic qualifications. This remark might gain much importance in the case of microstates with a composed operation of generation, so with inner quantum fields, if the assertion *Ass.3* can be extended to these.

-It seems very likely that Dirac's calculus of transformations has essential connections with the informational concept of mutual information.

7.2.6 On the Schrödinger equation of evolution and Born's postulate

We have noted that the result (7.17) frees of the necessity to write and solve the Schrödinger equation of a given problem concerning a microstate $ms(\text{free}, 1)_{G(n-c)}$ when this involves too much difficulty. This leads naturally to the following question: For *exclusively* predictive aims and for the case of microstates $ms(\text{free}, 1)_{G(n-c)}$, what – exactly – does Schrödinger's equation introduce *specifically* ? We go back to the *IQM* relation (3.7) $G^{(t)} = F(G, EC, (t - t_0))$ and its QM_{HD} specification (7.1) $G^{(t)} = F(G, \mathbf{H}, (t - t_0))$ where t_0 is the time when the *initial* operation of generation G of the studied microstate finishes (in particular one can have $G^{(t)} \equiv G$). As we have noted already, this relation absorbs the 'evolution' of the studied microstate into the operation of generation $G^{(t)}$ while '*one act of measurement MesA on a microstate*' is organically inserted into one realization of a *whole succession* $[G^{(t)}.Mes\mathbf{A}]$. So any process of individual contribution to a probabilistic description 3.8 ($D_{Mec}(ms_{G^{(t)}})$) starts with a corresponding operation of generation of a specimen of the studied

microstate and continues with a coding-measurement operation that finished with a registration of observable marks that code for one definite eigenvalue of the measured observable:

$$[(G^{(t_1)} \rightarrow \sigma_\Phi).Mes\mathbf{A}(\sigma_\Phi)] \rightarrow_{\mathbf{H}(\mathbf{A})} (\text{marks registered in } \Delta x \Delta t)_j \equiv 'a_j' \quad (7.21)$$

But as long as no equation of *individual* evolution is specified, nothing is specified concerning the way in which in (7.3) such an isolated thread of individual events develops in time, nor how the various inner individual time evolutions from a set of repeated successions (7.1) behave mutually in the exterior conditions (classical fields, ‘obstacles’) involved by the Hamiltonian operator \mathbf{H} that acts for constructing the state-ket $|\psi_{G^{(t)}}(t_0 \leq t \leq t_1)\rangle$ that verifies the Schrödinger equation of evolution $i(\hbar/2\pi)(d/dt)\psi_G(t) = \mathbf{H}|\psi_{G^{(t)}}\rangle$ (this is a particular consequence of the general fact that inside QM_{HD} the individual phenomena are not represented). Concerning this we make explicitly the following (rather trivial) assertion:

Ass.4. The Schrödinger equation of a problem that concerns a microstate $ms(\text{free}, 1)_{G^{(n-c)}}$ offers concerning the individual time-evolutions (7.1), directly and *exclusively* a collective numerical information constrained a priori to fit into a deterministic mathematical mould. This requirement is valid in particular for the representation from the initial state-ket $|\psi_{G^{(t)}}(t_0)\rangle$.

Arg(Ass.4). Suppose that the situation is Hamiltonian and that it has been possible to construct the equation of evolution of the problem, $i(\hbar/2\pi)(d/dt)|\psi_{G^{(t)}}\rangle = \mathbf{H}|\psi_{G^{(t)}}\rangle$, its general solution, as well as the initial state-ket $|\psi_{G^{(t)}}(t_0)\rangle$. This – postulated – equation of evolution is of first order with respect to time, so – itself and mathematically – it is ‘deterministic’. This means that a priori no unpredictable element is *formally* allowed to act during the representation of passage from the initial state-ket $|\psi_{G^{(t)}}(t_0)\rangle$ to the state-ket (6.11) $|\psi_{G^{(t)}}(t_0 \leq t \leq t_1)\rangle$, for any t and any t_1 (when measurements begin). So the specificity introduced by the Schrödinger equation of the state-ket $|\psi_{G^{(t)}}(t_0 \leq t \leq t_1)\rangle$ inside the framework $[IQM-QM_{HD}]$ consists precisely by the postulation of collective deterministic transformations. ■

Comment on the Ass.4. The initial state-ket $|\psi_{G^{(t)}}(t_0)\rangle$ is a mathematical representation in the sense of (7.17) of the description ($D_{Mec}(ms_{G^{(t_0)}})$) that is realized by the repetition of factual, individual, physical coding-measurement evolutions (7.3). So *it should be constructible factually for any sort of microstate, whether it is a microstate $ms(\text{free}, 1)_{G^{(n-c)}}$, or not*. Once the initial state-ket $|\psi_{G^{(t)}}(t_0)\rangle$ is ‘given’ in some way, it includes in it the inner individual structure of the studied microstate, even if it contains a non-null quantum potential or even active quantum fields, and then the equation of evolution – via the Hamiltonian operator \mathbf{H} that is involved – transforms deterministically the initial (ϵ, δ, N_0) -probabilities into those that are valid at subsequent times. That is *all* that the equation of evolution does. At a first sight one can even have an impression of triviality or even of tautology, since this equation can be read as: “The *change* of [that what you want to know (namely the set of statistics from (7.17) represented by the state-ket $|\psi_{G^{(t)}}(t)\rangle$)] measured in unities \hbar of minimal action, is equal to the *effect* upon [what you want to know] of [that what changes it (represented by the operator \mathbf{H})]”. But this, if one thinks of it, seems simply miraculous, not only in what concerns the capacity of \mathbf{H} to imply “*all*” the ‘legal’ transformations of an entity as diverse and complex as the set of statistics from (7.17), but even more, in what concerns the loading of the initial data to be transformed: *How is it possible that ‘all’ the ‘relevant’ factual data at t_0 be incorporated into a mathematically and globally worked out abstract initial descriptor $|\psi_{G^{(t)}}(t_0)\rangle$?* This seems astounding even when the finite (ϵ, δ, N_0) -character of the considered probabilities is taken into account: *What achieves the factual, individual, physical harvest of the initial data? What decides their ‘relevance’ or not? What does “all’ the ‘relevant’ factual data at t_0 mean?*

Here we find ourselves face-to-face with Wigner’s expression «the unreasonable power of mathematics.»

I hold that there is a unique way to understand what happens here: The list of possible ‘aspects’ enclosed in the smallest fragment of factual reality is *unlimited*, it cannot be exhausted by any description, it is unspeakable. On the contrary:

Any *description* of this fragment of factual reality – and in particular also any mathematical description – *filters out a finite number of possible qualifications, by relativizing to more or less explicit grids of qualification* (MMS [2002A], [2002B], [2006]). *Otherwise it could not be achieved.*

But there are not a priori reasons that the relativizations involved in the Schrödinger equation via the general axioms of the differential calculus, are those that optimize the use of this equation inside, specifically, a representation of microstates.

Inside the nowadays conceptualization, the initial relativization of any description to an only finite set of qualifications is not explicitly *researched and declared*. And in the case of the Schrödinger equation we are on just the edge where such a passage from unspeakable infinite singularity, to a finite set of pre-organized qualifications, – that quite certainly somehow preexists via the involved mathematical axioms – still remain to be *identified* and then organized explicitly. But an initial state-ket $|\psi_{G(t)}(t_0)\rangle$ of structure (7.17) permits already to *saturate* the possibilities *with respect to this set*, offered by the as yet unspecified grid for qualification involved by the considered Schrödinger equation. Which is a quite noteworthy fact..

Inside $[IQM-QM_{HD}]$, can be conceived to maximize a priori the efficiency of the equation of evolution by *always* ‘giving’ in an *entirely* factual, so non-restricted way, the initial state-ket $|\psi_{G(t)}(t_0)\rangle$, accordingly to the procedure from the *Arg(Ass.2)* that leads to (7.17).

And more generally

The relation (7.17) induces at any time and for any initial physical situation, a set of predictive expansions of a state-ket that is devoid of a known functional representation, but that suffices – and optimally, most entirely connected to factuality – for insuring all the predictational tasks of the functionally un-defined QM_{HD} state-ket.

Remarks on Born’s postulate. For reasons that likely are of the same nature as those from the above comment on the *Ass.4*, Born’s postulate has seemed to many to be miraculous, and there have been attempts at *deriving it* (cf. in Raichman [2003]: Destouches-Février [1946] et [1956], Ballentine [1973], Deutsch [1999]). Thereby these authors manifested non-perception of the unbridgeable abyss that separates logical-mathematical deduction, from the data that can be drawn only directly from facts, in a non-expressed a-rational manner. On the contrary Anandan [2001] has explicitly drawn attention upon the insurmountable hiatus between a conceptualization in ‘continuous’ mathematical terms and a probabilistic conceptualization, and he has proposed to dwell with the problem via a new sort of mathematical modelization ⁸³.

7.2.7 Conclusion on section 7.2

The main results from section 7.2 are the following ones:

- The specification of a succession $[G^{(t)}.Mes\mathbf{A}]$ for the particular case of microstates $ms(free, 1)$, accordingly to the postulate $P(cod)_{G(n-c)}$, expressed by the relation

$$[(G^{(t)} \rightarrow \sigma_{\Phi}).Mes\mathbf{A}(\sigma_{\Phi})] \rightarrow_{\mathbf{H}(\mathbf{A})} (\Delta x_j \Delta t_j : \sigma_{\Phi} \equiv |u_j(x, a_j)\rangle), j = 1, 2, .., \forall \mathbf{A} \in V_{Mec} \quad (7.22)$$

- The factual construction of the predictive equivalent

$$[\{\sum_j e^{i\alpha(\mathbf{A},j)} |c_j(t, \mathbf{A})| |u_j(x, a_j)\rangle\}, \forall \mathbf{A}, \forall t] \equiv_{\text{pred.-verif.}} |\psi_G(t)\rangle \quad (7.23)$$

⁸³ In its non-mathematical essence Anandan’s view is in agreement with our view, though we do not place the nature of the hiatus in the continuous character of the mathematical model that is made use of, but in the human *cognitive* choice of a grid of qualification that is deliberately posited to be finite in every respect. Such a choice is independent of the continuous mathematical representation of facts themselves.

of the state-ket $|\psi_G(t)\rangle$ of a microstate $ms(free, 1)_{G(n-c)}$.

On the cleaned ground left by the elimination of the measurement problem in section 7.1, and only for the particular category of microstates $ms(free, 1)_{G(n-c)}$, these results establish an acceptable representation of the measurements.

But furthermore *they introduce also a duplication of the basic formal descriptors and algorithms from QM_{HD} , by **factual**-formal corresponding descriptors that instil: possibility of factual control of the mathematical predictions; a noticeable degree of independence with respect to the mathematical formalism; an extended domain of applicability.*

Considered globally, the mentioned results transform already the initial adjunction of IQM and QM_{HD} inside the provisional framework [IQM - QM_{HD}], into a genuine merger of these two different approaches.

7.3 Coding problem for free microstates with internal quantum field and a possible solution tied with a crucial experiment

Consider now a microstate of one microsystem but with a composed operation of generation (cf. the definitions from section 3.1). Let us denote it $ms(free, 1)_{cG(q-f)}$ (the index ‘ $cG(q-f)$ ’ is to be read ‘composed operation G of generation involving quantum fields’). Such a microstate involves an internal quantum potential. In section 6.5 we have found that, while the approach $BBGPM$ presupposes implicitly that *any* measurement on any sort of microstate permits a coding-procedure of the type 6.13, in fact such a procedure can be more or less ‘explained’ only in the absence of quantum-fields. So in the case of a microstate $ms(free, 1)_{cG(q-f)}$ the coding-postulate (7.3) $P(cod)_{G(n-c)}$ – that requires a quasi-classical coding-measurement evolution – ceases to be acceptable a priori. The situation has to be re-examined.

7.3.1 Problem for the verifiability of the predictions of a state-ket of a free microstate $ms(free, 1)_{cG(q-f)}$ with composed operation of generation

From now on we write in three spatial dimensions. For simplicity we consider a microstate $ms(free, 1)_{cG(q-f)}$ of type (6.4) with only two components in the operation of generation:

$$|\psi_{\mathbf{G}(G_1, G_2)}(r, t)\rangle = \lambda_1 |\psi_{G_1}(r, t)\rangle + \lambda_2 |\psi_{G_2}(r, t)\rangle \quad (7.24)$$

Suppose that we want to measure the momentum-quantity \mathbf{p} represented by the momentum observable \mathbf{P} . The QM_{HD} prediction concerning an eigenvalue \mathbf{p}_j of \mathbf{P} is ⁸⁴:

$$\pi_{\mathbf{G}(G_1, G_2)}(\mathbf{p}_j) = \lambda_1 c_{j1} + \lambda_2 c_{j2}^2 = \lambda_1 c_{j1}^2 + \lambda_2 c_{j2}^2 + \lambda_1 c_{j1} (\lambda_2 c_{j2})^* + (\lambda_1 c_{j1})^* \lambda_2 c_{j2} \quad (7.25)$$

The Hamiltonian $\mathbf{H}(\mathbf{P})$ that commutes with \mathbf{P} has no potential term: $\mathbf{H}(\mathbf{P}) = -(\hbar/2\pi)^2 (d^2/dr^2)$ so no external macroscopic fields are involved. In this case the procedure required in (7.3) by the postulate $P(cod)_{G(n-c)}$ in order to verify (7.25) would be that of ‘time of flight’ (cf. note in section 6.6.1): Suppress any external macroscopic field and let the specimen σ_{Φ} of the studied microstate introduced by the performed operation of generation $\mathbf{G}(G_1, G_2)$ evolve freely accordingly to (7.3) until it reaches a space-time domain $(\Delta x \Delta t)_j$ that is found to be characteristic of the eigenvalue \mathbf{p}_j of \mathbf{P} . How is it found such? On the basis of the assumption that in absence of macroscopic exterior fields the de Broglie corpuscular-like singularity from the amplitude of the *physical wave* of the involved specimen

⁸⁴In this case we re-note the probabilities by ‘ π ’ in order to avoid confusion with the eigenvalues p_j of P , and we furthermore consider the three-dimensional eigenvalues ‘ $a_j \equiv p_j$ ’ for immediate comparability with the development from the subsequent section 7.3.2

$\sigma_{\Phi(G)}$ of the studied microstate, will be animated by a dynamic that can be assimilated to that of a classical mobile in the absence of exterior fields; so a dynamic with *constant momentum* \mathbf{p} . But a microstate of type (6.4) involves a non-null quantum potential and so in general it can involve also active quantum fields that cannot be always specified and predicted, nor suppressed physically by the human observer. In such conditions the coding-measurement-evolution supposed in (7.3) is not pertinent.

So in general the prediction (7.25) cannot be verified via (7.3).

Paradoxically, precisely the microstates $ms_{G(q-f)}$ might have strongly contributed to the choice of a linear Hilbert-Dirac mathematical framework for the representation of microstates. How can one imagine what happened in this respect? Let us go back to section 3.3 It has been very soon shown experimentally that the ‘presence’-probabilities involved in the Young’s two-slits experiment lead to *in-equalities*

$$\pi_{\mathbf{G}(G_1, G_2)}(\mathbf{r}_j) \neq \pi_{G_1}(a_j) + \pi_{G_2}(\mathbf{r}_j) \quad (7.26)$$

Now, at the first sight, the choice of a linear vector-space mathematical representation seems to be particularly convenient for dealing quite *generally* with this sort of cases, so also for the momentum observable \mathbf{P} , because, when associated with the use of complex functions, with expansions of the state-ket on a basis of eigen-functions of \mathbf{P} , and with Born’s postulate, it converts an inequality of type (7.26) into a numerical equality, via a linear calculus with complex expansion coefficients. For the momentum observable this yields the equality (7.25). But this conversion entailed by the formal choices enumerated above that represent the state-ket of a microstate $ms_{\mathbf{G}(G_1, G_2)}$ as a linear combination of the *virtual* state-ket of ms_{G_1} and ms_{G_2} , does not *entail* that the numerical equality asserted in (7.25) is *factually true*. And it comes out that the prediction (7.25) is not verifiable by use of (7.3). So as long as another conceptually acceptable coding-procedure is not specified, the ‘prediction’ (7.25) amounts in fact to *just a definition postulated to be endowed with factual truth*.

We detail this situation because of its conceptual importance.

No acceptable coding-measurement succession is defined as yet for the measurement of the momentum observable \mathbf{P} of *the studied microstate* $ms_{\mathbf{G}(G_1, G_2)}$ itself, so also for the probability from the left member of the equality (7.25). So from the point of view of momentum-measurements *the studied microstate* $ms_{\mathbf{G}(G_1, G_2)}$ *itself is devoid of any direct relation with factuality*. Indeed the right member involves factual verifications that concern exclusively the microstates ms_{G_1} and ms_{G_2} . But these have not been *physically individualized* by the unique effectively realized operation of generation $\mathbf{G}(G_1, G_2)$. The one-to-one relation (1.1) is asserted in [IQM-QM_{HD}] only for $ms_{\mathbf{G}(G_1, G_2)}$ (this is not visible inside QM_{HD} where the operation of generation of the studied microstates remains hidden). With respect to the studied microstate $ms_{\mathbf{G}(G_1, G_2)}$ – the only one that is physically realized in order to be studied – the two microstates ms_{G_1} and ms_{G_2} are just a sort of instillation from the ‘composed’ operation of generation $\mathbf{G}(G_1, G_2)$, lost in a non-specified way inside the global effect of $\mathbf{G}(G_1, G_2)$. They *could* be fully realized separately – by definition – and if this is done *then* the probabilities calculated by Born’s postulate from their respective separate state-ket $|\psi_{G_1}\rangle$ and $|\psi_{G_2}\rangle$ can be verified accordingly to (7.3) $P(cod)_{G(n-c)}$ because separately ms_{G_1} and ms_{G_2} do not involve any quantum field and so the dynamics of the corpuscular-like singularity from the wave-phenomenon of the involved specimens σ_{Φ} from (7.3) keeps entirely determined by the external macroscopic fields represented in the measurement-Hamiltonian operator $\mathbf{H}(\mathbf{P})$ ⁸⁵. But if indeed ms_{G_1} and ms_{G_2} were realized separately, then the asserted connection between the obtained result, and measurements on $ms_{\mathbf{G}(G_1, G_2)}$ would not be that asserted in (7.25). While effective factually realizable coding-measurement successions for achieving the verification, are not defined. That is why the writing (7.25), instead of expressing a ‘prediction’ is so far only a postulated definition⁸⁶.

One could believe that this whole problem can be eliminated by just refusing a posteriori the concept of a ‘composed operation of generation’ defined in section 2.1 and by accepting the QM_{HD} direct

⁸⁵It can be supposed that $\mathbf{H}(\mathbf{P})$ effaces ‘rapidly’ ‘significant’ initial differences with an eigenket of \mathbf{P} , if these existed (the quotation marks stress that we continue being immersed in approximations).

⁸⁶ It seems not unlikely that no verifications have ever been made for the momentum in a microstate $ms_{\mathbf{G}(G_1, G_2)}$, only the position-distribution has initially drawn attention upon it and then the corresponding representational solution has been confidently generalized. But the position-operator \mathbf{R} is a degenerate sort of ‘observable’ operator (in the theory of particles it is replaced by an operator of localization).

postulation of the existence of ‘superposition states of a microsystem’. But this is not the case. An a posteriori rejection of the concept would not in the least change the fact that the prediction (32) cannot be verified experimentally by coding procedures of the type (7.3). On the contrary, the fact that the concept of a ‘composed operation of generation’ reveals the situation examined above is a strong confirmation of its relevance.

In short, the formulation of an acceptable theory of quantum measurements started in section 7.2, is for the moment blocked with respect to microstates $ms(\textit{free}, 1)_{cG(q-f)}$. While this category of microstates is much more specific of *quantum*-mechanics than the category $ms(\textit{free}, 1)_{G(n-c)}$ where the mechanical behaviours still possess a quasi-classical character. So this situation must be solved explicitly inside $[IQM-QM_{HD}]$.

7.3.2 Recourse to the *dBB* interpretation of QM_{HD}

Preliminary remarks on beables and observables. It has become current to distinguish between ‘beable’ qualifications in the sense of the de Broglie-Bohm approach, and on the other hand QM_{HD} -‘observables’. The position vector-observable \mathbf{R} is considered more or less implicitly to behave like a ‘beable’, in this sense that its eigenvalues are observed such as they ‘are’ inside the physical wave represented by the wave-function $\Phi_G(\mathbf{r}, t) = a(r, t)e^{(i/\hbar)\beta(\mathbf{r}, t)}$ assigned to each specimen $\sigma_{\Phi(G)}$ of the studied microstate. But this is regarded as an exception. In general a QM_{HD} ‘observable’ – the momentum vector-observable-operator included – is considered quite generally to manifest eigenvalues ‘created by the measurement’. But this view expresses the belief that any act of measurement-interaction *changes* the initial value possessed by the measured quantity at the beginning of the measurement-interaction, so that what is observed and announced as the result of the considered measurement evolution – the ‘observable’ value – is always different from this initial ‘beable’ value.

But the analyses from section 6.5 have brought forth that in fact – under the influence of classical mechanics – a coding-measurement-evolution in the sense extracted in (7.3) is expressly conceived such as to favour the aim to freeze and *conserve unchanged the initial value of the measured quantity*, in order to export it into the realm of the observable such as it was when the measurement began (cf. the method time-of-flight for measuring the momentum-observable). And precisely this ceases being controllable for microstates $ms(\textit{free}, 1)_{cG(q-f)}$, because, in contradistinction to what happens for microstates $ms(\textit{free}, 1)_{G(n-c)}$, the dynamics of the corpuscular-like singularity from a specimen of a studied microstate $ms(\textit{free}, 1)_{cG(q-f)}$ can *never* be brought under the permanent dependence of *exclusively* ‘exterior’ macroscopic fields. In the *inside* of each specimen of such a microstate there subsist irrepressibly a possibility of forces that can change the value of the momentum at unpredictable times and to unpredictable degrees. The inside of the specimens of a microstate $ms(\textit{free}, 1)_{cG(q-f)}$ is out of the human observer’s control. There exists no exterior measurement-Hamiltonian-operator- $\mathbf{H}(\mathbf{P})$ that be able to insure conservation of the pattern of wave-movement around the involved corpuscular-like singularity from each specimen of a studied microstate $ms(\textit{free}, 1)_{cG(q-f)}$ so as to bring the corresponding eigenvalue \mathbf{p}_j of the momentum, such as it was when the measurement operation began, into a space-time domain that characterizes it via a one-one relation, accordingly to the content of the coding-measurement-evolution (7.3).

But when the inside of any specimen of the studied microstate does not contain any possibility to change the beable momentum-value – which is the case for microstates $ms(\textit{free}, 1)_{G(n-c)}$ – then (7.3) *can* be conceived to lead to an observed value that is ‘practically’ identical to the beable value such as it was when the considered act of measurement has begun. This is the very principle on which (7.3) is founded.

In short, *the essential difference between ‘observable’ values of the momentum, and ‘beable’ values, does not concern these values themselves but the way in which it is possible to bring them into knowledge.* In microstates without possibility of quantum fields the *beable* momentum can be frozen to remain practically unchanged during a coding-measurement-evolution (7.3), while in the case of a microstate with possibility of quantum fields this is not possible.

In this case the beable momentum-value \mathbf{p}_j is essentially an instantaneous value, so what is needed is a coding procedure of an instantaneous momentum value.

This is the essential point.

Measurability of de Broglie's guided value of the 'beable' momentum. The preceding remarks lead toward the *dBB* approach that penetrates explicitly into the inside of the microstates and takes into account the quantum potentials and the quantum fields that can act there instantaneously. The *dBB* approach posits quite essentially the well-known 'guiding relation' introduced by Louis de Broglie:

$$\mathbf{p}(\mathbf{r}, t) = -\nabla.\beta(\mathbf{r}, t) \quad (7.27)$$

where $\mathbf{p}(\mathbf{r}, t)$ is the 'guided' momentum of the corpuscular-like singularity and $\beta(r, t)$ is the phase-function from the wave-function $\Phi_G(\mathbf{r}, t) = a(\mathbf{r}, t)e^{(i/\hbar)\beta(\mathbf{r}, t)}$ that represents each specimen $\sigma_{\Phi(G)}$ of the studied microstate (cf. sections 6.2 and 6.5) The guidance law (7.27) is asserted *deductively* and *with full generality*, in the presence of quantum fields as well as in their absence. *But this law is quasi-unanimously considered to be un-observable.* Even de Broglie and Bohm themselves adhered to this view. It is believed that as soon as one would try to register the guidance-trajectory in a specimen $\sigma_{\Phi(G)}$ of the studied microstate, the beginning of the interaction would immediately destroy the phase represented by the phase-function $\beta(\mathbf{r}, t)$, which would compromise any relevance of the data drawn from the interaction. This idea however is asserted on the basis of only a qualitative and absolute reasoning. Nobody analysed whether yes or not it is possible to choose the values of the involved parameters such that – in *theoretical* agreement with the the *dBB* assumptions – the registered data shall *permit* to construct from them the value of the guided momentum $\mathbf{p}(\mathbf{r}, t)$ from (7.27) *at the time t when the interaction begins.* But when such a theoretical examination is achieved (cf. the Appendix II) it leads to a proof of the following proposition denoted Π_{guid} :

Π_{guid} . For a *stable* interference microstate (with non-null quantum potential but with null permanent quantum fields) it is possible – *in full compatibility with de Broglie's theory of 'double-solution'* – to register data that do permit to calculate from them the corresponding momentum-value from (7.27) for the time t when these registrations have *begun* ⁸⁷.

So in the specified conditions – contrarily to an un-critical belief of general impossibility – *nothing* of logical or mathematical nature withstands the idea of principle that the *dBB* momentum-value (7.27) for a free interference-microstate can be determined *experimentally*.

However this possibility of principle still remains to be proved experimentally.

On a proposed experiment. The mentioned theoretical proof idealizes the situation from (7.24) into a physical superposition of two plane waves. I summarize immediately below the essence of Π_{guid} because it contains indications for an effective experimental realization – *EXP* – with a conveniently chosen Young-like-interference-state (7.24). (The notations on the figure do not distinguish between physical wave and state-function, etc.).

One starts with a free precursor state of which the state-function ψ has as much as possible the structure of a plane wave. This precursor-state encounters a divider of the front of the corpuscular wave that splits it into two practically plane wave-packets of state-ket $|\psi_1(\mathbf{r}, t)\rangle$ and $|\psi_2(\mathbf{r}, t)\rangle$ that then superpose inside a *delimited* but comfortably big space-time domain where is thus realized an interference-state from the same general category as (7.24). This is the state-ket of the microstate to be studied. The directions of propagations from ψ_1 and ψ_2 make a mutual angle α , while with the axis $0z$ they make angles θ of the same absolute value. According to *QM_{HD}* the state inside the space-time domain where there is interference is represented by a superposition state-ket

$$|\psi_o(\mathbf{r}, t)\rangle = |\psi_1\rangle + |\psi_2\rangle = \sqrt{2} \cos(2\pi(\nu/V) \cos \theta.z + \delta/2) e^{2\pi\nu(t - (x/V \sin \theta))} e^{i(\delta/2)} \quad (7.28)$$

where δ designates the phase-difference. With respect to the introduced referential, the guidance relation asserts for the corpuscular-like singularity in the amplitude of the de Broglie wave-function

⁸⁷Even if this result is established only for a particular case, it represents a first destructive intrusion into a belief of general impossibility

$\Phi(\mathbf{r}, t) = a(\mathbf{r}, t)e^{(i/\hbar)\varphi(\mathbf{r}, t)}$ a velocity with the following components

$$v_x = v_0 \sin\theta = \text{const}, \quad v_y = v_z = 0 \quad (7.29)$$

So the momentum-components are

$$p_x = Mv_x = Mv_0 \sin\theta, \quad p_y = p_z = 0 \quad (7.30)$$

where M designates the ‘quantum mass’ of the electron, in the sense of de Broglie (1956).

For times t that exceed the space-time domain of factual superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$ the state-ket $|\psi_o(r, t)\rangle$ describes a mathematical superposition of two plane wave-packets that do not superpose physically.

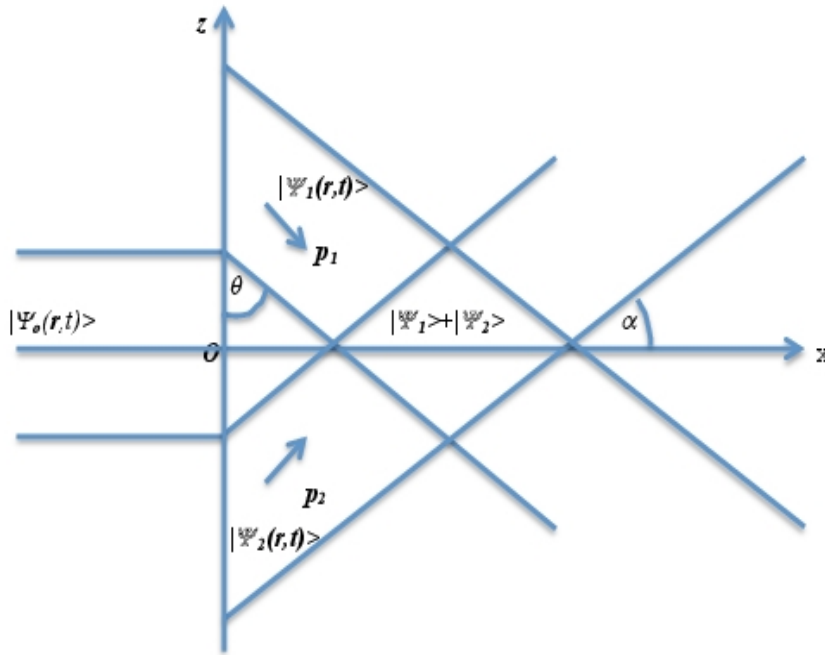


Figure 7.1: The microstate for experimental study

The proof Π_{guid} shows that if the global dynamical structure of the experiment is adequately conceived and the experimental parameters are chosen adequately, the values from (7.30) *can* be established experimentally, as follows.

The presence probability inside the space-time domain covered by the interference phenomenon is representable by a pattern of fringes of high presence-probability (‘brilliant’ fringes), all parallel to the Ox axis and mutually separated by fringes of quasi-zero presence-probability (dark fringes). If the corpuscular-like singularity suffers a perturbation that is energetically sufficiently small with respect to its kinetic energy, then a *quantum-force* emerges that *is parallel to Oz* and acts very briefly. This force might displace the singularity on another brilliant fringe, *but it does not suppress the phase relation from (7.28)* that determines the momentum-value from (7.30).

The experiment can be structured as a sequence of distinct tests:

At a distance Ox_1 , near the entry into the zone of interference is placed on Ox a very thin layer L_1 of sensitive substance permitting with maximal probability at most 2 successive initial acts of ionization. At a second distance Ox_2 placed near the end of the interference domain is placed a thick layer L_2 of photographic emulsion with high density of molecules. When the first ionization occurs in L_1 at a time t_1 a chronometer registers this time. As soon as the corpuscular singularity reaches the second layer it produces there nearly certainly, practically on the entry-edge, a third ionization that

is recorded at a time t_2 . Then other ionizations follow until the energy of the corpuscular-like energy is consumed.

We keep all the cases in which either one or two initial ionizations have been registered.

* When the two first ionizations are available they permit to establish via the small segment of line that unites them whether the perturbing quantum-force has effectively displaced the corpuscular singularity on another fringe of high presence-, or not (this datum verifies the existence of the perturbing quantum field and the strength of its effects).

* The two registered times t_1, t_2 permit a first estimation of the velocity (7.29) so of the momentum (7.30) (a sort of time-of-flight method ‘internal’ to the involved specimen of the studied microstate).

* The ending set of ionizations permits to calculate the absolute value of the momentum.

* The statistic of the positions at the time t_2 permits to know whether the position distribution after the first one or two ionizations is still organized in maxima and minima indicating interference fringes, so *it verifies the conservation of the initial phase-relation*.

* The statistic of the registered momentum-values permits confrontation with the QM_{HD} -prediction (7.25).

* Since the first impact defines also the initial position \mathbf{r} with respect to the referential, **such a registration would violate Heisenberg’s principle**⁸⁸.

This would prove that the *validity of Heisenberg’s principle is relative to the experimental procedure*; it would also permit to delimit clearly the domain of validity of the uncertainty theorem from QM_{HD} , namely the dependence of the theorem on the implicit assumption of a coding procedure (7.3) that can be realized **only** for microstates $ms(free, 1)_{G(n-c)}$.

* The statistic of the registered momentum-values permits now confrontation with the QM_{HD} -prediction. And in the case examined here (supposing that *EXP* succeeds) this prediction is *invalidated*. Indeed – exactly insofar that the two plane waves approximation is acceptably realized immediately after the action of the device that splits the front of the incident wave – the QM_{HD} -prediction asserts for the studied microstate a spectrum of two vector-values \mathbf{p}_{1b} and \mathbf{p}_{2b} , but no vector-value along the axis $0x$. While the *dBB* guided momentum is asserted to be parallel to $0x$.

In the particular case (7.28) the QM_{HD} -prediction simply *circumvents* momentum-measurements performed at times interior to the space-time domain from the evolution of the state-ket where the physical interference phenomenon is realized.

Indeed the QM_{HD} -prediction would be *verified* for pairs (\mathbf{r}, t) that exceed the space-time domain covered by a physical phenomenon of interference. An acceptable theory of microstates should interdict such arbitrary exclusions of space-time zones that the equation of evolution does assert. But even if this last remark were contested, the *general situation stays unchanged* because in a usual Young-interference with spherical waves, strictly speaking, one *never* comes out of the space-time domain of *physical* superposition, while in a mathematical theory of Physics the very *principles* of the processes of verification of the predictions on measurements cannot be *founded* upon limit-approximations like in pure mathematics. *Verifiability must be an effective possibility involving exclusively effective physical operations*. A mathematical theory of Physics is not pure mathematics. So:

If the guided-momentum value (7.27) is shown to be measurable, then it appears that in general, for microstates $ms(free, 1)_{cG(q-f)}$, the QM_{HD} ‘predictions’ are incompatible with the inner structure of the studied microstates and with the momentum-values determined by this structure.

And let us notice that *indirect* measurements (Compton-interactions, etc.) would lead to the *same conclusion*, since the same beable values of the momentum would emerge. Furthermore, this conclusion entails similar conclusions on also the predictions concerning the other observables, with only the exception of the position-observable \mathbf{R} .

These considerations establish the very particular stake of the experiment *EXP*.

⁸⁸ Such a violation – of which the possibility has been very explicitly asserted for heavy microstates in MMS [1964] – has been recently *proved* experimentally for photons (cf. Piacentini & altera [2015]).

For *photonic* interference states a guided trace has already been experimentally registered (A. Steinberg [2011]). This is a very strong indication that an experiment with heavy microsystems would also succeed.

But we stress that – strictly and fundamentally – QM_{HD} concerns *heavy* microsystems. So in the present context only an experiment with microsystems endowed with non-null rest-mass would possess a full significance of principle. It seems likely that the best choice would be to work with a *neutron-Young-state* that would from the start introduce relatively high kinetic energies even for moderate velocities and would involve exclusively quantum fields, thus avoiding any possible effect produced by electromagnetic fields during the ionizations⁸⁹.

In order to achieve the started construction of an acceptable representation of quantum measurement, in what follows we just admit by hypothesis that the *EXP* has been performed and has established the possibility to observe the ‘beable’ *dBB* momentum-values (7.27).

7.3.3 Postulate for coding-measurement-successions of the momentum-value of microstates $ms(free, 1)_{cG(q-f)}$

From now on the framework [*IQM-QM_{HD}*] is transmuted into the more complex framework [*IQM-QM_{HD-dBB}*].

A coding-postulate for microstates $ms(free, 1)_{cG(q-f)}$. Inside *dBB* the guiding-law (7.27) is asserted without restriction. So we now admit that for *any* sort of free microstate (in the sense of the definitions from section 2.1) the beable value of the fundamental dynamical quantity of momentum can be calculated from the registration of observable data concerning the *dBB*-guidance-trace. This amounts to the general assertion of the following coding-postulate $P(cod)_{\forall ms_G}$:

P(cod)_{∀ms_G}. The beable momentum-value of any free microstate can be determined by coding-measurement-successions that obey the representation

$$\left([(G^{(t)} \rightarrow \sigma_{\Phi(G^{(t)})}) \cdot Mes(\mathbf{r}_t \cdot \mathbf{p}_t)](\sigma_{\Phi(G^{(t)})}) \rightarrow_{dB \text{ guid. trace}} (\mathbf{r}_{k,t}, \mathbf{p}_{j,t}) \right), \quad k = 1, 2, \dots, K; \quad j = 1, 2, \dots, J \quad (7.31)$$

In this writing $G^{(t)}$ is posited to generate a specimen $\sigma_{\Phi(G^{(t)})}$ of the studied microstate $ms_{G^{(t)}}$ that is initially represented by an unknown individual wave-function $\Phi_{G^{(t)}}$; one act $Mes(\mathbf{r}_t \mathbf{p}_t(\sigma_{\Phi(G^{(t)})}))$ of measurement of the beable momentum \mathbf{p}_t (7.27) permits to calculate the beable vector-momentum-value $\mathbf{p}_{j,t}$. While the space-time point where the registration of a guiding-trace begins defines also the beable value $\mathbf{r}_{k,t}$ of the position-vector $\mathbf{r}(t)$ of the corpuscular-like singularity from the involved specimen $\sigma_{\Phi(G^{(t)})}$ of the studied microstate. So:

The coding postulate (7.31) *violates Heisenberg’s principle* as well as the Heisenberg theorem from QM_{HD} .

This illustrates the relativity of this principle to the involved circumstances.

7.4 On the mathematical representation of $ms(free, 1)_{cG(q-f)}$

The problem. For the moment, with respect to microstates $ms(free, 1)_{cG(q-f)}$ we are left with a void of a predictive mathematical algorithm. Indeed the coding-postulate (7.31) brings us to the following situation.

* *IQM* as a whole, the general preliminary specifications from section 6.5, the critical conclusion from section 6.6, and the basic initial constructive steps from section 7.1, remain as valid for microstates $ms(free, 1)_{cG(q-f)}$ as they are for microstates $ms(free, 1)_{G(n-c)}$. But:

* The assertions *Ass.2-Ass.4* from section 7.2 that are established by use of the coding postulate (7.3) are *not* valid for microstates $ms(free, 1)_{cG(q-f)}$. For such microstates with possibility of

⁸⁹If this first step is gained, it will have to be somehow extended to cases with quantum fields that are active from the start and permanently, if it is desired to obtain full insight into the microphysical substrata of our physical reality. But since the progress in the domain of nanotechnologies and informatics is galloping, this might become quite attainable.

inner quantum fields, inside $[IQM-QM_{HD}-dBB]$ the unique sources of *theoretical*, mathematized predictive information, consist so far of only: The Schrödinger equation of the problem, its solution $|\psi(\mathbf{r}, t)\rangle = a(\mathbf{r})e^{\varphi(\mathbf{r}, t)}$ and the coding-postulate (7.31) $P(cod)_{\forall ms_G}$. As long as out of these sources is not yet drawn explicitly a comfortable mathematical predictive *algorithm*, a generally valid 'theory of microstates' is not yet genuinely achieved.

Indeed in the initial wave-mechanics the operators of observables permitted already expansions of $|\psi(\mathbf{r}, t)\rangle$ on bases introduced by operators representing the mechanical quantities, and these expansions, associated with Born's postulate, insured the prediction of the corresponding distributions of observable eigenvalues. As for QM_{HD} , the statistic-probabilistic predictions are obtained again via expansions and Born's postulate, but represented in the Hilbert space of the involved state-ket where Gleason's theorem is acting and confirms the form of Born's postulate. But inside $[IQM-QM_{HD}-dBB]$, in the present stage of our elaboration, such a predictive mathematical algorithm is lacking for microstates $ms(free, 1)_{cG(q-f)}$:

IQM considered as a whole and the assertion *Ass.1*, do permit to construct for also microstates $ms(free, 1)_{cG(q-f)}$ the probability laws 3.8 for the beable momentum-values⁹⁰

$$(D_{Mec}(ms_{G(t)}))/\mathbf{p} \equiv [\{(\epsilon, \delta, N_0) - \{\pi(\mathbf{p}_j)\}_{G(t)}, (Mpc(G^{(t)}))_{(\mathbf{A}, \mathbf{B})}], \forall (\mathbf{A}, \mathbf{B}) \in V_{Mec}^2, j = 1, 2, \dots, J \quad (7.32)$$

at any time t , initial or not, by use of the new specifically appropriated coding-postulate (7.31). But from the description 3.8 one *cannot* draw – for the dBB -values of the mechanical quantities – a mathematical predictive algorithm that perform in the way expressed in (7.17) for microstates $ms(free, 1)_{G(n-c)}$. That is so because in the arguments *Arg(Ass.2)-Arg(Ass.4)* the relation (7.17) – just like in QM_{HD} – is founded upon expansions of the state-ket $|\psi_G(\mathbf{r}, t)\rangle$ (unknown but supposed to be known) *on bases of eigenstates of operators representing the qualifying quantities*. While with the QM_{HD} additive representation of type (7.24) of the state-ket $|\psi_{G(G_1, G_2)}(\mathbf{r}, t)\rangle$ of a microstate $ms(free, 1)_{cG(q-f)}$, and with the bases of eigenket of the momentum *as they now stand inside QM_{HD}* , the predictions via Born's postulate and Gleason's theorem for the momentum-values have been found above not to be verifiable, nor always factually true. Therefore for the microstates $ms(free, 1)_{cG(q-f)}$ the Hilbert-space formalism from QM_{HD} – such as it now stands – is not adequate for defining inside $[IQM-QM_{HD}-dBB]$ a predictive algorithm. So the problem to be solved is:

Find a class of bases of corresponding eigenket such that a mathematical Born-Gleason algorithm be possible for representing the predictive probabilities $\pi(\mathbf{p}, t)$ of the values of the dBB momentum quantity inside a Hilbert-space representation of the microstates $ms(free, 1)_{cG(q-f)}$

I do not doubt that this can be achieved. The problem is technical, not of principle. The essential point of principle is the measurability of guided instantaneous momenta (7.27), so the result of *EXP*.

However in the present state of my own understanding of the problem I cannot assert a worked out solution. But I take the liberty to express a brief conjecture that, at least, will convey a clearer idea of what is researched, and possibly even the way toward a solution.

Inside the dBB approach the gradient operator ∇ works on the phase-function $\beta(\mathbf{r}, t)$ from the involved individual, physical corpuscular wave-function $\Phi_G(\mathbf{r}, t) = a(\mathbf{r}, t)e^{(i/\hbar)\beta(\mathbf{r}, t)}$, and it determines the guided trajectory of the corpuscular singularity from its amplitude. Up to a multiplicative constant, the QM_{HD} momentum-operator of the observable \mathbf{P} also is the gradient operator ∇ , and its eigenfunctions $|u(\mathbf{p}_n)\rangle = a.exp((i/\hbar)\mathbf{p}_n \cdot \mathbf{r})$ too have been found in section 6.1 to characterize the *phase* of the individual physical wave-movement around the space-time location of the corpuscular-like singularity that is involved: The quantum mechanical momentum 'observable' \mathbf{P} and de Broglie's concept of guided 'beable' momentum (7.27) are in essence the same conceptual-mathematical representation of the momentum of a specimen of the studied microstate. But while the quantum mechanical operator \mathbf{P} can be associated with the coding postulate (7.3), de Broglie's operator in general *cannot*, because its nature is in general essentially *instantaneous*. Furthermore QM_{HD} introduces – explicitly – for each one act of measurement only *one* eigenfunction of \mathbf{P} , while de Broglie's guiding law introduces in

⁹⁰ The sign ' $/\mathbf{p}$ ' is to be read: with respect to the momentum quantity \mathbf{p} .

general – virtually – two or more eigenvalues of the momentum operator – in the sense of QM_{HD} – out of which the effectively registered \mathbf{p} value is conceived to be composed and it introduces the values additively, so in possible relation with a product eigenfunction $\prod_n [a.exp(i/\hbar(\mathbf{p}_n \cdot \mathbf{r}))]$ of the gradient operator.

Now, inside QM_{HD} where the *physical significance* of the mathematical concept of eigenfunction is ignored, the eigenfunctions of \mathbf{P} are researched from the start as the most elementary sufficient solutions of the corresponding equation, and these consist of single eigenfunctions. But functions of the general form

$$\prod_n [a.exp(i/\hbar(\mathbf{p}_n \cdot \mathbf{r}))] = a.exp[i/\hbar \left(\sum_n (\mathbf{p}_n \cdot \mathbf{r}) \right)], \quad n = 1, 2, \dots, N$$

with N an integer, do equally satisfy the equation for eigenfunctions of the quantum mechanical operator $\nabla \approx \mathbf{P}$. However for the expansion (6.3) of the state-ket of a microstate of type $ms(free, 1)_{G(n-c)}$ and in association with the implicitly admitted *general* efficiency of the coding-postulate (7.3), such product-eigenfunctions might have never been supposed to be useful. And on the other hand the full adequacy of an additive representation of the state-ket of type (7.24) of a microstate $ms(free, 1)_{cG(q-f)}$ has probably never been questioned.

But let us consider the state-ket $|\psi(\mathbf{r}, t)_{\mathbf{G}(G_1, G_2)}\rangle$ that represents a microstate $ms_{\mathbf{G}(G_1, G_2)}$ with non-null quantum potential, so with possibility of quantum fields. And suppose that instead of the additive QM_{HD} -representation of type (7.24) we choose to assign it – directly and *exclusively* – a *one-term* representation $|\psi_{\mathbf{G}(G_1, G_2)}(\mathbf{r}, t)\rangle$ instilled by the operation of generation $\mathbf{G}(G_1, G_2)$ and the equation of evolution. It is not because historically the human mind, advancing top-down from the classical level of conceptualization, has encountered first the primordially statistical character of what we are able to represent scientifically concerning microstates and, upon that, has plastered the mathematical representations that seemed the most appropriated at that time, that we have now to neglect definitively the new views and understandings that become perceivable when one proceeds down-top; namely the specific effects of composed operations of generation and the non-analysable unity instilled by any operation of generation reflected in the basic posit (1.1), (6.1). *Hilbert-space superpositions are quintessentially useful for spectral decompositions of state-ket, not for the representation of the state-ket themselves.*

The implicit criterium that works inside an optimal Hilbert-space representation QMHD seems to be that:

Each involved micro-*system* introduces one corresponding space of representation, while each micro-*state* that is only virtually involved as a useful mental reference *introduces its corresponding value of the measured quantity* so that the unique physically existent value has to be constructed out of these virtual reference-contributions (a sort of generalization of Descarte's procedure for representing positions in a reference space via components on reference-axes).

So suppose that we write the expansion of $|\psi(\mathbf{r}, t)_{\mathbf{G}(G_1, G_2)}\rangle$ on a basis of eigenstates of the form, posited to be eigenstates of the de Broglie “instantaneous guided-momentum” \mathbf{p} from (7.27):

$$\prod_n [a.exp(i/\hbar(\mathbf{p}_n \cdot \mathbf{r}))] = a.exp[i/\hbar \left(\sum_n (\mathbf{p}_n \cdot \mathbf{r}) \right)], \quad n = 1, 2, \dots, N, \quad a \text{ constant} \quad (7.33)$$

For each pair $(\mathbf{p}_{(1j)}, \mathbf{p}_{(2k)})$, $j, k = 1, \dots, J$, (J a big integer), we have the corresponding eigenket

$$|u_{jk}(\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})\rangle = a.exp(\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})\mathbf{r} \quad (7.34)$$

and we consider the corresponding set of all the possible realizations of a pair (7.34):

$$\{|u(\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})\rangle\} = \{a.exp[i/\hbar[(\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)}) \cdot \mathbf{r}]]\} = \{a.exp[i/\hbar(\mathbf{p}_{(jk)} \cdot \mathbf{r})]\}, \quad j, k = 1, \dots, J \quad (7.35)$$

where in the last expression we have noted $\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)} = \mathbf{p}_{(jk)}$ and the discreet and finite spectrum of $\mathbf{p}_{(jk)}$ is posited to *exclude* the possibilities $\mathbf{p}_1 = 0$, $\mathbf{p}_2 = 0$ and to include the possibility $\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)} = \mathbf{p}_{(jk)} = 0$ ^{91,92}.

The following mathematical questions have to be checked:

- (a) Can an orthonormal Hilbert-space basis be constructed with the set (7.36)?
- (b) Can Dirac's calculus of transformations include bases (7.36)?
- (c) Can Gleason's theorem hold for bases (7.36)?
- (d) Can the expansion postulate (6.3), (6.11) be extended to bases (7.36):

$$|\psi(\mathbf{r}, t)_{\mathbf{G}(G_1, G_2)}\rangle / \mathbf{p} = \sum_{jk} c_{jk}(t) |u(\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})\rangle = \sum_{jk} c_{jk}(t) |u(\mathbf{p}_{(jk)\mathbf{b}})\rangle, \quad j, k = 1, \dots, J \quad (7.36)$$

If all this is found to work, then an extended Born postulate can assert that the probability to obtain via the coding-measurement procedure from (7.31) the value $(\mathbf{p}_{\mathbf{b}_j} + \mathbf{p}_{\mathbf{b}_k})$ of the beable momentum value \mathbf{p} , is

$$\pi(\mathbf{p}_{(jk)}) = \pi((\mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})) = |c_{jk}(t)|^2 \quad (7.37)$$

Then Gleason's theorem (7.6) permits to place the numbers $|c_{jk}(t)|^2 = |Pr_{.jk}|\psi(\mathbf{r}, t)_{\mathbf{G}(G_1, G_2)}\rangle|^2$ on the axes of the Hilbert-space of $|\psi(\mathbf{r}, t)_{\mathbf{G}(G_1, G_2)}\rangle$ endowed with the basis (7.36).

The relation (7.34) can be extended to more than two terms in the exponential.

Via (6.11) and (7.36) and by use of the coding postulate (7.31) instead of (7.3), the $[IQM-QM_{HD}]$ assertions Ass.2-Ass.4 become applicable to also the microstates $ms(free, 1)_{cG(q-f)}$ that involve quantum fields.

On the other hand, from the *dBB* approach we have that the probability of the eigenvalue $\mathbf{p}_{(jk)} = \mathbf{p}_{(1j)} + \mathbf{p}_{(2k)}$ of the momentum-vector $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2$ defined by (7.27) can be written as

$$\begin{aligned} \pi(\mathbf{p}_{(jk)}) &= \int [\pi(\mathbf{r})\pi(\nabla\beta(\mathbf{r}, t) = \mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})] d\mathbf{r} \\ &= \int [|\psi_{\mathbf{G}(G_1, G_2)}(\mathbf{r}, t)|^2 \pi(\nabla\varphi(\mathbf{r}, t) = \mathbf{p}_{(1j)} + \mathbf{p}_{(2k)})] d\mathbf{r} \end{aligned} \quad (7.38)$$

Inside *dBB* the writing (7.38) possesses a purely conceptual utility, it cannot be verified factually. However therefrom one draws now:

$$|c_{jk}(t)|^2 = \int [|\psi_{\mathbf{G}(G_1, G_2)}(\mathbf{r}, t)|^2 \pi(\nabla\beta(\mathbf{r}, t) = (\mathbf{p}_j + \mathbf{p}_k))] d\mathbf{r} \quad (7.39)$$

which yields a conceptual significance for Born's postulate, a definition of the meaning that is its source. While furthermore inside $[IQM-QM_{HD}-dBB]$ this conceptual significance of Born's postulate can be verified factually via the assertions Ass.2-Ass.4 fulfilled by use of the coding-postulate (7.31) instead of (7.3).

The above writings (7.33)-(7.39) express just a conjecture. But they explicate what sort of solution is researched.

Consequences If the question of a convenient Hilbert-Dirac representation of the microstates $ms(free, 1)_{cG(q-f)}$ is settled in the way conceived above this would entail essential consequences:

(a) Accordingly to both the *dBB* approach and QM_{HD} , any dynamical quantity A is defined as a function $A(\mathbf{r}, \mathbf{p})$. So there appears a *general* stratum of representation that can start inside a *deeper* stratum of the microphysical reality than that involved by a microstate $ms(free, 1)_{cG(n-c)}$ for which the quasi-classical coding-postulate (7.3) is valid. Namely it can reach the radically non-classical region where quantum potentials involved by microstates $ms(free, 1)_{cG(q-f)}$ do appear, for which the coding relation (7.31) is required. This introduces a probability tree of the type from the figure 3.2

⁹¹ Here we do not research some truth. We want to construct a convenient technique of Hilbert-space representation. A technique is subjected only to its aim.

⁹² Inside de Broglie's approach ([1956]) in a microstate obtained by reflection on a mirror of an incident state there are places where the corpuscular-like singularity – endowed with a 'quantum mass' – is at rest.

with a trunk common to *all* the classical mechanical quantities $A(\mathbf{r}, \mathbf{p})$, topped by a crown of only conceptually worked out probability spaces united by a meta-statistical level of correlations. Though such a representation is necessary only for microstates $ms(\text{free}, 1)cG(q - f)$, it is possible for any microstate. So:

With respect to this generally possible representation, the probability trees of the type represented in the figure 8.1, with branches, appear as a particular second possibility characteristic of exclusively the more superficially rooted microstates $ms(\text{free}, 1)_{G(n-c)}$.

(b) If this succeeded it would also entail the existence of a channel

$$[MP(ms_{G,cw}), (6.1) G \Leftrightarrow ms_{G,cw}, (6.2) ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\}, (7.31) P(\text{cod})_{\forall ms_G}] \quad (7.40)$$

of adduction of the whole *dBB* approach, into the 'scientific' knowledge (i.e. communicable, consensual, observable and verifiable knowledge). This channel entails in particular a radical transmutation of the conceptual status of the *dBB*-conceptualization, with respect to that of QM_{HD} : an *inversion* of their relative status.

(c) It seems likely that one microstate of two or more microsystems *too* does involve quantum-fields, while bound microstates do certainly involve quantum fields. But in the case of each one of these categories the predictions researched up to now are not very sensitive to the observable deviations from the QM_{HD} -predictions produced by the involved quantum-fields (cf. below in sections 7.5 and 7.6.

Along these lines it should be possible to achieve inside the framework [*IQM-QM_{HD}*], a Hilbert-space representation of the results of any sort of quantum measurements, *entirely duplicated by a factually generated and genetically relativized Hilbert-space representation of all the potential predictive contents of QM_{HD}*.

7.5 One free micro-state of two or more micro-systems

We consider only the case of one microstate of two microsystems that is involved in Bell's theorem on non-locality. I have exposed elsewhere (MMS [2013]) what I call a 'conceptual invalidation' of Bell's proof (namely the fact that the conclusion, as it is expressed verbally, *does not follow from the mathematical proof*). I have also constructed a counterexample to Bell's formulation (MMS [1987]) that has been confirmed as factually possible (Bordley [1989]). But these features are not relevant in the present context. So here I confine to the following remarks.

According to the modelling postulate $MP(ms_{G,cw})$ from section 6.2 every specimen $\sigma_{\Phi(G)}$ of one microstate of two microsystems (as defined in section 2.1) involves two 'corpuscular-like' de Broglie singularities. The *dBB* wave-function Φ_G is common by definition to these two micro-systems because only one operation of generation G comes in. So it seems natural to conceive that, while the two involved singularities recede from one-another, Φ_G subsists everywhere throughout the space-time domain covered by the coding-measurement successions where one specimen $\sigma_{\Phi(G)}$ is involved: The corpuscular wave of the involved specimen $\sigma_{\Phi(G)}$ of the studied microstate is conceived *not to be delimited in space-time*. So the spins of the two involved singularities – that can be imagined as features specific of only these singularities – can be conceived to stay permanently connected via the common wave.

The QM_{HD} tensor-product representation of one free micro-state of two or more micro-systems can be now regarded as a convenient transposition of the representation proposed above for one microstate of one microsystem with composed operation of generation. I repeat the fundamental remark that the general but implicit criteria that work inside an optimal Hilbert-space representation QM_{HD} seem to be that each involved micro-*system* introduces one corresponding space of representation, while each micro-*state* that is only virtually involved as a useful mental reference *introduces its corresponding value of the measured quantity* so that the unique physically existent value has to be constructed out of

these virtual reference-contributions (a sort of generalization of Descartes's procedure for representing positions in a reference space via components on reference-axes). It is interesting to watch how the human mind introduces implicitly methodological features for favoring intelligibility.

It might also be interesting to consider the predictions concerning the momentum in one free micro-state of two or more micro-systems and to make experiments in order to establish which coding-procedure does work, (7.3) or (7.31) or both in mutual agreement⁹³. This might show whether yes or not quantum-fields are involved.

In any case there is no a priori reason whatever for expecting that the spin-values registered for S_1 and S_2 will come out non-correlated; quite on the contrary. And the *existence* of a meta-correlation in the sense of section 3.2 has *nothing to do* with the orientations of the two involved *apparatuses*. These orientations have to be conceived to determine a priori *what can be registered* in each case concerning, respectively, the two spins, so which *general category* of correlation will become *manifest*. The orientation of the apparatus are features that belong to the chosen grid for qualification. But if the orientation of the apparatus would determine the very *existence*, or not, of the correlation itself, that would indicate a bad apparatus, to be thrown away. So *what is the point* in hastening for changing the orientations of the apparatuses just at the last moment before the registration?

And more generally what is the point in imposing so dramatically an Einstein condition of locality that has a definite significance and role for macroscopic mobiles directly perceived with definite finite contours by the human observers via signals of light? When two or more human observers perceive simultaneously such mobiles, a scientific representation does indeed require some consensus concerning the dynamics of the mobiles, some invariants that generate sense for the assertion that these observers are all perceiving the same mobiles via the qualifiers. But for microstates on which each human observer gathers knowledge as it is stated in *IQM*, so indirectly, and alone inside his own Laboratory, without perceiving anything else than marks out of which he draws some significance only via solitary conceptual-mathematical treatments and on the basis of a model of a microstate that involves an unlimited wave, the a priori importation of all the requirements of the macroscopic relativistic mechanics is far from being an obvious necessity. It is just an interesting subject for study: *What* – specifically – has to be required for a scientific representation of the physical reality *at any given scale*, and on the basis of *which* epistemological-methodological principles? In *what a sense*, and how, on the basis of *which* facts, considerations, possibilities, principles, is it necessary or useful in some definite sense to insure global inner 'unity' between the various physical theories?

Coming back to Bell's work, the problems raised by this work seem to be entailed by a model of two bowls that are receding from one another.

Of course all the preceding considerations are founded on *models*. But by now, I think, it has become clear that without models one cannot even try to construct a theory of microstates; one cannot reason, prove, *conceive*. And indeed the whole non-locality problem concerns some model. And Bell's proof only eliminates the model implied by him.

7.6 Bounded microstates

With respect to the essential specificities of the descriptions of microstates such as these have been organized first inside *IQM* and then inside the framework the frameworks [*IQM-QM_{HD}*] and [*IQM-QM_{HD-dBB}*], the case of bounded microstates cumulates all the limiting conceptual characters that expose to confusions. Among these the following two are major sources of confusion:

- The state is permanently captured inside a small domain of physical space-time (not of an abstract space of representation) that is included into a bigger abstract representation space. In a classical 'configuration' space this happens currently and nobody wonders why more than only four dimensions are involved. If in the case of a bound microstate one begins to wonder concerning this, then much place for confusions is introduced.

- In this case, like in the case of a free state (7.24), the state-function from the statistical state-ket $|\psi_{G(t)}\rangle$ and the de Broglie wave-function Φ superpose nearly exactly (only the variable location

⁹³ It seems possible that such experiments have not yet been performed.

inside the wave-function makes the difference) (which is the source of de Broglie’s “double solution” interpretation of QM_{HD}).

- The state-ket is from the start conceived to be *also* an eigenket of the total energy observable \mathbf{H} , which in this case is possible formally, but conceptually is a huge confusion, namely an identification of a descriptor of a set of sets of numbers, with the descriptor of a physical phenomenon).

- The human observer did not himself achieve deliberately the involved operation G of generation; this operation has been achieved naturally before the beginning of the human investigation. So from the point of view of the basic operation G a bound microstate is like a classical “object”, it just pre-exists ‘outside there’. So the measurements can be conceived in the classical manner, i.e. outside successions $[G.MesA]$, just via repetitions of only indirect acts of measurement $MesA$, via test-microstates. These measurements are often achieved indirectly, via test-elements (photons, Compton collisions, etc.; and then the test microstate, after the interaction, is usually of the type $ms(free, 1)_{G(n-c)}$ that does accept a coding procedure (7.3). While when ‘effects’ in classical fields (Stark, Zeeman) are made use of, these entail a merger with classical atomic and molecular physics. Thereby the measurements on bound states introduce no active difference between inexistence or existence of quantum fields inside the studied state.

These features blur the frontier between classical physics and the representation of microstates, they tie the problem of measurements operated upon bound states, to classical physics as much as to quantum mechanics. The bound states are likely to keep hidden inside them many specific problems that are not even supposed. These – in principle – could probably be identified and integrated into a re-organized theory of quantum measurements (MMS [2014B]). But in fact such a theory of quantum measurements is not imperatively necessary for measurements on bound states, it is not genuinely useful.

7.7 Conclusion on chapter 7

Inside the chapter 7 we have constructed – inside the framework $[IQM-QM_{HD}-dBB]$ – a factual formal theory of quantum measurements. The von Neumann’s representation of measurements has been dropped from the start⁹⁴. We have first considered the particular case of microstates $ms(free, 1)_{G(n-c)}$. This has been achieved by an approach founded on the assertions *Ass.1* to *Ass.4* that has led to the expression (7.17) that summarizes – both – a process of constructive statistical prediction and of verification of this prediction, like in the case of a description 3.8 from *IQM*. This approach has furthermore brought forth a **duplication** of the mathematically generated statistical predictions from QM_{HD} , by a **factual-formal procedure** (7.17) that can be conceived to be extended to *any* sort of microstate and *any* dynamical quantity. This procedure insures for the new representation of microstates proposed here, a high degree of independence with respect to the purely mathematical depictions yielded by Schrödinger’s equation (when it can be constructed and solved) and it also permits to control them.

But the principle construction recalled above is valid for also microstates $ms(free, 1)_{cG(q-f)}$ only on the basis of two fundamental assumptions *that still remain to be confirmed*. The first assumption consists of the measurability of de Broglie’s guided momentum (7.27) and it remains to be confirmed by the realization of the of the experiment *EXP* defined in section 7.3.2. The second assumption is that for microstates with quantum fields, the Hilbert-space representation of the factual formal constructions proposed here is confirmed from a mathematical point of view. If the whole structure elaborated here succeeds then the set

$$[MP(ms_{G,cw}), (6.1) G \Leftrightarrow ms_{G,cw}, (6.2) ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\}, (7.31) P(cod)_{\forall ms_G}] \quad (7.41)$$

acts a channel for the of adduction of the whole *dBB* approach into ‘scientific’, communicable, consensual, observable and verifiable knowledge.

⁹⁴ Imagine what would have happened all along the path followed inside the chapter 7 if von Neumann’s representation would have been accepted. The whole false problem of ‘decoherence’ would have come in, where physical phenomena of coherence and decoherence (Cohen-Tannoudji 1973, 1996) are confounded with only formal mathematical writings.

However the hypothetical features of the mentioned assumptions – that seem themselves rather safe – do not threaten the already certainly accomplished results from the chapter 7:

- The various criticisms and clarifications;
- the explicit introduction of an individual level of conceptualization;
- the global conceptual organization of the representation of quantum measurement founded on the assertion *Ass.1*;
- the revelation of the coding problem and of the solutions that it can accept;
- the *strategy* founded on the assertions [*Ass.1* → *Ass.4*] that permits to connect the individual, physical, factual level of conceptualisation, with the statistical-predictive conceptualisation.

All these results, I think, are already stable gains.

8 INTEGRATION OF $QM2$

The framework [$IQM-QM_{HD}$] has played the role of a scaffold for constructing $QM2$. It has permitted and guided the insertion of QM_{HD} into the general structure imposed by IQM . The process of insertion has hit a limit for the case of microstates $ms(free, 1)_{cG(q-f)}$ and this has drawn into play also the dBB approach, so a new conceptual scaffold has been used, [$IQM-QM_{HD}-dBB$]. In what follows we drop any scaffold and, by a final integration of the results obtained up to now, we shall define the neighbourhood, the content and the essence of the inner structure of $QM2$ itself.

8.1 The three source-domains and their respective roles in the generation of $QM2$

So the prime matter for constructing $QM2$ has been drawn out of IQM , QM_{HD} and the dBB approach. The respective roles of these source-domains have been the following ones.

The Infra-(Quantum Mechanics), IQM . This has been constructed with the overt aim to generate the whole epistemological-operational-methodological structure of $QM2$ in a sense similar to that in which the structure of an organic entity is generated by its genetic code. It has acted as a morphological-functional mould, as a void receptacle in which have been poured the factual substance and data involved by $QM2$. Thereby the role of IQM has been both selective and constructive. It has eliminated as non-conformal with it, the general mathematical representation of quantum measurements; it has explicitly called for a general model of a microstate and for coding-measurement successions founded on this model; and throughout the chapters 6 and 7, step by step, it has dictated refusals, specifications and re-organizations.

The de Broglie-Bohm approach, dBB . This approach started in Louis de Broglie's Thesis, with the Jacobi formulation of classical mechanics where the conditions that restrict to the 'geometric approximation' have been suppressed (de Broglie [1924], [1956], [1963]). This representation has been then progressively specified. But the process of specification did not individualize a definite concept of microstate, neither conceptually nor in a physical-operational way. Though from the start de Broglie's representation has been mathematical, nevertheless it remained global, continuous, a-observational. It concerned as a whole the general substratum of the physical reality. Individualizing words did appear ('particle'), as well as a current recourse to 'the observer', but these occurrences have been kept independent of any worry for consensus in the sense of operational and verifiable inter-subjectivity. In this sense de Broglie's global approach – notwithstanding the experimental confirmation of his seminal relation $p = h/\lambda$ ⁹⁵ – remained basically a sort of metaphysical insertion into mathematical physics, that specifies in a physical-mathematical language Spinoza's 'substance'. Even Louis de Broglie's theory of measurements inside his subsequent theory of double solution formulated so much later (de Broglie [1956]), [1957]) remains just an explanation of the fully accepted QM_{HD} theory of measurements, juxtaposed as a device for connecting explicitly QM_{HD} with the double-solution interpretation of QM_{HD} . In short, Louis de Broglie did not offer his own representation otherwise than just an elaborated interpretation of QM_{HD} . Nor did he desire for it another status.

Bohm's 1952 work possessed the same sort of characters.

Inside [$IQM-QM_{HD}$] and [$IQM-QM_{HD}-dBB$] however, de Broglie's work has offered the model of a microstate and the coding-measurement successions (7.31) founded on the guiding law (7.27). Thereby, and on the basis of the hypothesis that the experiment EXP has established the calculability

⁹⁵Very notably and curiously, this relation has been derived from precisely a condition of consensus between two imagined observers!

from factual data of the values (7.27) of the momentum, dBB – globally – remains converted into a vast and rich reservoir of deep and mathematically worked-out views and representations that can connect $QM2$ explicitly and in detail to the classical mechanic, to the classical optic, and – most preciously – to the whole basic physical factuality.

The nowadays Quantum Mechanics, QM_{HD} . The specific convenience of the Hilbert-space representation of microstates concerns the intuitive and efficient way of representing statistic-probabilistic predictions. This representation has been impressively developed into a very powerful mathematical and complex system of mathematical tools. Therefore the preservation of a non-restricted Hilbert-space representation constitutes a considerable practical aim. But the additive representation of type (6.8) of the state-ket of a microstate $ms(\text{free}, 1)_{cG(q-f)}$ that involves a non-null quantum potential, has been found to lead to predictions that are not verifiable, nor factually correct, at least in general. So this sort of state-ket call for an extended Hilbert-space representation. The process of integration of $QM2$ has to deal with this situation.

8.2 The basic assumptions of $QM2$

IQM is globally conserved inside $QM2$:

1. *The structural immersion-postulate in IQM.*

IQM as a whole is conserved inside $QM2$ in the role of a pre-organized epistemological-operational-methodological mould that constraints a priori the structure of the theory. The possibility of a structure that operates in this way is by itself a procedural novelty that insures intelligibility and control.

A massive but selective importation of QM_{HD} postulates:

2. *All the QM_{HD} are conserved*

with only the following exceptions :

- The measurement-postulates are all suppressed.
- For microstates with possibility of quantum fields the QM_{HD} representation is suspended and the representation from 7.4 is considered as a candidate for replacing it.

New postulates

3. *The individual modelling postulate*

$$[MP(ms_{G,cw}) + (6.1) (G \Leftrightarrow ms_{G,cw}) + (6.2) (ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\})]$$

(valid for any sort of individual microstate).

4. *The definition (7.27) of the beable instantaneous momentum $\mathbf{p}(\mathbf{r}, t)$ – de Broglie's 'guiding law' posited to be valid for any sort of microstate.*

5. *The coding-measurement postulates (7.3) (for microstates $ms(\text{free}, 1)_{G(n-c)}$) and (7.31) (for microstates $ms(\text{free}, 1)_{cG(q-f)}$).*

Via the postulates 3 and 4 the whole dBB representation is unified with $QM2$.

8.3 The main characters of the inner structure of $QM2$

Throughout the chapters 5, 6, and 7, various features of what has been a priori named $QM2$ have emerged scattered chaotically. Now they will just be summarized in an organized structure. The summary, moreover, will be synthetic to the extreme. Any elaboration will be banished. Inside this work the aim is *not* to offer a fully achieved new theory of microstates. It is only to identify the conceptual loci wherefrom unintelligibility is spouting inside QM_{HD} , to clean these away, and to open up a well-defined general framework for a subsequent exhaustive elaboration of a genuinely achieved new representation of microstates.

The global inner structure of the formalism from $QM2$ is strictly subjected to *all* the *general* – qualitative but syntactic – requirements of IQM that are recalled beneath:

* The representation has to be rooted directly in the factual physical-operational reality. This establishes the zero-level of conceptualization. Therefrom the process of conceptualization progresses step by step on the direction down-up (fig. 1.1).

* The individual level of conceptualization, and the statistical one, are explicitly and radically distinguished from one-another.

* The microstates are *classified* according to the definitions from section 2.1

* The passage from the individual level of conceptualization, onto the statistical one is webbed by repeated individual coding-measurement-successions [*G.MesA*].

* The description of any microstate is a primordially transferred $[\epsilon, \delta, N_0]$ -probabilistic description (3.8) and it is always inserted into a tree-like structure of the general type defined in chapter 3, of which the detailed content varies with the type of the considered microstate (in the sense of the definitions from section 2.1. This tree-like structure defines a deep and complex unity.

In short: *The general structural aspects are defined a priori inside IQM, so outside QM2.*

We reproduce beneath the figure 8.1 of the *IQM-QM2* probability tree :

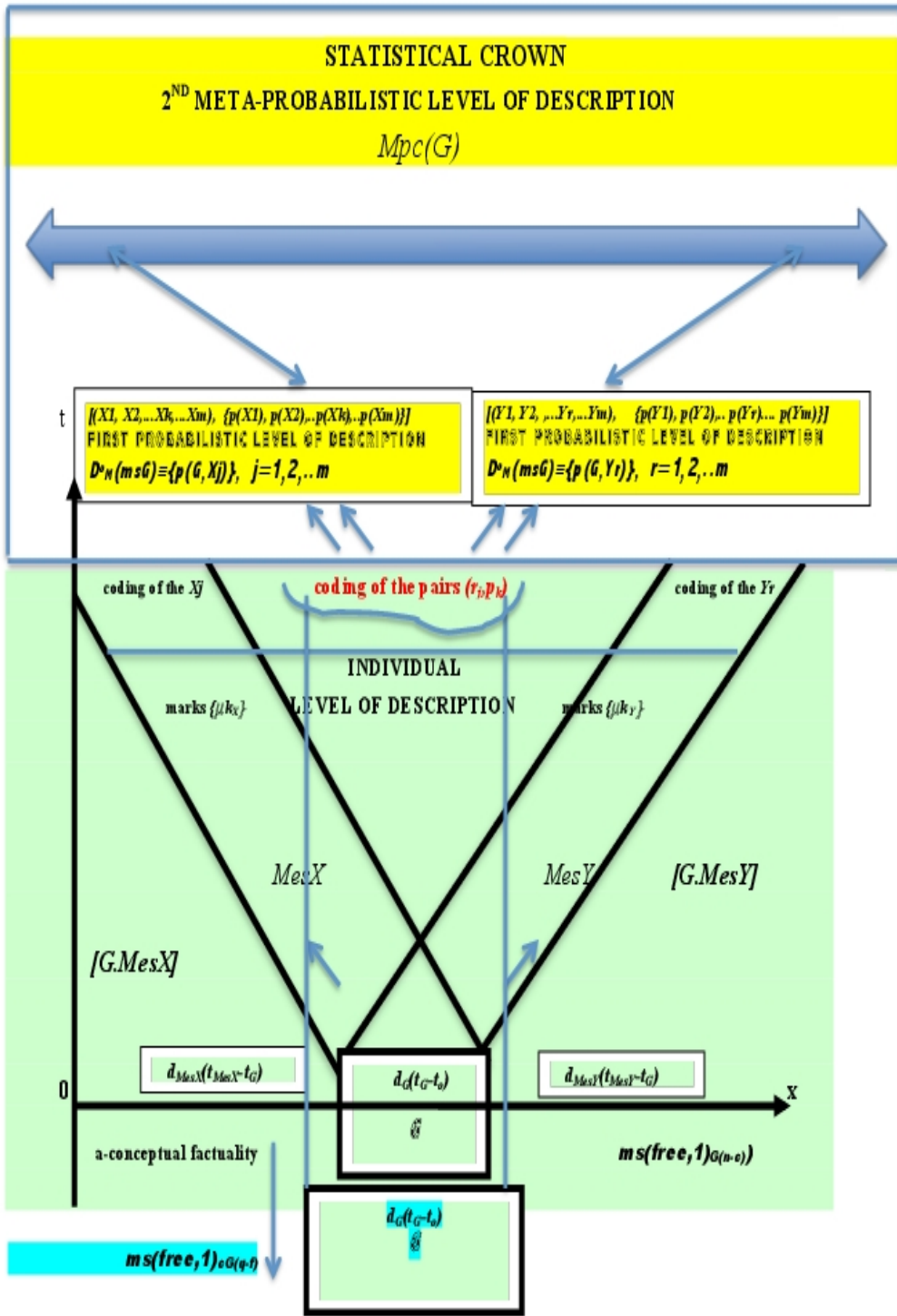


Figure 8.1: The coding postulate (7.31) is generally valid; it generates a tree consisting of just a trunk of measurement surmounted by the two conceptualized probabilistic crowns. The probability-tree of a microstate $ms(free, 1)_{cG(q-f)}$ is rooted deeper into the a-conceptual factuality than the trunk-probability-tree of a microstate $ms(free, 1)_{G(n-c)}$

* Inside $QM2$ the representation of the quantum measurements is entirely reorganized in strict agreement with the structural commands induced by IQM :

** Throughout the sections 7.1.1-7.1.3, and the section 7.2, IQM has instilled the deeply organizing distinction between the individual level of *conceptualization and the statistical or meta-statistical levels*. Furthermore, via the assertions *Ass.1, Ass.2, Ass.3* and *Ass.4*, IQM has instilled a fusion between, on the one hand the purely mathematical Hilbert-space predictive representation, and on the other hand the factual-mathematical predictive constructs (7.17). The factual-mathematical constructs (7.17) double and control factually the purely mathematical outputs of the Schrödinger equation, *when these can effectively be generated*; while when these cannot be obtained in fact, the constructs (7.17) offer factual independence with respect to the purely mathematical outputs. From a quite restrictive necessity, the specification and the solution of the Schrödinger equation of the problem have become a historical relic that can be dispensed with nowadays, in the era of the accumulated effects of Moore's law:

IQM introduces structured constraints that permit to convert the *void form* (7.9) of the predictive expansion of the unknown 'state-ket of the problem' on the basis of eigenket of any observable A , into the effectively realized expansion of this unknown state-ket, without having to first obtain it mathematically via the Schrödinger equation of the problem that often is non-specifiable or unsolvable.

** The IQM request of an explicit model of a microstate has led in section 6.1 to the modelling postulate $MP(ms_{G,cw})$ associated with (6.1) $G \Leftrightarrow ms_{G,cw}$ and (6.2) $ms_{G,cw} \equiv \{\sigma(ms_{G,cw})\}$ and has permitted in chapter 7 to specify the coding-postulates (7.3) and (7.31). Considered together, these new descriptonal elements act as a channel of adduction of elements from the interpretive dBB approach, into the 'scientific' representation of microstates from $QM2$, required consensual, predictive and verifiable. Thereby the dBB mathematical formalization becomes a reservoir of deeply worked out representations, precious for a more worked out future development of $QM2$.

We now come to QM_{HD} . Notwithstanding the radical modification of the representation of quantum measurements, the very powerful operational and algebraic mathematical representation of the predictions concerning measurements on microstates developed inside QM_{HD} is entirely conserved inside $QM2$, *but claned of mixtures with the question of verification*, that is treated separately. As for the representation of the process of *verification* of the statistical predictions, it is fully sketched out for microstates $ms(free, 1)_{G(n-c)}$; and if the experiment EXP succeeds and the conjecture from section 7.4 resists examination, the same scheme is valid – in essence – for also the microstates $ms(free, 1)_{cG(q-f)}$. So, in its essence, the representation in $QM2$ of the process of verification of the statistical predictions is treated for any sort of microstates.

Moreover: Inside $QM2$ the assertions *Ass.1, Ass.2, Ass.3* and *Ass.4*, while they web the Hilbert-Dirac representation to factuality, they also connect it explicitly with the whole individual level of conceptualization introduced by IQM .

So, as announced, $QM2$ is sketched out as an intimate synthesis between IQM , QM_{HD} and the dBB approach.

8.4 Interpretation problems

The inclusion of IQM protects $QM2$ of all the interpretation problems that have consumed such quantities of ink (cf. MMS [2013]v3).

8.5 Conclusion on the chapter 8

We have outlined very succinct indications on the sources, the inner structure and the contours of $QM2$. The embryo constituted here can now be developed. Those who feel critical with respect to QM_{HD} might draw some profit from the existence of this sketch of a new representation of microstates.

9 QM2 CONSIDERED FROM ITS OUTSIDE

9.1 Universality

It is often perceived that QM_{HD} is endowed with a remarkable ‘universality’ and it is believed that this is entailed by the fact that any material entity is a structure of microstates. But this belief is illusory, for two distinct reasons, an epistemological reason and a formal one.

The epistemological reason. Though in a non-explicit way, inside what is called ‘quantum mechanics’ the origin of the creation of new scientific knowledge⁹⁶ that is *specific* to a given fragment of physical reality, is placed *just upon* the extreme boundary between the still strict absence of *such* knowledge, and the previously conceptualized in scientific terms (fig. 8.1). Inside QM_{HD} this origin is not immediately perceivable because the individual level of scientific conceptualization is not yet reached and so it is occulted, only the already statistical level that is *first* encountered by an up-down progression is explicitly represented. Nevertheless the individual level also is irrepressibly present and active. Now, in an epistemological situation of such an ultimate nature, placed on the frontier between what has never as yet been conceptualized, and what is subject to a primordially first conceptualization, *each* condition for generating some knowledge acquires necessity, it becomes a radical *sine qua non* condition; and under the pressure of logical-mathematical constraints of coherence, and via trials and errors, it becomes separately perceptible. But even if it is not yet explicated, ‘purified’ and put under magnifying glasses as we have tried to do in this work, nevertheless it is *felt* to be there.

Those who perceive universality in the formalism of quantum mechanics, in fact, more or less clearly, perceive the presence of the new and specific concept of description that we have called a ‘transferred primordial description’ of a microstate and have denoted $D/G, ms_G, V/$.

And they also feel more or less faintly that this concept, with its *primordially* statistical character – notwithstanding the individualized operations that generate it – and with the basic unsuppressible relativities to the trio $/G, ms_G, V/$, is not confined to the case of microstates; that the symbol ‘ ms_G ’ can be replaced by the symbol of a quite general entity, say ‘ α_G ’ (for ‘object-of-study-entity’) generated as such by the operation ‘ G ’⁹⁷. They somehow perceive that the study of microstates introduces an instance of a *general* epistemological *method* that is necessary and sufficient for *starting* at no matter what *local* but **total** *relative zero of knowledge*, a process of creation of new and communicable local knowledge that, by its structure, permit consensus and verification. And indeed transferred descriptions emerge quite currently inside the classical processes of conceptualization, as much as when micro-entities are involved⁹⁸. But in the case of micro-entities *all* the involved descriptive features are *radicalized*, non-degenerate, mutually separable, and that is why the concept of primordially transferred description has revealed itself for the very first time only inside microphysics, and has entailed new and striking questions of intelligibility as well as mathematical specificities.

The formal reason. The concept of Hilbert-vector-space – via Gleason’s theorem – offers a very expressive framework for just *lodging* inside it “*factual probability laws*” (MMS [2014]) that have been established *outside* this framework. And the concept of *probability is omnipresent inside human thought*. But:

⁹⁶That is, communicable, consensual and verifiable knowledge, not a subjective one, for instance imagined or metaphysical.

⁹⁷cf. MMS [2002A], [2002B], [2006], [2014], etc.

⁹⁸Henri Boulouet, private communication and Ph.D thesis 2014 Univ. of Valenciennes.

This circumstance – little known and thoroughly understood – **has no necessary connection whatever with, specifically, microstates**. It is illusory to believe that there exists a direct logical relation between, for instance, social sciences in general, and on the other hand the concept of microstate; or even between the *psychology of classical conceptualization*, and microstates. In this sense expressions like “quantum social science” or “quantum cognitive science” are utterly misleading.

The specific descriptive capacities of the Hilbert-space representations have to be strictly separated from the concept of microstate. For this reason, speaking, for instance, of ‘quantum’ social science is misleading.

Similar considerations can be made concerning the descriptonal relativizations. The construction of *relativized descriptions* and of the representational structures generated by these constitute the object of a discipline that is independent of the study of microstates, notwithstanding that this discipline has been suggested by the examination of the formalism of the quantum mechanics. I have called this new discipline – a method of relativized conceptualization denoted *MRC*. I have developed *MRC* into a rather complex whole where logic, probabilities and information theory are *unified*; and *this whole incorporates IQM as a particular application to the case of microstates*. But this does not in the least entail that *MRC* is a ‘*quantum method*’.

As for mathematical physics, and in general for mathematical science, we are still far from thoroughly understanding the conditions that restrict the ‘acceptable’, or fertile, or optimal association between, on the one hand, this or that mathematical formal system, and on the other hand a given domain of what we call ‘reality’, physical or social or economical reality, etc. And the same assertion holds for the techniques, in particular for engineering. This introduces the following last point.

9.2 Facts, mathematics, knowledge

The approach developed in this work brings into evidence very general and fundamental questions concerning the relations, inside a mathematical theory of a domain of *physical* facts, between: the nature of the considered physical factuality; the cognitive situation that is involved; the sort of descriptive *aims* that act; and the mathematical framework that is made use of.

In this work, in order to compensate for this lacuna, I have tried to create a contrast between two different and mutually independent sorts of descriptive systems, *IQM* and *QM_{HD}*, and to bring into evidence the conditions that can organize a convergence. *IQM* is a qualitative but *formalized* structure determined exclusively by the cognitive relations and the cognitive aims that are involved when one wants to generate knowledge on microstates. Whereas *QM_{HD}* has been directly elaborated as a mathematical theory ‘of microstates’. The contrast has permitted to establish to *what a degree* detailed semantic contents *must* be poured into a mathematical theory of microstates *in order to insure the possibility of consensual and verifiable prediction*.

More generally, it has permitted to become fully aware of the crucial role of channels of adduction of semantic contents into any mathematical theory of physical facts. Indeed in the absence of such channels – and well formed accordingly to clearly elaborated criteria *specific to the particular aim that is involved* – the mathematical ‘theory’ claimed to concern a given domain of physical entities, in fact remains disconnected from that domain of facts. It simply does not..... ‘*make sense*’.

The semantic void from a mathematical theory of ‘real’ entities is always *felt*, it is apprehended as an unintelligibility of which the source cannot be specified nor located. And when this sort of unintelligibility works, the human mind secretes in a reflex way a tendency to consider the mathematical formalism as if it were itself a physical reality of some superior essence, out of reach and immutable like a galaxy or like gravitation. In consequence of this reflex-tendency, a mathematical formalism that is applied to an important domain of physical entities, but is not understood, is – both – reified and divinised into an ‘extraordinary’ entity that slips out of conceptual control and is transmuted into an idol. Jung would have had much to say about this sort of effects of the collective unconscious.

This process of divinisation can also generate a superposed tendency to generalize the ‘extraordinary’ mathematical representation, to the representation of *everything*. Which leads to arbitrary, long

and very difficult elaborations devoid of any clear utility.

Therefore I think that in the present stage of development of human thought, the first urgency is a scientific and methodological general *epistemology of the processes of generation of scientific knowledge* where – in particular – be stated the conditions of connectivity between the aim to create mathematized knowledge, and on the other hand, the way in which the adequate mathematical tools can be generated and handled optimally.

INSTEAD OF A GENERAL CONCLUSION

It is likely that the reader of this attempt has been often surprised and also repelled. In as much as this is so the reason might be that it is very unusual that a physicist bring in epistemological and methodological points of view.

Anyhow, now, since the approach and its results are exposed, it has become possible to accept or to reject them advisedly.

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A APPENDIX 1

REFLEXION SUR LE PROBLEME DE LOCALITÉ

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 (EXTRAIT)

Goal

For the past eight years, the so-called locality problem has garnered more and more attention. Theoreticians, experimentalists, and multi-disciplinary thinkers have all made considerable efforts to clarify the problem. The technical aspects — mathematical and experimental — have already been examined in a large number of works and are well known to those who carry out related research. But the relevant conceptual framework seems to me to be less well defined. The goal of this section is to examine this conceptual framework. I shall attempt to carry out this examination in as simple and striking a way as possible, almost poster-like given the extensive use of diagrams and tables. This technique seems to me the most suited to the task of highlighting the insufficiencies that I see in the very definition of the locality problem.

Brief Review.

The EPR Paradox (1935). The locality problem was considered in a well-known theorem by J. Bell (1), formulated in response to an argument made in 1935 by Einstein, Podolsky and Rosen (2). This argument, known as the “EPR paradox”, was devised to demonstrate that the quantum mechanical formalism does not provide a complete description of individual microsystems. The hypotheses that form the point of departure for the EPR paradox are given in the table below (abbreviated notations are associated with them):

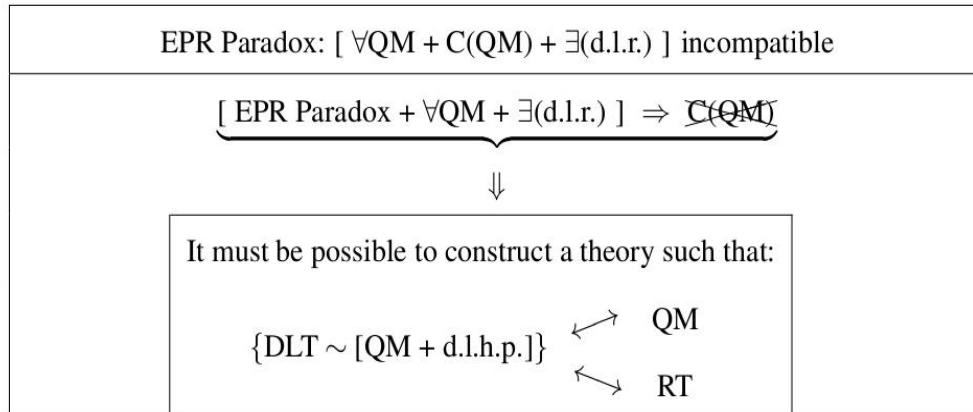
Hypothesis	Representation
All quantum mechanical predictions are correct.	$\forall\mathbf{QM}$
Quantum mechanics provides a complete description of microsystems	$\mathbf{C}(\mathbf{QM})$
Physical reality exists independently of observation. It is “deterministic” and local (or “separable”).	$\exists(\text{d.l.r.})$

The “EPR paradox” consists of proving that the hypotheses listed are not compatible.

The interpretation proposed by Einstein, Podolsky and Rosen of this proof is as follows:

The predictions of the quantum formalism are correct. As a result there is no basis for abandoning hypothesis $\forall\mathbf{QM}$. Hypothesis $\exists(\text{d.l.r.})$ expresses a metaphysical belief that we are free to accept or reject. But if we accept it, it must be added to the predictions of quantum mechanics. In this case, the proof of the incompatibility of the system of hypotheses $[\forall\mathbf{QM} + \mathbf{C}(\mathbf{QM}) + \exists(\text{d.l.r.})]$ requires us to abandon the completeness hypothesis $\mathbf{C}(\mathbf{QM})$. In other words, this proof requires us to accept the possibility of a deterministic local theory (DLT) of micro-phenomena, in which the quantum formalism would be completed by additional descriptive elements, hidden (with respect to the

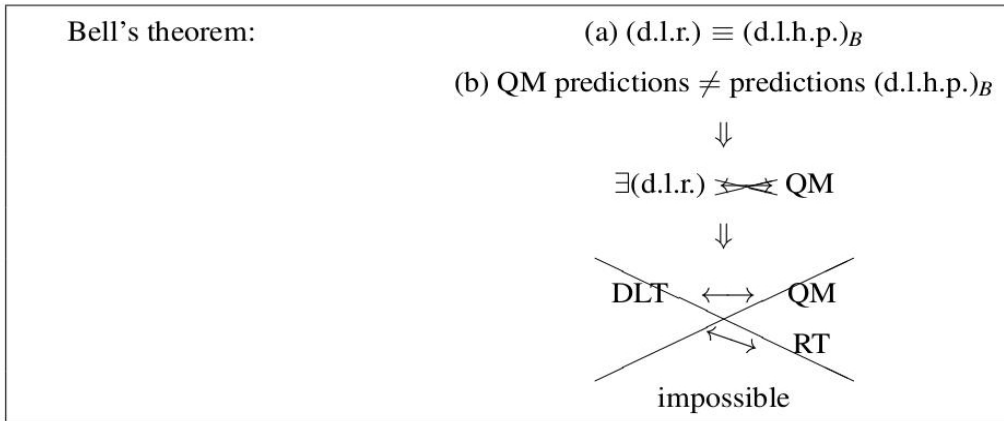
quantum formalism) deterministic and local parameters (d.l.h.p.), which make it possible to produce a complete description of individual microsystems. The complete description provided by a DLT must be compatible with quantum mechanics—given hypothesis $\forall\text{QM}$ —and with relativity theory—given hypothesis $\exists(\text{d.l.r.})$, which is an integral part of relativity theory. This set of ideas can be represented by the following diagram:



Thirty Years’ Worth of Reactions. Reactions have varied, but the dominant note has clearly been positivist. The “realist” hypothesis $\exists(\text{d.l.r.})$ is devoid of operational significance. It is essentially metaphysical, and therefore external to the scientific approach. The incompatibility of the so-called “EPR paradox” only exists with respect to this non-scientific hypothesis and so is not a scientific problem. It is a false problem as far as science is concerned.

J. Bell’s Theorem (1964). Thirty years after EPR, John Stewart Bell presented a theorem that seemed to contradict the interpretation made by Einstein, Podolsky, and Rosen of their own proof. The conclusion of Bell’s theorem can be stated as follows (and in other equivalent ways): it is not possible in every case using deterministic and local hidden parameters to obtain the same predictions as quantum mechanics; in some cases these parameters lead to different predictions. In order to re-establish compliance with the quantum mechanics predictions, the local nature of the introduced hidden parameters must be removed, which would contradict the $\exists(\text{d.l.r.})$ hypothesis that is part of relativity theory. Consequently the deterministic theory DLT, compatible with both quantum mechanics and relativity theory, which Einstein, Podolsky, and Rosen thought they had shown to be possible, is in fact impossible.

The proof is based on an example. Consider two systems S_1 and S_2 with non-null and correlated spin created by the disintegration of an initial system S with null spin. Suppose that spin measurements are taken in three directions a , b , and c on S_1 using apparatus \mathcal{A}_1 , and in the same directions on S_2 using apparatus \mathcal{A}_2 which may be located at an arbitrarily large distance from \mathcal{A}_1 . The $\exists(\text{d.l.r.})$ hypothesis is then formalized: hidden parameters are introduced subject to the condition that they provide a mathematical translation of “deterministic” and “local” constraints. In this way the conceptualization introduced earlier at the level of clear but qualitative semantics is raised to the level of syntactisized semantics. Such a procedure is often important because it can make mathematical deductions possible from quantitative conclusions. And in fact Bell showed that the $\exists(\text{d.l.r.})$ hypothesis, formalized in this way, necessarily brings with it an inequality in the statistical correlations between the spin measurements recorded by apparatuses \mathcal{A}_1 and \mathcal{A}_2 . And yet this inequality is not satisfied by the statistical correlations predicted by quantum mechanics. One might be able to recover the quantum correlations by removing the condition that mathematically translates the “local” characteristic of the hidden parameters introduced, i.e., by giving up part of the $\exists(\text{d.l.r.})$ hypothesis. This can be expressed by stating that, in the given circumstance, “quantum mechanics is non-local” or “implies non-local effects”, which render it incompatible with $\exists(\text{d.l.r.})$. Bell’s contribution can be expressed schematically as follows (note that $(\text{d.l.h.p.})_B$, are the hidden parameters subject to Bell’s conditions).



Since the statistical results in question are observable, it is possible in principle to establish experimentally whether the physical facts correspond to the predictions of quantum mechanics or to those that result from Bell's deterministic and local hidden parameters. This is one of the most significant points of Bell's theorem.

If the experiment proved quantum mechanics wrong, the conceptual situation created would be clear. The possibility of a deterministic and local theory of micro-phenomena, different from that envisioned by Einstein, Podolsky, and Rosen, would have to be accepted, because EPR would not comply with the requirement for equality with quantum mechanics in every case.

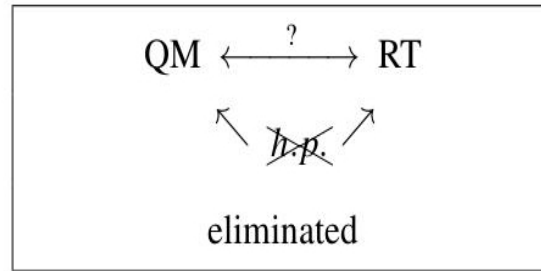
A certain number of verification experiments, however, have already been conducted and the results obtained up to the present—even though they are not yet definitive—strongly support the supposition that the predictions made by quantum mechanics are correct. The question, therefore, is one of understanding the conceptual situation that seems to have arisen and that is generally referred to as the “locality problem”.

Interpretations

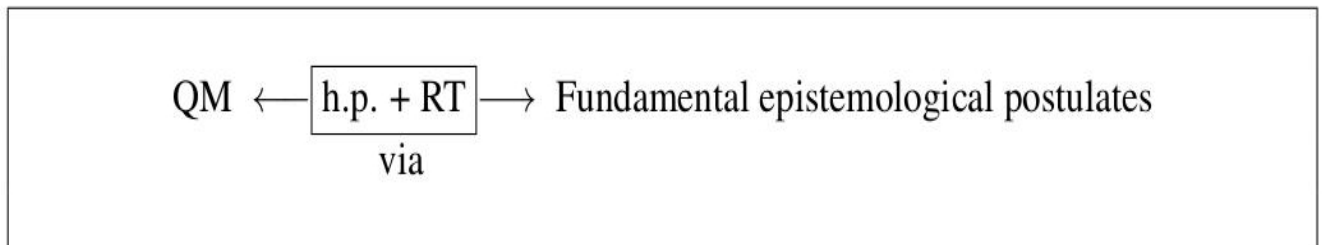
Reactions to the locality problem have varied. By omitting or glossing over nuances, they can be reduced to three main schools of interpretation. **1-** The *Rejectionists*. A certain number of physicists seem to think once again that it is a metaphysical problem that exists only with respect to the non-operational concept of the hidden parameters. Reject this concept and the problem disappears. Others think the problem does not exist because it is posed incorrectly (3).

2- The Minimalists. According to other physicists⁹⁹ ((4), (5), (6), (7), etc.) the problem this time satisfies the most draconian of positivist norms, because it leads to experimental tests. Nonetheless they refuse to consider anything beyond what such tests involve. They restrict themselves exclusively to the statistical correlations between measurement events which are separated by a space-like distance and which can manifest either “instantaneous independence”, i.e., locality, or, on the contrary, “instantaneous dependence”, i.e., non-locality. Any reference to the underlying “explanatory” concepts is avoided. From this point of view the concept of hidden parameters only plays a conceptually revelatory (or catalyzing) role for a problem to which it remains external. That is because the problem, once perceived, persists without needing to refer to the hidden parameter concept. In fact, it represents a conflict between quantum mechanics and relativity theory.

⁹⁹I apologize in advance to those who do not accept that they fall into this category.



3- The *Epistemologists*. And finally, there is a tendency (8) to link the problem of locality to the most widespread way of thinking about reality, which postulates the existence of independent objects that have intrinsic and permanent properties. Violation of Bell's inequalities would be incompatible with such suppositions. In the final analysis, then, we are dealing with a conflict between quantum mechanics and fundamental epistemological postulates, which is centered on the concept of hidden parameters and relativity theory.

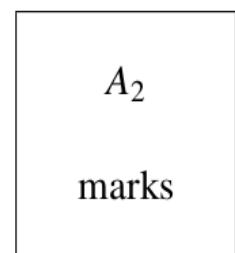
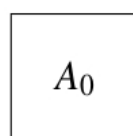
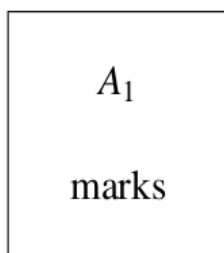


I shall refrain from examining the rejectionist interpretation, since it can contribute nothing new.

As for the two conflicts implied by the two other interpretations, neither of them seems to be viable given the current state of the debate. Only one question stands out clearly: What exactly does the locality problem involve?

To find an answer, the investigation that follows will show that current ways of thinking and tests of Bell's inequality cannot be sufficient. Certainly alternate ways of thinking and new tests based on them will have to be used. Otherwise no definitive conclusion can be drawn, even if Bell's inequality is clearly violated.

The locality problem and the underlying conceptual base. Let us take another look at the locality problem trying to keep separate what is directly observed during experiments, what is calculated, and the intermediaries that link what is observed to what is calculated. **A. What is observed during experiments.** We observe (omitting all of the details) an object \mathcal{A}_0 located between two pieces of equipment, \mathcal{A}_1 and \mathcal{A}_2 located, respectively, at equal distances to the left and right of \mathcal{A}_0 . Every now and again, visible marks appear on certain parts of \mathcal{A}_1 and \mathcal{A}_2 .



B- What is calculated. Statistical correlations are calculated using three types of probability distributions that lead to three correlation functions: a function $F_{(DLT)B}$, characteristic of a deterministic local theory in the sense of Bell, a function F_{QM} that functions according to quantum mechanical algorithms, and a function F_{obs} that corresponds to observation statistics. Bell's inequality distinguishes $F_{(DLT)B}$ from F_{QM} . The experiment should show whether the observed reality reproduces F_{QM} or $F_{(DLT)B}$.

	$F_{(DLT)B}$,	F_{QM}	,	F_{obs}
Bell's inequality	\Rightarrow		$F_{(DLT)B} \leq \text{bound}$,	$F_{QM} > \text{bound}$
Experiment	\Rightarrow		$F_{obs} \stackrel{?}{\approx}$		bound

C- The intermediaries between what is observed and what is calculated. The set of these intermediaries is rich and complex. It does not make sense to attempt an exhaustive list and description. Instead I present a sampling, while distinguishing between the words used, the ideas linked to these words, and the syntactical organizations into which these ideas are integrated. (See the table on the next page)

The central column of the table may be a little shocking from a positivist point of view. But in any case Bell's deterministic and local hidden parameters violate the semantic restraints dictated by positivism. So we may as well continue and acknowledge all of the semantic questions related to interpretations 2 and 3 of the locality problem as laid out above.


I shall begin with the minimalist interpretation. I see two questions.

Firstly, does the semantic content assigned to the qualifiers "deterministic" and "local", implied by Bell's mathematical modeling, permit the most general representation imaginable of the process of observing a "microstate" using a macroscopic "apparatus"?

Secondly, supposing that Bell's modeling of an observation process does not really introduce any unnecessary restriction, exactly what kind of non-locality would violation of Bell's inequality demonstrate? Is it the non-locality that relativity theory clearly prohibits? Or is it spontaneous and still fuzzy extensions of this prohibition that might turn out to be contrary to reality?

Since for the time being I lack the elements needed to delve into the first question, I'll pass directly to the second:

To the extent that it exists, the so-called "system" that disintegrates into \mathcal{A}_0 , must include some original non-null spatial extension $\Delta x_s(t_0) \neq 0$. (Is what populates this spatial domain an "object", a "process", or both at once? The very definitions needed to answer the question are missing.) How are we to understand the concept described by the terms "disintegration" or "creation of the pair S_1 and S_2 "? In the conceptual substratum, the words suggest a process, a real entity that is undergoing change. In order to exist, such a process must take place somewhere and must last, it must occupy a certain non-null space-time domain $\Delta s_c(t) \cdot \Delta t_c \neq 0$ (where the subscript c denotes creation), within which "the original system S " still exists but is changing, while S_1 and S_2 do not yet exist but are in formation.

Words	Concepts	Syntactical Organization
A system	Macro-object, object	Logic of object classes and predicates
Creation of a pair	Change, process, SUCCESSIVE EVENTS, DURATION, TIME	
Two correlated systems	Objects $\begin{matrix} \leftarrow \rightarrow \\ \leftarrow \rightarrow \end{matrix}$ correlated isolated	
Apparatus	Macro-object capable of reacting meaningfully with a micro-system	
Spin	Observational property	
Measurement	Observation process, Event connecting macro- and micro-objects	
Hidden parameters	Intrinsic and permanent properties (of an object or a process)	
Deterministic	Predictable? On what basis?	
Local, signal	Propagating at a speed $v < c$	
Statistics Probabilities	Random phenomena (events, processes, objects)	Probability theory
Quantum predictions	State vectors, quantum algorithms	Quantum mechanics
Action at a distance	Change carried at a speed $v < c$?

In the writing that designates the space-time domain, the duration factor $\Delta t_c = t_{12,0} - t_0$ extends – by definition – from a supposed “initial time value” to where the creation change begins, to a “final time value” $t_f = t_{12,0}$ at which “the correlated system pair S_1 and S_2 ” begin to exist (objects? processes as well? both at once?). As for the spatial extension factor

$$\Delta s_c(t)$$

, since we are dealing with a process, it seems we are obliged to suppose that it changes as a function of the “time value” t , with ($t_0 < t < t_f$), but nonetheless meta-stably staying connected as long as $t < t_f$ (i.e., as long as S subsists and S_1 and S_2 have not yet been created). For every $t > t_f$, however, this spatial domain ought to have become non-connected via a more or less “catastrophic” fission leading to a new form of stability referred to as “the pair S_1 , S_2 of correlated systems”.

Let me pause for a moment and look at what I have just written. What a mixture of the “necessary” and the arbitrary, of signs and words that seem to point towards a precise designation and yet behind which one finds only blurred and moving images hooked onto these signs and words in an entangled way. I wrote in inverted commas “time value”, for example, because every time that I think about the level of unexploredness of the concepts of duration and time and their relationship, I am reluctant to write anything absent from an algorithm that will set the rules of the game. Parameterization of the basic property of duration using the time variable t (similar to the parameterization used in existing theories and even in relativity) is still doubtless very simplifying and often falsifying, too rigid, and somehow mechanizing. Changes are not always movements of internally stable entities. To be able to fully account for the entire diversity of types and intensities of change, we would need a kind of vector scale, a process time field defined at each point of abstract space framed by the duration axis and by the envisioned change axes.

But would such a time comply with the Lorenz transformation? What role does the speed of a light “signal” play *vis-à-vis* the propagation speed of “influences” (?) in such a process space? What does relativity theory really impose on any process and what does it leave unspecified? When there is a locally very “intense” process, like the “creation of a pair” probably is, what becomes of “time”?

In the general relativity theory of gravitation, for example, a non-null gradient in the gravitational field is linked to the impossibility of defining a unique time for two observers in the same reference frame, when these observers are separated from one another in space. As for the invariance of the speed of light itself (rather than the speed of other kinds of “influences”) when moving from one reference frame to another, it is only postulated locally, because there is no uniform definition of distances and times in variable gravitational fields (curved space-time) (9). How can we know what sort of local space-time “curvature” results (or not) from the—essentially variable—process of pair creation?

And finally, relativity theory does not introduce any quantification in the quantum mechanical sense; its description is continuous. When we write [speed = distance/time], time is a continuous variable.

If we go on to ask how we can find the value of t , we notice that it is of the form NT_H , where N is an integer and T_H is a “clock” period (supposedly constant!) which brings us back to the discrete. In macroscopic terms that can be negligible as much on the principal level as on the numeric level. But when we consider quantum and relatively short processes, how much significance does a condition such as the one below have?

$$v = \frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{NT_H} = \text{const?}$$

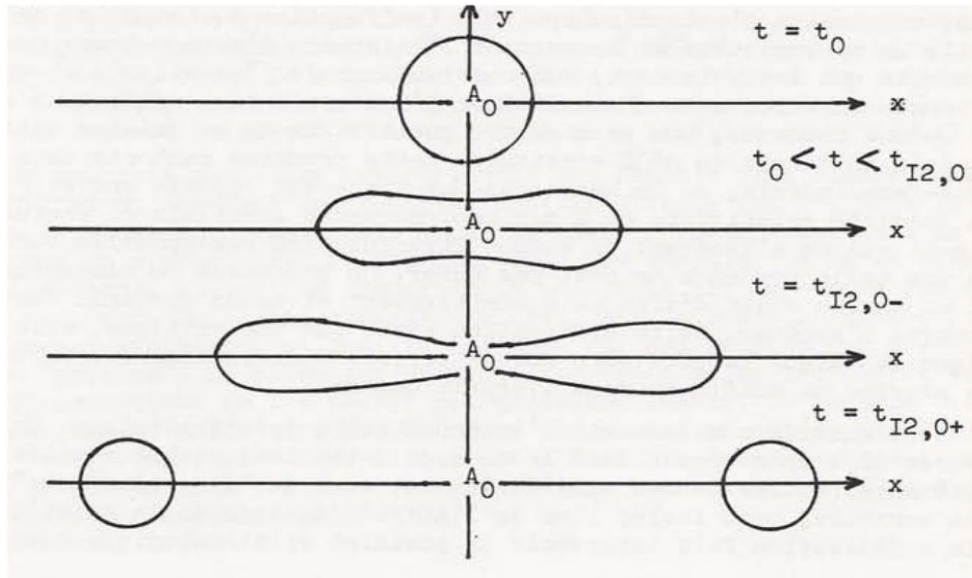
What clock should we choose, with what T_H , and besides that how can we be sure, when writing $t = 10^{-x}$, that we are doing anything more than an empty calculation?

Faced with such questions, it is understandable that positivist prudence and norms advise keeping within the healthy zone of the operationally defined and of the “syntactisized”, where thought circulates on well-worn and fixed tracks. Beyond this, we plunge into a veritable semantic swamp. Nevertheless it is only there, in that murky swamp, where one must force the eye to discern moving forms, that perhaps one perceives something new. In fact the locality problem forces us to do just this: it is a very fundamental problem, where any inertial behavior, not analyzed or approximate,

leads inevitably either to suspending the capacity to reason or to illusory problems and perspectives. At this point we cannot follow a path just because it has been laid. We have to choose and follow the appropriate direction.

Let me now return to the creation of a correlated pair S_1 and S_2 . I envisage this process as having analogies with drop formation. (*This may be wrong, but is not a priori impossible, and I only need one possible example.*) So I draw the spatial projection (in two dimensions) of the space-time domain $\Delta s_c(t) \cdot \Delta t_c, t_0 < t < t_f$, for four stages:

- * $t = t_0$;
- * $t_0 < t < t_f$
- * $t_0 < t < t_f^-$ (i.e., immediately before t_f);
- and $t = t_f^+$ (i.e., immediately after t_f).



Now suppose that the distance d_{12} between apparatuses \mathcal{A}_1 and \mathcal{A}_2 is smaller than the maximum projection $\Delta s_c(t)$ on the x-axis corresponding to $t = t_f^-$.

Apparatuses \mathcal{A}_1 and \mathcal{A}_2 will not therefore be impacted by “ S_1 ” and “ S_2 ” respectively, but by “ S in the process of disintegration”, which is nonetheless capable of registering an impact on \mathcal{A}_1 and \mathcal{A}_2 . Further suppose that the duration of the measurement events is such that with respect to d_{12} the space-time distance between the measurement events is spatial. And finally, suppose that in spite of everything, the measurement events are not independent in the sense of Bell; in other words let us suppose that a change in \mathcal{A}_2 can act at a speed $v > c$ on the result of one of the \mathcal{A}_2 recordings. The statistics related to recordings on \mathcal{A}_1 and \mathcal{A}_2 will therefore be “not locally correlated” and Bell’s inequality will have been violated. But in this case, is it justifiable to conclude that a contradiction of relativity theory has been proved? Relativity theory only concerns itself with “signals” (how exactly are they defined?) propagating “in a vacuum”. It does not say anything at all about the transmission of “influences” (definition?) across a “system” (object? process?). In particular, in no way does it constrain “the temporal order” (?) (“causal” or “not causal”) (?) of events located in different places in “the same system”. The example given — the “pair creation” model — simply does not belong to the factual domain described by relativity theory. No established theory has yet described it. And yet this example, whatever its inadequacies when faced with unknown reality, certainly is characteristic in an essentially acceptable way of what deserves to be called a process of pair creation: such a process must occupy a non-null space-time domain whose spatial projection, although initially dependent, eventually evolves to become independent.

This example of the possible seems to me sufficient as a basis for the following conclusion: results of tests meant to verify Bell’s inequality, even if these results definitively violate the inequality, can never alone establish the fact that the relativity principle of locality has been infringed. To be more precise

about what is at stake, Bell's model and the corresponding test should be used with other models and other tests of both non-observable ("creation") and observable (measured) extensions of space-time that occur. The minimalism of the minimal interpretation is no more than prudence, a remaining positivist fear of letting oneself go too far beyond what has already been established. Such prudence runs into an indecisive confrontation in which quantum mechanics is opposed vaguely to relativity's locality and to inertial and confused extensions of the latter which do not fit into any established conceptual framework. But this kind of prudence cannot last. A whole series of thoughts has been surreptitiously set in motion which no artificial obstacle can stop. This claim is not a criticism; it is simply a way of highlighting the soundest value that I see in Bell's approach and of expressing my confidence in the human mind.

Let us now consider the epistemological interpretation. This brings us immediately to the inevitable supplementary modeling. The terms under consideration are "a single system" and "two systems that are correlated but isolated one from the other" (in the relativity theory sense). The supplementary modeling referred to brings up the usual epistemological postulate on the existence of intrinsic properties for real, separated entities. From this postulate we deduce the same type of inequalities as those of Bell with respect to the statistics of measurement results on supposedly isolated entities. As a result we establish a connection between tests of observable inequalities on the one hand and, on the other, the epistemological postulate on the existence of intrinsic properties for separated objects in the sense of relativity theory. On this basis, we must accept (it seems to me?) that the violation of Bell's inequality in and of itself invalidates any significance of this way of thinking in terms of separated entities that have intrinsic properties. As it happens I have already shown elsewhere (10) (in terms that are too technical to be repeated here) that that is not possible. Here I will limit myself to some qualitative remarks. First of all, the points made above about the creation of a pair can also obviously be transposed onto the case of epistemological interpretation. But extending such thoughts further, this time let us begin by positioning ourselves at that instant in time $t = t_0$ at which S_1 and S_2 are created. For $t > t_0$, S_1 and S_2 now occupy two disjoint spatial domains $\Delta_{s_1}(t)$ and $\Delta_{s_2}(t)$, which move away from one another and then encounter, respectively, apparatuses \mathcal{A}_1 and \mathcal{A}_2 , producing measurement interactions. The measurement interaction of S_1 with \mathcal{A}_1 is itself an event that occupies a non-null space-time domain $\Delta_{s_{m1}}(t_{m1}) \cdot \Delta t_{m1} = 0$ (the subscript m is for measurement), where $t_{m1} \in \Delta t_{m1}$ and the duration factor Δt_{m1} depends on the spatial extension $\Delta_{s_{m1}}(t_{m1})$ linked to the stage $t_{m1} \in \Delta t_{m1}$ (assuming that the spatial extension remains constant in the stage $t_{m1} \in \Delta t_{m1}$). The same is true of the measurement event on \mathcal{A}_2 , whose spatial extension is $\Delta_{s_{m2}}(t_{m2}) \cdot \Delta t_{m2} = 0$. How should we define the space-time distance between these two measurement events? No matter what the fixed spatial distance is between \mathcal{A}_1 and \mathcal{A}_2 , how can we know if the corresponding space-time distance between the measurement events is spatial or not? Because that determines whether or not the crucial condition of reciprocal "separation" of these measurement events exists or not; and it is on the basis of that condition that we expect Bell's inequality in the statistics of the results recorded. Whether or not the space-time distance between measurement events is spatial obviously depends (among other things) on the spatial extension factors $\Delta_{s_{m1}}(t_{m1})$ and $\Delta_{s_{m2}}(t_{m2})$. But what do we know about the value of these factors? Do S_1 and S_2 move "in tandem" or "mechanically" as the de Broglie model and the recent soliton idea suggest? Or do they spread out as the standard quantum model of Schrödinger's linear evolution of wave packets suggests?

We might possibly hope to have a clearer response for the case in which S_1 and S_2 are photons "whose speed is c ". But the speed of what? Of the front of the photonic wave, yes, but what should we conclude about the rest of the photon? How is a photon made? Is it like a de Broglie microsystem with a singularity and a more extensive presence surrounding it? The behavior shown by radio waves would suggest so. What kind of extension then? In the current phase, what exactly do we know individually about these entities that we call "photons"? Newtonian quantum mechanics does not describe them; electromagnetism does not describe them individually. Quantum field theory has been marked in recent years by "semi-classical" experiments, whose goal is quite simply to eliminate the notion of a photon in order to avoid conceptual difficulties linked to renormalization algorithms (11).

We can therefore conclude completely generally that, whatever the fixed spatial distance between

\mathcal{A}_1 and \mathcal{A}_2 (whether we are dealing with microsystems that have non-null mass or photons), in order to know whether the measurement events on these microsystems are separated or not by a space-time distance of a spatial nature, we need to know (among other things) the spatial extension of the states of these microsystems as a function of time.

Without going into detail on inessential logical chains of thought, these few remarks should be sufficient to indicate the basis for the following statement.

By themselves the tests of Bell's inequality will never make it possible to reach a final conclusion on the significance of assigning intrinsic properties to separated real entities as defined by Einstein's relativity theory. So, for the moment, there is no conflict between quantum mechanics and the epistemological postulates of our standard way of thinking about reality. Only a line for future inquiry has been sketched, which indicates why further research into the space-time structure of so-called individual microsystems would be of interest. This line of thought seems to me both courageous and important, but only to the extent that it is clear-eyed and context-aware. It fits naturally with recent research on the extension of microsystems with non-null mass at rest (12), (13) and on the concept of the photon (11). It is quite remarkable to see that all of this research concentrates on interference phenomena and concepts. In fact it is here that the individual emerges from the mass of statistics; here that we see the failure to distinguish between the mathematical interference of standard statistics on the one hand, and on the other the statistics of physical interference of an individual entity superposed on itself (14), (15).

By addressing the locality problem I have intentionally directed attention to the semantic layer that underlies the words we use. The nature of this layer is to some extent the main subject of these remarks. The semantic sludge in which we happily slide from algorithm to algorithm, attached only to the safety cord of words, seems to me to be worthy of closer attention. We have to dive into it to forge the new concepts that we lack and to draw their outlines in such a way that we can advance to expressing the "syntactizations".

The idea of an object in the macroscopic sense of the word is rigorously situated — albeit qualitatively — within the logic of object classes and predicates. This is in essence an explicitly structured theory of macroscopic objects that is of maximum generality. But this theory is fundamentally unsuited to an unrestricted description of changes. Indeed, the logic of object classes and predicates is based on the membership relationship \in : if for an object x the predicate f is true, then x is a member of class C_f defined by $f : f(x) \rightarrow x \in C_f$. But this fundamental membership relationship \in is conceived from the very beginning in a static, hypostasized way. No subsequent adjustment can compensate for the rigidity introduced at the outset. Probability theory on the one hand, and on the other the various physical theories (mechanics, thermodynamics, field theories, quantum mechanics, and relativity theory) have managed to compensate for this shortcoming to varying degrees. But each of them for a particular category of facts and each by implicit and diverse methods. No general and specific theory of events and processes, no logic of absolute changes using an explicit and unified methodology has yet been devised¹⁰⁰.

Take another look at the logic of object classes and predicates. It is fundamentally incompatible with the individual, since it describes classes. It would seem therefore naturally suited to numeric quantification of the statistical or probabilistic type via probability measurements of classes. Nevertheless, so far, such numeric quantification of logic has not been achieved. The logical "quantifiers" \exists , \forall , and \emptyset remain qualitative in nature!

In a complementary manner, probability theory to date has failed to explicitly develop a classification theory. The fundamental concept used is one of a probability space $[U, \tau, \pi(\tau)]$ where $\pi(\tau)$ designates a measure of probability imposed on a sigma-algebra of events τ , defined in a universe $U = \{e_i, i = 1, 2, \dots\}$ of elementary events e_i . This algebra may, in particular, reflect a classification of the elementary events e_i governed by a predicate f . In this case, specific "logical" properties follow for the probability space $[U, \tau, \pi(\tau)]$. Via these classifying properties, an initial connection between logic and probability could be developed. But no such attempt has yet been made and so for the moment the connection remains unspecified.

¹⁰⁰I have learned about an original and courageous attempt to formalize duration (16). So far, only values associated with duration ("time") have been the object of formalization attempts.

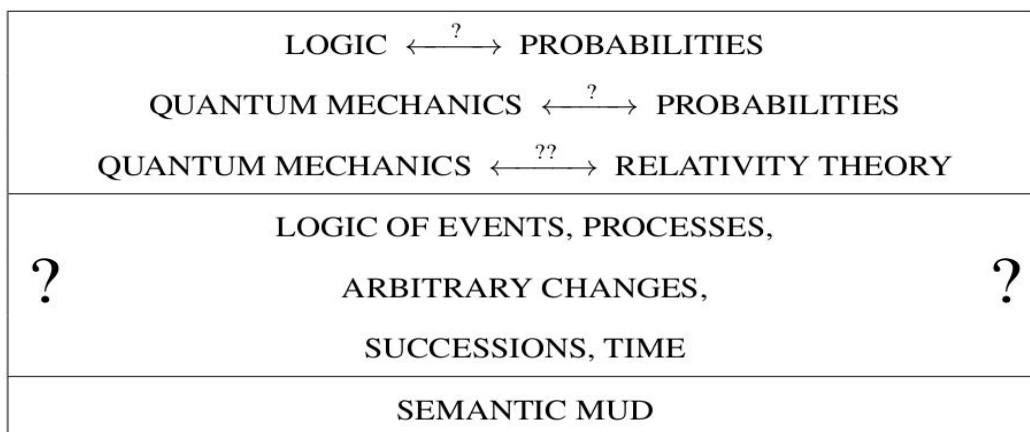
Now consider quantum mechanics. It introduces probability spaces, but the relationships between these spaces are such that some mathematicians state that “quantum mechanics is not a probability theory”. The connection between probability theory and quantum mechanics also remains very obscure for the time being.

On the other hand, the relationship of quantum mechanics to the various concepts suggested by the language it uses — one system, one system of two correlated systems, etc. — is also very obscure. In fact, quantum mechanics does not say anything at all about these concepts to the degree they might be thought of as beyond observation. Even the probability of presence is only a probability resulting from observation interactions: in quantum mechanics we can suppose that a “system” that makes a mark on a screen at time t is itself as far away as we like from this mark at as short a time as we like before time t . Quantum mechanics is perfectly silent about “the reality” whose observable manifestations it codifies so richly and in such detail via measurement interactions.

And finally, consider relativity theory, which at bottom is discrete, non-statistical, and continuous, i.e., non-quantified. Furthermore, it describes “what is”, although in relation to the state of observation. Its relationship to the quantum mechanical probability space with fundamentally observational and quantified events raises very well-known and very thorny problems.

Thus, we currently have several well-constituted rule-based constructions, each very complex, rich, and rigorous; but they are like the tips of icebergs emerging from the sea of semantic sludge, below the surface of which the edges and bases disappear. As for the set of concepts related to the fundamental property of duration, the concepts of process, event, change, permanence, succession, and TIME, they only act freely in a very sparse, primitive, and subjective state induced in our minds in varying ways by experience and language. Because the ways in which these concepts have been organized (within relativity theory, probability theory, or some other physical theory) are all particularizing and limiting. The situation remains that described by Bergson, “Deduction is an operation regulated by the processes of matter, modeled on matter’s mobile articulations, and finally, implicitly given with the space that underpins matter. As long as it moves in space or in spatialized time, all it needs to do is let go. It is duration that puts spokes in the wheels.” (17)

Once again, I have summarized the above in a diagram:



When there is still no unification between the statistical, discrete, and observational approach oriented towards the microscopic area of quantum mechanics and the individual, continuous, and realistic approach oriented towards the cosmology of relativity theory; when everything that touches upon duration and time is still barely elucidated; when everything that concerns the nature of those entities referred to as microsystems—or better still, microstates—is still so unexplored, then what sense does it make to maintain that—on the basis of “non-locality” tests—we are faced with a determinative conflict, direct or not, between quantum mechanics and relativity theory? Or between quantum mechanics and our conception of reality?

Conclusion

Personally, I feel I must set aside the conflict that other physicists think they see. For me, the value of Bell’s theorem resides elsewhere: this theorem and its fallout illustrate in a striking fashion

the active power of *mathematized* modeling when linked to experiments. For decades positivist taboos have been an obstacle to models. The result is this vertiginous void of syntactic and even qualitative models that one finds today in quantum algorithms. And yet Bell's model has triggered a conceptualization and rule-based dynamic that might even arrive at the positivist position. Perhaps it will even shake up quantum mechanics and relativity theory. This is because it attracts attention to and keeps it on the state of the conceptual milieu in which current theories are immersed. Out of this prolonged contact new theories will perhaps emerge that are more unified and have more breadth and depth. Here as in information theory I can see the first movements towards formalization of epistemology, the first outlines perhaps of a mathematized methodology of knowledge. And that could prove to be more fertile than any particular theory of a given reality-based domain.

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B Appendix 2

Proof of the compatibility of the experiment EXP.1 with de Broglie's theory of double solution

Let a free microsystem in a state ψ^0 "of interference with itself" be obtained as indicated in the figure below:

$$\psi^0 = \psi_1 + \psi_2 = \sqrt{2} \cos \left(2\pi \frac{v}{V} \cos \theta^0 z + \frac{\delta}{2} \right) e^{2\pi i v (i - \frac{x}{V} \sin \theta^0)} e^{i \frac{\delta}{2}}. \quad (1)$$

According to the theory of the double solution, and thus also the theory of two measurement types, the considered microsystem consists of a corpuscle for which the probability to be found is given by $|\psi^0|^2$, and for which the guidance law $\vec{v} = -c^2 \left\{ \left[\Delta \varphi + (\epsilon/c) \vec{A} \right] / (\partial \varphi / \partial t) - \epsilon U \right\}^{(5)}$ defines in ψ^0 the velocity \vec{v}^0 with components

$$v_n = \frac{c^2 \sin \theta^0}{V} = v_{12} \sin \theta^0 = \text{const}, \quad v_y = v_z = 0$$

$[v_{12} = |\vec{v}_1| = |\vec{v}_2|]$ is the common magnitude that the guidance law assigns to the velocity of the corpuscle in ψ_1 and ψ_2 from (1) and the figure below. Since the quantum potential $Q = (\hbar^2/8\pi^2 m_0) \square a/a$ ⁽⁵⁾ is constant on ψ^0 , the quantum forces $\vec{F}_Q = -\Delta Q$ ⁽⁵⁾ which act on the corpuscle are zero in ψ^0 . The corpuscle thus moves in ψ^0 in the direction Ox of the bisector of α , along a maximum of the amplitude of ψ^0 where $|\psi^0|^2 \neq 0$. *The spectrum of the momentum \vec{p} in ψ^0 of (1) reduces, according to the theory of two measurement types, to a unique vector quantity \vec{p}^0 corresponding to the velocity \vec{v}^0 defined above, while the quantum spectrum of \vec{p} consists of the two vector quantities \vec{p}_1 and \vec{p}_2 shown in the figure.* Let us show that \vec{p}^0 is measurable by a measurement $M_G(\psi)$ by trace: during the first interaction with the sensitive environment, the corpuscle undergoes, on the one hand, the diffusing effect of a Coulombian collision – directly, like a classical mobile – and, on the other hand, the effect of quantum forces that give rise to the modification of the wave of the microsystem, that the expressions of the guidance law of the quantum potential and forces associate with the modification produced on the dynamics of the corpuscle by the Coulombian diffusion. Under fixed external conditions, for any given angle of diffusion, the amplitude of the diffusion is a decreasing function of the momentum of the diffused microsystems only. As a result we can choose v and θ^0 in (1) such that $\vec{p}^0 = (hv/c^2)v_{12} \sin \theta^0 = p_{12} \sin \theta^0$ assures a negligible deviation of the corpuscle due to the first Coulombian collision. In this case, *in the state ψ^1 which follows the first interaction the direction of displacement of the corpuscle is practically the same as in ψ^0 .* This clarifies the effect of the Coulombian diffusion on the dynamics of the corpuscle of the microsystem. What then is the modification of the wave associated to this effect? In ψ^1 the probability of presence is $|\psi^1|^2$, so the direction of the amplitude maxima is that of the displacement of the corpuscle: *the direction of the amplitude maxima also suffers a negligible variation during the passage from ψ^0 to ψ^1 .* During the first interaction the amplitude of the wave is implicitly a function of time:

$$[a(z, t)]^{0 \rightarrow 1} = \sqrt{2} \cos \left[2\pi \frac{v}{V}(t) \cos \theta(t) z + \frac{1}{2} \delta^{0 \rightarrow 1}(t) \right]. \quad (2)$$

If we accept that v/V and θ vary independently with respect to time, then the term $(1/c^2)(\partial^2 a/\partial t^2)$ of the d'Alembertian in the quantum potential is a second-degree polynomial function of z , which leads to a quantum force field which depends linearly on z . This result is physically unacceptable, since the wave phenomenon described by the amplitude of (1) and (2) is periodic with respect to z . We must thus accept that v/V and θ vary in such a manner that the coefficient of z^2 in $\partial^2 a/\partial t^2$ is zero.

This leads to the condition

$$2\pi \frac{v}{V}(t) \cos \theta(t) = \frac{p_{1z}}{\hbar} = \frac{p_{2z}}{\hbar} = \chi = \text{const} \quad (6).$$

If we introduce this condition in (2), the result is that *the interfringe distance does not change either: the first interaction does not modify the symmetry ($\theta_1^1 = \theta_2^1$) or the structure of the microsystem* (although the waves composing ψ_1 and ψ_2 change their propagation direction and frequency). The quantum forces corresponding to (2) are

$$\vec{F}_a^{0 \rightarrow 1} = \frac{-\hbar^2}{c^2 8\pi^2 m_0} \frac{d^2 \delta^{0 \rightarrow 1}}{dt^2} \frac{\chi}{|[a(z, t)]^{0 \rightarrow 1}|^2} \vec{O}z, \quad (3)$$

where $\chi = \text{const}$. The location of the fringes with respect to Oz , however, *will* have changed if $d\delta^{0 \rightarrow 1}/dt \neq 0$. In this case, we must accept that $d^2 \delta^{0 \rightarrow 1}/dt^2 \neq 0$ as well, since, in agreement with the meaning of $|\psi^1|^2$, the corpuscle will be found in ψ^1 on one of the maxima of the new amplitude which, if $d\delta^{0 \rightarrow 1}/dt \neq 0$, are all *displaced* with respect to the maxima of ψ^0 , and the non-zero forces (3) must have acted to accomplish this change. In this case, then, the forces (3) create a component $v_{cz}^{0 \rightarrow 1}$ according to Oz of the velocity of the corpuscle during the duration Δt_i of the first interaction, which brings the corpuscle, in ψ^1 , onto a maximum of the amplitude of the wave, displaced by $\Delta z_c^{0 \rightarrow 1} = \langle v_{cz}^{0 \rightarrow 1} \rangle \Delta t_i$ with respect to the amplitude of ψ^0 that it was previously on. Given the nature of the mechanism that develops $v_{cz}^{0 \rightarrow 1}$, and in particular its brevity, we must accept that $v_{cz}^{0 \rightarrow 1}$ is of a Newtonian order of magnitude. On the other hand, the corpuscle is found at any given moment during Δt_i , with the highest probability, in a point of the field (3) where the amplitude (2) is maximal. Hence, at the location where the forces (3) find an object to act on they take a value corresponding to $\{\max[a(z, t)]^{0 \rightarrow 1}\} = \sqrt{2}$. If we take into account these facts we obtain, for the displacement $\Delta z_c^{0 \rightarrow 1}$, the expression $\Delta z_c^{0 \rightarrow 1} = (-\hbar^2 \chi / 4c^2 m_0^2) \Delta \delta^{0 \rightarrow 1}$, where $\Delta \delta^{0 \rightarrow 1}$ is the variation of δ during Δt_i . $\Delta z_c^{0 \rightarrow 1}$ is thus proportional to $\Delta \delta^{0 \rightarrow 1}$, its direction being random, as well as the sign of $\Delta \delta^{0 \rightarrow 1}$. We can thus conclude that, *following the first interaction, the corpuscle conserves the direction of its displacement, while the amount of displacement changes if $\Delta \delta^{0 \rightarrow 1} \neq 0$* . If we now consider the second interaction, which leads from ψ^1 to a state ψ^2 , it immediately seems that all the preceding conclusions are can be transposed, point by point, since the new initial state ψ^1 is of the same type as ψ^0 . Gradually, for a large enough initial v and sufficiently reduced thickness and density of the sensitive environment, the conclusions concerning the first interaction are transposable to all the interactions undergone during the entire crossing of the sensitive environment: *the direction of the displacement of the corpuscle is conserved, while the amount changes by $\Delta z_e^{l \rightarrow l+1} = (-\hbar^2 \chi / 4\pi c^2 m_0^2) \Delta \delta^{l \rightarrow l+1}$ during the l th interaction if $\Delta \delta^{l \rightarrow l+1} \neq 0$* . The form of the trace of the microsystem is determined by the relative positions of the successive ionizing interactions. These relative positions are described by the angles with Ox :

$$\begin{aligned} \gamma^{l \rightarrow l+1} &= \arg \tan \sum_{j=l}^{l+i+1} \frac{\Delta z_e^{j \rightarrow j+1}}{\lambda_{j \rightarrow j+1}} \\ &= \arg \tan \frac{-\hbar^2 \chi}{4c^2 m_0^2} \sum_{j=l}^{l+i-1} \frac{\Delta \delta^{j \rightarrow j+1}}{\lambda_{j \rightarrow j+1}} \quad (l = 1, 2, \dots, N; 1 \leq i \leq N - l), \end{aligned} \quad (4)$$

where $\lambda_{j \rightarrow j+1}$ is the distance separating the interactions j and $j + 1$, and j indicates the elastic interactions produced between the two successive ionizing interactions l and $l + i$. If, in place of a continuous sensitive environment we first use a sensitive strip that is fine enough that the probability of a further ionization occurring there is very small, followed by a sensitive layer placed at a distance λ and thick enough that the energy of the microsystem vanishes within it, then the first term of (4) is $\gamma^{0 \rightarrow 1} = \arg \tan[-\hbar^2 \chi \Delta \delta^{0 \rightarrow 1} / 4c^2 m_0^2 \lambda]$ and its average value is inversely proportional to λ . By

increasing λ one must find, to the right which combines the first two ionizations, an average direction closer and closer to the direction of the bisector of α , while the directions \vec{p}_1 and \vec{p}_2 of (1) predicted by quantum mechanics must be *independent of λ* . *This allows one to measure, with controllable approximation, the direction \vec{p}^0 of \vec{p} corresponding to the guidance law, different to the quantum directions of \vec{p}_1 and \vec{p}_2 , and considered to be hidden in the theory of the double solution.*

(*) Session on the 25 March 1968.

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